Proakis digital communications solution pdf

 1. Digital communication edits, 4th edition1. Page 31, equation (2,1-54) First row: yl instead of y2 Second line: gn instead of g12. Page 163, equation (4.2 to 30∞ must be: s(t) = ao/2 + Σ k=13. Page 163, equation (4.2-31) T must be: ak = (2/T) ∫ s(t) cos 2πkt/T dt , k>0 0 T bk = (2/T) ∫ s(t) sin 2škt/T dt , as>1 04. Page 178, 7 lines from above, should be: sqrt (2ε) instead of ε sqrt (2)page 5, the equation (5.1 to 19) should be: h(T-τ) in place of h(T-τ) h(t-τ)6. On page 20, two lines below the equation (5.1-20) must be: y2 (T) instead of y2(t) n7. On page 244, the equation (5.1– 45) should be: m = 1,2,... The M8. Page 245, the equation (5.1-48) must be: sqrt (εb) sqrt (εn) 2.9. Page 309, Equation (5,4-39) R1 sqrt (2εs/N0) instead of sqrt (2εsR1/N0)10. Page 318, Equation (5.5-17)Added term: – (N0)dBW/Hz11. P. 367, Equation (6.4-6) Replace + Sign with – sign in the summing second term13. On page 367, equations (paragraphs 6.4-8 and 6.4-9) loggs and probability 14 add subscript L to page 422, rows 2 and 3 above (equation (equation 8.1-14) not more than 15. On page 468, 12 lines from the top and 5 lines from the bottom should be: b < b<16. Figure 8.2 to 15, the hard line corresponds to the broken line of the fine decision decoding, which corresponds to the hard decision in the case of decoding17. Page 500, Equation (8.2-41) Denominal Mk must be MJ and Mj must be MJ18. P. 591, P9.9. 19. Page 609, 6 rows above the equation (10.1-34) ε must be ε k+1– L-1 k+l – L-120. On page 646, Figure 10.3-5 removes the hat from I(z)21. Page 651, 4 rows from the top is replaced above by par22. On page 651, 2 lines above section 10.6 Turob must be Turbo23. Figure 11.1-6 Lower delay line elements: z1 must be z-124. Figure 750, Figure 13.2-8 Adders is replaced by multipliers25. Figure 752, Figs. 13.2-9 Replace add-ons with multipliers26. Page 856, equation (14,6-5) Replace K with k27. Figure 885, Figs. 14.7-7 Input should be 02310 4.28. Page 894, Problem 14.16 r1 = h1s1 + h2s2 + n1 r2 = h1s2* + h2s1* + n229. Page 895 Delete a probability30 error 2k. on page 915, at the top of the page (15.47) Page 925, 6 rows of the top T0 must be Tp32. Page 936 at the top of the page of the first equation, the second term must be: ln cosh { [r2 sqrt (ε2) – b1ρ sqrt (ε1ε2)]/N0 } 5.34. Page 936, the second equation from the top of the page, divide each argument into a showy function with N035. 936. lpp., problēmai 15.10 jābūt ηk = [ ]236. 936. lpp., 15.11. problēmas pēdējais vienādojuma termins ir: ε + ε -2 ⎪ ρ ⎪ sqrt (ε ε) (1/2) Q { sqrt[1 2 1 2]} N0/2 6. CHAPTER 2Problem 2.1 : 3 P (Ai) = P (Ai, Bj), i = 1, 2, 3, 4 j=1Hence : 3 P (A1) = P (A1 , Bj) = 0.1 + 0.08 + 0.13 = 0.31 j=1 3 P (A2) = P (A2 , Bj) = 0.05 + 0.03 + 0.09 = 0.17 j=1 3 P (A3) = P (A3 , Bj) = 0.05 + 0.12 + 0.14 = 0.31 j=1 3 P (A4) = P (A4 , Bj) = 0.11 + 0.04 + 0.06 = 0.21 j=1Similarly : 4 P (B1) = P (Ai , B1) = 0.10 + 0.05 + 0.05 + 0.11 = 0.31 i=1 4 P (B2) = P (Ai , B2) = 0.08 + 0.03 + 0.12 + 0.04 = 0.27 i=1 4 P (B3) = P (Ai , B3) = 0.13 + 0.09 + 0.14 + 0.06 = 0.42 i=1Problem 2.2 :The relationship holds for n = 2 (2-1-34) : p(x1 , x2) = p(x2 |x1)p(x1)Suppose it holds for n k , i.e. p(x1 , x2 , ..., xk) p(xk |xk−1, ..., x1)p(xk−1 |xk−2, ..., x1) ... p(x1)Then n = k + 1: p(x1, x2 , ..., xk , xk+1) = p(xk+1 |xk , xk−1 , ..., x1)p(xk, xk−1..., x1) = p(xk+1 |xk, ..., x1)p(xk−1, ..., xk−1 ,..., xk−1 |xk−1 |xk−2 | p(x1)So the ratios belong to n = k + 1, and the induction there for any n. 1 7. Issue 2.3: Following the same procedure as the example 2-1-1, we will prove: 1 y−b pY (y) = pX |a | aProblem 2.4: Relationship (2-1-44) gives: ⎡ ⎤ 1/3 1 y−b pY (y) = pX ⎣ ⎦ 3a [(y − b) /a] 2/3 a 2 /2X is gaussian r.v. with zero mean and unit deviation: pX (x) = √1 e-x 2šHence : 1 1 y-b 2/3 pY (y) = √ e-mail 2 (a) 3a 2š [(y − b) /a]2/3 pdf from Y 0.45 0.4 a=2 0.35 b=3 0.3 0.25 0.3 2 0.15 0.1 0.05 0 −10 −8 −2 0 2 4 6 8 yProblem 2.5 :(a) Whereas (Xr, Xi) are statistically independent : 1 -(x2 +x2)/2σ2 pX (xr, xi) = pX (xr)pX (xi) = e r i 2πσ 2 2 8. Arī : Yr + jYi = (Xr + Xi)ejφ ⇒ Xr + Xi = (Yr + jYi) e−jφ = Yr cos φ + Yi sin φ + j(−Yr sin φ + Yi cos φ) ⇒ φ) φ <3> Xr = Yr cos φ + Yi sin φ Xi = -Yr sin φ + Yi cos φJējēkams no iepriekš minētās transformācijas ir: ∂XR ∂Yr ∂Xi ∂Yr cos φ − sin φ J= ∂XR ∂Xi = =1 ∂Yi ∂Yi sin φ cos φHence ar (2-1-55): pY (g., yi) = pX ((Yr cos φ + Yi sin φ) , (−Yr sin φ + Yi cos φ)) 2 2 2 = 2πσ2 e−(yr +yi)/2σ 1(b) Y = AX un X = A−1 Y 2Now, pX (x) = (2πσ1)n/2 e−x x/2σ (nejaušo mainīgo x1 kovariācijas matrica M , ..., xn ir 2 2M = σ I because they are i.i.d) and J 1 December 2004 det(A)|. Thus: 1 1 −1 −1 2 pY (y) = 2)n/2 | det(A)| e-y (A) A y/2σ (2šσTo x and Y pdf is identical, we need: | det(A)| = 1 and (A-1) A-1 = I = ⇒ A-1 = AHence, A must be a uniform (orthogonal) matrix . Problem 2,6 :(a) n n n n ψY (jv) = E ejvY = E ejv i=1 xi =E ejvxi = E ejvX = ψX (ejv) i=1 i=1Bet, pX (x) = pδ(x − 1) + (1 − p)δ(x) ⇒ ψX (ejv) = 1 + p + pejv n ⇒ ψY (jv) = 1 + p + pejv 3 9.b) dψY (jv) E(Y) = -j |v=0 = -jn(1 − p + pejv)n−1 jpejv |v=0 = np dvand d2 ψY jv) d E(Y 2) = − 12v |v=0 = − jn(1 − p + pejv)n−1 pejv = np + np(n − 1)p d dv v=0 ⇒ E(Y 2) = n2 p2 + np(− p) Problem 2.7 : ψ (jv1 , jv2 , jv3 , jv4) = E go(v1 x1 +v2 x2 +v3 x3 +v4 x4) ∂ 4 ψ(jv1 , jv2 , jv3 , jv4) E (X1 X2 X3 X4) = (-j)4 |v1 =v2 =v4 =v4 =v4 =0 ∂v1 ∂v2 ∂v3 ∂v4No (2-1-151) text, and specific rv s : 1 ψ(jv) = e-2 v Mvwhere v = [v1, v2 , v3 , v4] , M = [μij]. We get the desired result by combining the exponent in a scalar shape and then performingquadruple differentiation. We can simplify the procedure by stating that: ∂ψ(jv) 1 = -μi ve− 2 v Mv ∂viwhere μi = [μi1 , μi2, μi3, μi4] . Also note that: ∂ μj = μij = μji ∂viHence: ∂ 4 ψ(jv1, jv2 , jv3 , jv4) | V=0 = μ12 μ34 + μ23 μ14 + μ24 μ13 ∂v1 ∂v2 ∂v3 ∂v4Proble 2.8:For central hies with n freedom degress: 1 ψ(jv) = (1 − j2vσ 2)/2 4 10. Now : dψ(jv) jnσ 2 dψ(jv) = n/2+1 ⇒ E (Y) = -j |v=0 = nσ 2 dv (1 − j2vσ 2) dv d2 ψ(jv) −2nσ 4 (n/2 + 1) d2 ψ((2 + 1) jv) = ⇒E Y2 =- |v=0 = n(n + 2)σ 2 dv 2 dv2 (1 − j2vσ 2)n/2+2 dv 2 Dispersion is σY = E (Y 2) − [E (Y)]2 = 2nσ 4 2Neatsausk. degrees of freedom : 1 2 2 ψ(jv) = ejvs /(1-j2vσ) (1 − j2vσ 2)n/2where by definition : s2 = n i =1 m2 . i dψ(jv) jnσ 2 js2 2 2 = + ejvs /(1−j2vσ) dv (1 − j2vσ 2) n/2+1 (1 − j2vσ 2) n/2+2Hence, E (Y) = −j dψ(jv) |v=0 = nσ 2 + s2 dv d2 ψ(jv) −nσ 4 (n + 2) −s2 (n + 4)σ 2 − ns2 σ 2 −s4 2 2 2 = n/2+2 + n/2+3 + n/2+4 ejvs /(1−j2vσ) dv (1 − j2vσ 2) (1 − j2vσ 2) (1 − j2vσ 2)Hence, d2 ψ(jv) E Y2 =− |v=0 = 2nσ 4 + 4s2 σ 2 + nσ 2 + s2 dv 2and σY = E Y 2 − [E (Y)]2 = 2nσ 4 + 4σ 2 s2 2Problem 2.9 :The Cauchy r.v. has : p(x) = a/π x2 +a2 , −∞ < x < ∞(a) ∞ E (X) = xp(x)dx = 0 −∞cince p(x) is a pair function. ∞ ∞ x2 E X2 = x2 p(x)dx = dx -∞ π -∞ x2 + a2 x2Note that for large x, x2 +a2 → 1 (i.e. a non-zero value). Thus, E X 2 = ∞, σ 2 = ∞ 5 11.b) ∞ a/π jvx ∞ a/π jvX ψ jv) = E = e dx = ejvx dx −∞ x 2 + a2 +∞ (x + if) (x − if)This integral can be assessed, residue theory in a complex changing theory. Then, for v ≥ 0: a / π jvx ψ (jv) = 2πj e = e−av x + if x = ifFor v < 0: a / π jvx = -2šj e = eav v x − if x=−jaTherefore : ψ(jv) = e-a|v|Note: An alternative way to find the characteristic function is to use the Fourier transformation relationship between p(x), ψ(jv) and Fourier pairs : 1 c e-b|t| ↔ 2 + f2 , c b/2π, f = 2šv šcProblms 2,10 : 1(a) Y = n n i =1 Xi , ψXi (jv) = e-a|v| n 1 n n ψXi (jv/n) = e-a|v|/n = e−a|v| v| v ψY (jv) = E ejv n i=1 Xi = E ej n Xi = i=1 i=1 (b) Since ψY .jv) = ψXi (jv) ⇒ pY(y) = pXi (xi) ⇒ pY (y) = a/π y 2 +a2 . (c) as n→ ∞, pY (y) = y2 +a2 other than Gaussian; he/he/she does not have a central boundary theorem to hold.

The reason is that cauchy distribution is not limited dispersion. Problem 2.11: We assume that x (t), y (t), z (t) are real valued stochastic processes. Processing complex value processes is similar. (a) φzz (τ) = E {[x(t + τ) + y(t + τ)] [x(t)+ y(t)]} = φxx (τ) + φxy (τ) + φyx (τ) + φyy (τ) 6 12.b) If x(t), y(t) is not interconnected: φxy (τ) = E [x(t + τ)y(t)] = E [x(t + τ) E [2] [4] [4] [4] [4] [4] [4] [4] [4] [4] [4] [4] [4] [4] [4] [4] [4] [4] [4] [4] [4] [4] [4] [4] [4] [4] [4] y (t)] = mx mySimilarly: φyx (τ) = mx myHence: φzz (τ) = φxx (τ) + φyy (τ) +2mx my(c) When x(t), y(t) is unrelated and has zero meaning: φzz (τ) = φxx (τ) + φyy (τ) Problem 2,12: power spectral density random process x (t) is: ∞ Φxx (f) = φxx (τ) e-j2 τf τ = τ = N0 /2. -∞suggling spectral density at the filter outlet will be : N0 Φyy (f) = Φxx (f)| H(f)|2 = | H(f)|2 2Stence, total power at filter output will be: ∞ N0 ∞ N0 φyy (τ = 0) = Φyy (f)df = | H(f)|2df = (2B) = N0 B -∞ 2 -∞ 2Problem 2,13 : ⎡ ⎤ X1MX = E [(X − mx)(X − mx)], X = ⎣ X2 ⎦ , mx is the vector corresponding to the average value. ⎦ ⎤ X3 7 13. Then : MY = E [(Y − my)(Y − my)] = E [A(X − mx)(A(X − mx))] = E [A(X − mx)(X − mx) A] = AE [(X − mx) (X − mx)] A = AMx AHence: ⎡ ⎤ μ11 0 μ11 + μ13 ⎤ ⎤ MY = ⎣ 0 4 μ22 0 ⎦ μ11 + μ3 1 0 μ11 + μ13 + μ31 + μ33Problem 2,14:Y (t) = X 2 (t), φxx (τ) = E [x(t + τ)x(t)] φyy (τ) = E [y(t + τ)y(t)] = E x2 (t + τ)x2 (t)Let X1 = X2 = x(t), X3 = X4 = x(t + τ). Then out of the problem 2.7 : E (X1 X2 X3 X4) = E (X1 X2) E (X3 X4) + E (X1 X3) E (X2 X4) + E (X1 X4) E (X2 X3)So: φyy (τ) = φ2 (0) + 2φ2 (τ) xxProblem 2.15 : m 2 /Ω 2 1pR (r) = Γ(m) m Ω r 2m-1 e-mr , X= √ R Ω 1 √ p x √We know ka : pX (x) = 1/ Ω R 1/ Ω . Consequently: 1 2 m √ 2m−1 −m(x√Ω)2 /Ω 2 2 pX (x) = √ x Ω e = mm x2m−1 e−mx 1/ Ω Γ(m) Ω Γ(m) Problem 2.16: filter transfer function is: 1/jωC 1 1 H(f) = = = R + 1/jωC jωRC + 1 j2šf RC + 1 8 14. a σ2 Φxx f = σ 2 ⇒ Φyy f = Φxx f | H(f)|2 = (2πRC)2 f 2 + 1(b) 1 σ2 ∞ φyy (τ) = F −1 {Φxx (f)} = 1 RC ej2πf τ df RC −∞ (RC)2 + (2πf)2 : a = RC, v = 2ff. Then : σ2 a/π must σ 2 −a|τ | σ 2 −|τ |/RC φyy (τ) = e dv = e = e 2RC −∞ a2 + v 2 2RC 2RC, where the last integral is assessed in the same way as in problem P-2.9 . Finally: σ2 E Y 2 (t) = φyy (0) = 2RCProblem 2,17: If ΦX (f) = 0 | (f) | > W, the ΦX (f) e-j2πf a is also bandlimited. The corresponding autocor-ratio function can be represented as (remember that ΦX (f) is deterministic): ∞ n sin 2šW τ − 2W n φX (τ − a) = φX (− a) (1) n=−∞ 1 2W 2πW τ − n 2WLet we define: n sin 2πW t − 2W ∞ n à x(t) = X() n=−∞ 2W 2šW t − n 2WM must be demonstrated ka : E | X(t) − X(t)|2 = 0 àor ⎛ ⎝ ⎞⎤ m sin 2šW t − 2W ⎠⎦ ∞ m E ⎣ X(t) ⎝X(t) − à x() =0 (2) m=−∞ 2W 2šW − m2W First time we have: n − m sin 2πW t − 2W ∞ n m m E X(t) − X(t) X() = φX (t −)- φX () 2W 2W n=−∞ 2W 2šW t − n 2W 9 15. But the right side of this equation is equal to zero, applying (1) with = m/2W. As it applies to any m, it follows that E X(t) − X(t) X(t) = 0. Also, ∞ n sin 2šW t − 2W n E X(t) − X(t) X(t) = φX (0) − φX (− t) n=−∞ 2w 2šW t − n 2WAgain, applying (1) with = t anf τ = t, we observe that the right side of the equation is zero. So (2) there. Problem 2.18: ∞ 2Q(x) = √1 x e−t /2 dt = P [N ≥ x], where N is gaussian r.v with zero average and unit 2švariance. From Chernoff binding: P [N ≥ x] ≤ e-àx E evN v à(1)where v is the solution √ √ √ √ √ ∞ √ √: 2/2 E evN = 2π -∞ evt e-t dt 2 /2 √1 ∞ −(t-v)2 /2 = ev 2π -∞ e dt 2 //2 = evand d d 2 E NevN = E evN = vev /2 dvHence (2) gives: v = x 2 2 /2 2/2(1) ⇒ Q (x) ≤ e−x ex ⇒ Q(x) ≤ e-xProblem 2,19: ∞Since H(0) = -∞ h(n) = 0 ⇒ my = mx H(0) = 0 10 16. The auto-corporation of the output sequence is ∞ 2 φyy (k) = h(i)h(j)φxx (k − j + i) = σx h(i)h(k + i) i j i=-∞ where the last equality derives from the function of the X(n) car corporation: 2 2 σx , j = k + i φxx (k − j + i) = σx δ(k − j + i) = 0, o.w. 2 2 2Sality, φyy (0) = 6σx , φyy (1) = φyy (−1) = −4σx , φyy (2) = φyy (−2) = σx , φyy (k) = 0 otherwise. Finally, the discrete time system frequency response is: ∞ -j2šf n H(f) = -∞ h(n)e = 1 − 2e-j2πf + e-j4πf 2 = 1 − e-j2 πf 2 = e-j2πf ejπf − e-jf = -4e-jπf sin 2 πfwhich gives power density spectrum output: Φyy (f) = Φxx (f)| H(f)|2 = σx 16 sin 4 πf = 16σx sin 4 πf 2Problem 2.20 : |k| 1 φ(k) = 2The power density spectrum is ∞ Φ(f) = k=-∞ φ(k)e-j2πf k −1 −k ∞ k 1 1 = k=-∞ 2 e-j2πf k + k=0 2 e-j2πf k ∞ 1 jš ∞f 1 −j2πf k = k=0 (2 e) + k =0 (2 e) −1 1 1 = 1-ej2πf /2 + 1−e-j2πf /2−1 2−cos 2šf = 5/4−cos 2πf -1 3 = 5−4 2šf 11 17. Issue 2.21: We will denote the discrete time process with subscript d and continuous time (analog) process with subscript a. Also f will denote analog frequency and fd discrete time frequencies. a φd (k) = E [X ∗ n)X(n + k)] = E [X ∗ nT)X(nT + kT)] = φa (kT)) Thus, the auto-correction function of the sample signal is equal to x(t) auto-correction function. b) ∞ φd (k) = φa (kT) = -∞ Φa (F)ej2πf kT df ∞ (2l+1)/2T = l=−∞ (2l−1)/2T Φa (F)ej2πf kT df ∞ 1/2T l = l=−∞ −1/2T Φa (f + T)ej m df df 1/2T ∞ l = −1/2T l=−∞ Φa (f + T) ej2šF kT dfLet fd = f T. Tad : ⎡ ⎤ 1/2 ∞ ⎣ 1 φd (k) = Φa ((fd + l)/T)⎦ ej2πfd k dfd (1) −1 /2 T l=−−∞Mēs zinām, ka diskrēta laika procesa autokoretācijas funkcija ir apgrieztā Fouriertransform no tās jaudas spektrālā blīvuma 1/2 φd (k) = Φd (fd)ej2πfd k dfd (2) −1/2Salīdzinošs (1),(2) : ∞ 1 fd + l Φd (fd) = Φa ( ) (3) T l=−−∞ T(c) No (3) mēs secinām that: 1 fd Φd (fd) = Φa ( ) T Tiff: Φa (f) = 0, ∀ f : |f | > 1/2T 12 18. Otherwise, the sum of the copy copies of the Φa shift (point 3) will overlad and a pseudonym will take place. Problem 2.22 :(a) ∞ φa (τ) = -∞ Φa (f)ej2πf τ df W = −W ej2πf τ τ τ = πτBy apply 2.21. problem result, we have sin 2πW kT φd (k) = fa (kT) = πkT 1 (b) If T = 2W, then : 2W = 1 / T, k = 0 φd (k) = 0, otherwiseS, sequence X (n) is the order of white noise. The fact that this is a minimum value can be shown from the following sample process power spectral density message: −fs − W −fs −fs + W −W W fs – W fs fs + W æWe can see, that the maximum sampling rate fs giving a spectral flat sequence of derived bone: 1 W = fs − W ⇒ fs = 2W ⇒ T = 2W (c) Triangular spectrum Φ(f) = 1 − |f | , |f | ≤ W can be obtained with √ wing rectangular spectrum Φ1 (f) = 1/ W , |f | ≤ W/2. Thus φ(τ) = φ2 (τ) = 1 13 19. Therefore, sampling X(t) at a rate T = W samples in seconds constitutes a white sequence with an autoretreafunction: 2 2 1 sin πW kT sin πk W, k=0 φd (k) = W = W πkT πk 0, otherwiseProblem 2,23:Let's note: y(t) = fk (t)fj (t). Then : ∞ ∞ fk (t)fj (t)dt = y(t)dt = Y (f)|f =0 −∞ −∞where Y (f) is the Fourier transformation y(t). Since : y(t) = fk (t)fj (t) ←→ Y (f) = Fk (f) ∗ Fj (f). But : ∞ 1 −j2πf k/2W Fk (f) = fk (t)e-j2πf t dt = e -∞ 2WThe : ∞ Y (f) = Fk (f) ∗ Fj (f) = FK (a) ∗ Fj (f − a)∞ da <3> <3> <3>and at f = 0: ∞ Y (f)|f =0 = -∞ Fk (a) ∗ Fj (-a)da 2 ∞ 1 -j2π(k-j)/2W = 2W −∞ e da 1/2W, k = j = 0, k = jProblem 2,24: 1 ∞ | H(f)|2df Beq = G 0Thest filter shown in the box below. P2-12 we have G = 1 and ∞ Beq = | H(f)|2df = B 0Wems for low-rise filter. P2-16 we 1 1 H(f) = ⇒ | H(f)|2 = 1 + j2šf RC 1 + (2šf RC)2 14 20. So G = 1 and ∞ 2 Beq = 0 | H(f)| df 1 ∞ = 2-∞ | H(f)|2df 1 = 4RWhere the last integral is assessed in the same way as in the problem P-2.9 . 15 21. Chapter 3.1 : P (Bj | Ai) P (Bj , Ai) I(Bj ; Ai) = log 2 = log 2 P (Bj) P (Bj)P (Ai)Also: ⎧ 4 ⎪ 0,31, j = 1 ⎨ ⎬ ⎪ P (Bj) = P (Bj , Ai) = 0,27, j = 2 ⎪ ⎩ ⎭ ⎭ i =1 0,42, j = 3 ⎧ , ⎪ 0,31, ⎪ i=1 ⎪ ⎪ 3 ⎪ ⎨ ⎬ ⎪ ⎬ 0,17, i=2 P (Ai) = P (B,Ai) = ⎪ ⎪ ⎪ 0,31, ⎪ i=3 ⎪ ⎪ j=1 ⎩ ⎭ 0,21, i=4Hence : 0,10 I(B1; A1) = log 2 = +0,057 bees (0,31) (0,31) 0,05 I(B1); A2) = log 2 = -0.076 bits (0.31) (0.17) 0.05 I(B1); A3) = log 2 = -0.943 bees (0.31) (0.31) 0.11 I(B1; A4) = log 2 = +0.757 bees (0.31) (0.21) 0.08 I(B2; A1) = log 2 = -0.065 bees (0.27) (0.31) 0.03 I(B2; A2) = log 2 = -0.614 bit (0.27) (0.17) 0.12 I(B2); A3) = log 2 = +0,520 bees (0,27) (0,31) 0,04 I(B2; A4) = log 2 = -0.503 bees (0.27) (0.21) 0.13 I(B3; A1) = log 2 = -0.002 bees (0.42) (0.31) 0.09 I(B3; A2) = log 2 = +0,334 bits (0,42) (0,17) 16 22. A3) = log 2 = +0,105 bees (0,42) (0,31) 0,06 I(B3); A4) = log 2 = -0.556 bees (0.42) (0.21) (b) Average cross-information will be: 3 4 I(B; A) = P (Ai, Bj) I (Bj; Ai) = 0.677 bits j =1 i=1Problem 3.2: 3 H (B) = − j = 1 P (Bj) log 2 P (Bj) = − [0.31 log 2 0.31 + 0.27 log 2 0.27 + 0.42 log 2 0.42] = 1.56 bits/letterProblem 3.3: Let f (u) = u − 1 − ln u. F(u) the first and second derivatives are df 1 =1− du uand d2 f 1 2 = 2 > 0, 0. ∀u > 0 du u 17 23. Thus, this function reaches the minimum level du = 0 ⇒ u = 1. The minimum value is f (u = df1) = 0 so ln u = u − 1, at u = 1. For all other values from you: 0 < u < ∞, u = 1, we have (u) > 0 ⇒ u − 1 > ln u.Problem 3.4: We will show that −I (X; Y) ≤ 0 P (x, y) -I(X; Y) = − i j i j P (xi, yj) log 2 P (xi)P (yj) = 1 ln 2 i j P (xi, yj) ln PP (x)P (yj)) (xi i,yjWe use inequality ln u ≤ u − 1. We only need to consider those terms for which P (xi, yj) > 0;then applying inequality to each term I (X; Y): 1 P (xi) P (yj) −I(X; Y) ≤ ln 2 i j P (xi, yj) P (xi,yj) −1 1 = ln 2 i j [P (xi)P (yj) − P (xi, yj)] ≤ 0The first inequality becomes equality if and only if P (xi) P (yj) = 1 ⇐⇒ P (xi)P (yj) = P (xi, yj) P (xi , yj)when P (xi , yj) > 0. Moreover, since the summations [P (xi) P (yj) − P (xi , yj)] i jcontain only those terms for which P (xi, yj) > 0, this term is equal to zero, if and only if P (Xi)P (Yj) = 0, when P (xi, yj) = 0. Therefore, the two inequalities become the same, and therefore I(X; Y) = 0if and only if X and Y are statistically independent. 18 24. Problem 3.5: We will prove that H (X) − log n ≤ 0:1 H (X) − log n = n i = 1 pi log pi − log n 1 = n i = 1 pi log pi − n i = 1 pi log n n n 1 = i = 1 pi log NPI n 1 = ln 2 i=1 pi ln npi 1 1 ≤ ln 2 n i=1 pi npi −1 = 0Hence, H(X) ≤ log n. Also, if pi = 1/n ∀ i ⇒ H(X) = log n.Problem 3.6: By definition, differential entropy is ∞ H(X) = − p(x) log p(x)dx -∞About the evenly distributed random variable: 1 1 H(X) = − log dx = log a 0 a(a) About a = 1, H(X) = 0(b) For = 4, H(X) = log 4 = 2 log 2 (c) For a = 1/4, H(X) = log 1 = -2 log 2 4Problem 3,7 :(a) The following figure shows a triple Huffman code (we follow the convention that a lower probability branch has been assigned 1) : 19 25. Codeword Probability 01 0.25 1 11 0.20 0.58 1 . 0 001 0.15 1 0.42 1 100 0.12 0.22 0 101 0.10 1 0.33 00 01 0.08 1 0 0.18 0.05 0 0 00000 0.1 0 00001 0.05 1 æ (b) The average number of binary digits per source letter is :¯R= P (xi)ni = 2 (0,45) + 3(0,37) + 4(0,08) + 5(0,1) = 2,83 bees /letter i(c) Source entropy is: H(X) = − P (xi)logP (xi) = 2,80 bits/letter iAs, expected that the source entropy is less than the average length of each code. Problem 3.8: the source of entropy is: 5 1 H (X) = pi log = log 5 = 2.32 bits / letter i = 1 pi () When we encode one letter at a time we request R = 3 bits / letter . Thus, the efficiency is ̄2.32/3 = 0.77 (77%). So, we need 5 bits ̄per 2-letter symbol, or R = 2.5 bits / letter; efficiency is 2.32/2.5 = 0.93. So we ̄need 7 bits to the 3-letter symbol to R = 7 / 3 bits / letter; efficiency is 2.32/(7/3) = 0.994.Problem 3.9 :(a) P (xi |yj) I(xi; yj) = log P (xi) P (xi, yj) = log P (xi)P (yj) P (yj) P (yj |xi) = log P (yj) 1 = log P (yj) − log P (y1|xi) j = I(yj) − I(yj |xi)(b) P (xi |yj) I(xi; yj) = log P (xi) P (xi, yj) = log P(xi) P(yj) 1 1 = log P (xi) – log P (x1,yj) + log P (yj) i = I(xi) + I(yj) − I(xi, yj)Problem 3 ,10 :(a) ∞ H(X) = − p(1 − p)k−1 log2 (p(1 − p)k−1) k=1 ∞ ∞ = -p log2 (p) (p) (1 − p)k−1 − p log2 (1 − p) (k − 1)(1 − p)k−1 k=1 k=1 k=1 1 1−p = −p=−p=-p) − p log2 (1 − p) 1 − (1 - p) (1 − (1 − p))2 1−p = − − log2 (p) − log2 (1 − p) p 21 27.b) Clear P (X = k| X > K) = 0 for k ≤ K. If k > K, P (X = k, X > K) p(1 − p)k−1 P (X = k| X > K) = P (X > K) P (X > K)But, ∞ ∞ K P (X > K) = p(1 − p)k−1 = p (1 − p)k−1 − (1 − p)k−1 k=K+1 k=1 k=1 k=1 1 1 - (1 − p) K = p − = (1 − p)K 1 − (1 - p) 1 − (1 - p)so that p(1 − p)k−1 p (X = k| X > K) = (1 − p)KIf we let k = K + l with l = 1, 2, . . . , then p(1 − p)K (1 − p)l−1 P (X = k| X > K) = p(1 − p)l−1 (1 − p) Kthat is P (X = k | X > K) is geometrically distributed. Thus, using the results of the first part, we obtain ∞ H(X|X > K) = − p(1 − p)l−1 log2 (p(1 −) l-1 1-p -log2 (p) - log2 (1 x p) pProblem 3. 11 :(a) Margin-lo sadal-jumu P (x) apr-ina P(x) and P(x, y). T.D. H(X) . . . . P(x)P (y)(b) Izmantojot nevienl.dz.bu ln w ≤ w ≤ . . . . Reizinot iepriek-j ar P (x, y) a pievienojot vair-k nek-x, y, m-s ieg-tam P (x, y) ln P(x)P(y) - P(x, y) ln P (x, y) ≤ P(x) P(y) - P (x, y) x yHence, H(X, Y) ≤ ? P (x, y) ln P (x)P (y) ? P (x, y)(ln P(x) + ln P (y)) x,y x,y s P (x, y) ln P (x) ? P (x, y) ln P (y) ? H(X) + H(Y) x,y x,y P (x)P (y)Equality holds when P (x,y) s 1, t.i., when X, And go neatkar-gi. c H(X, Y) ? H(X) + H(Y) X) ? H(Y) + H(X-Y)Ar-no b), H(X, Y) ≤ H(X) + H(Y). Apvienojot divas attiec-bs, ms ieg-tam H (Y) + H (X) Y) ≤ H (X) + H (Y) s ⇒ H (X) Y) ≤ H (X) Pie-emsim, ka tagad, ka iepriek-j-attiec-ba ir ar vienl-daz-bu. P'c tam P(x) ? P(x) log P (x'y) ' P(x) log P (x) ⇒ P(x) log()'0 x x X P (x'y)Tom'r P (x) vienm.r ir liel.ks vai vien.ds ar P (x-y), lai-urn-ls(P(x)/P (x-y)) b-tu nenegat-vs. T. k. P (x) > 0, iepriek.min.t. vienl.dz.ba ir tad, ja un tikai tad, ja log(P (x)/P (x-y)) - 0 vai equivalentlyif un tikai tad, ja P (x)/P (x-y) - 1. Tas noz.m., ka P (x-y) - P(x) noz-m-, ka X a Y nav atkar-gi. Problama 3.12: Robe-ile varb-t-bu apr-ina ar 2 P (X-0) - P (X-0, Y-k) -P (X-0, Y-0) + P (X-0, Y-1) -k 3 1 P (X-1) -P (X-1, Y-k) -P (X-1, Y s 1) s k 3 1 P (Y s 0) s P (X s k, Y s 0) s P (X s 0, Y s 0) k 3 2 P (Y s 1) s P (X s k, Y s 1) s P (X x 0, Y s 1) + P (X s 1, Y s 1) s k 3 23 29. 1 1 1 1 H(X) ? Pi log2 Pi ? (log2 + log2) . . 9183 i .0 3 3 3 1 1 1 H(X) . . . . Y) - log2 - 1,5850 i-0 3 3 H(X-Y) - H(X, Y) - H(Y) - 1,5850 - 0.9183 - 0.6667 H(Y - X) ? H(X, Y) ? H(X) ? 1,5850 ? 0.9183 ? 0.6667Problem 3.13: H ? lim H(Xn) X1 , ... , Xn-1) n→∞ - lim - P (x1 , . . , xn) log2 P (xn x1 , . . , xn -1) n→∞ x1 ,...,xn . . . . , xn) log2 P (xn xn-1) n→∞ x1 ,...,xn ⎡ ⎤ -lim ⎣-P (xn, xn-1) log2 P (xn-xn-1)⎦ n→∞ xn ,xn-1 - lim H(Xn- Xn-1) n→∞Cik par stacion-ro procesu P (xn, xn-1) a P (xn xn-1) nav atkar-gas no n, lai H - lim H(Xn - Xn-1) - H(Xn- Xn-1) n→∞Problem 3.14 : H(X, Y) - H(X, g(X)) - H(X) + H(g) X) ? H(g(X)) + H(X-g(X)) 24 30. Bet, H(g(X) X) s 0, jo g(a) go deterministiska. H(g(X)) + H(X-g(X)))T-K-katrs iepriek-j-vien-dojuma termins nav negat-vs, m-s ieg-tam H(X) ≥ H(g(X)) Vienl-dz-ba, kad H(X-g(X)) Tas noz.m., ka v.rt.bs g(X) unik.li nosaka that g (·) is a one-to-one mapping. Problem 3.15: I (X; Y) = n i =1 m j=1 P (xi, yj) log PP (x)P (y)j) (xi i,yj n i=1 m j=1 P (xi, (xi, yj) − n i=1 m j=1 P (xi, yj) log P (xi) = − i=1 m P (xi , yj) log P (xi) − i=1 m P (xi , yj) log P (yj) n j=1 n i=1 m j=1 P (xi, yj) log P (xi, yj) − n i=1 P (xi) log P (xi) = − j =11 P (yj) log P (yj) m = −H(XY) + H(X) + H(Y)Problem 3,16: m1 m2 mn H(X1 X2 ... Xn) = − ... P (x1, x2, ..., xn) log P (x1, x2, ..., xn) j1 =1 j2 =1 jn =1Since (xi } is statistically independent: P (x1, x2, ..., xn) = P (x1) P (x2)... P (xn) and m2 mn ... P (x1)P (x2)... P (xn) = P (x1) j2 = 1 jn =1 (similar to other xi). Then: H (X1X2... Xn) − m1 j1 = 1 m2 j2 = 1 ... mn jn =1 P (x1)P (x2)... mn jn = 1 P (x1) P (x2)... P (xn) log P (x1)P (x2)... P (xn) = − m1 j1 =1 P (x1) log P (x1) − m2 j2 =1 P (x2) log P (x2)... − mn jn =1 P (xn) log P (xn) n = i=1 H(Xi) 25 31. Problem 3.17: We believe n − input, n − output channel. As it is noiseless: 0, i = j P (yj |xi) = 1, i = jHence: H(X|Y) = n i=1 n j=1 P (xi, yj) log P (xi |yj) = n i=1 n j=1 P (yj |xi)p(xi)log P (xi |yj)But it is also true that: 0, i = j P (xi |yj) = 1, i = jHence : n H(X|Y) = − P (xi) log 1 = 0 i=1Problem 3,18: The condition between x3 and x2 is defined as : P (x3, x2 |x1) P (x3 |x2 x1) I(x3; x2 |x1) = log = log P (x3 |x1)P (x2 |x1) P (x3 |x1)So : I(x3; x2 |x1) = I(x3 |x1) − I(x3 |x2 x1) and I(X3); X2 (, CA) X1) = x1 x2 x3 P (x1, x2, x3) log PP (x|x2 x)) (x3 |x1 1 − x1 x2 x3 P (x1, x2, x3) log P (x3 |x1) = x1 x2 x3 P (x1, x2, x3) log P (x3 |x2 x1) = H(X3) | X1) − H(X3 | X2 X1) since I(X3); X2 (, CA) X1) ≥ 0, it follows that: H (X3 | X1) ≥ H(X3 | X2 X1)Problem 3.19 : 26 32. Let's say > 0. Then we know ka linear transformation Y = aX + b: 1 y−b pY (y) = pX () aHence: ∞H (Y) = − −∞ pY (y)) log pY (y)dy ∞ 1 1 = − y−b y−b -∞ pX (a) log a pX (a) dy y−bLet u = a . Then dy = adu, and: ∞ 1 H (Y) = − -∞ pX (u) [log pX (u) − log a] adu ∞ ∞ = − -∞ pX (u) log pX (u) du + -∞ pX (u) log adu = H(X) + log aln in a similar manner, we can prove that about < 0:H (Y) = −H(X) − log aProblem 3.20: Linear transformation creates symbols: yi = axi + b , i = 1, 2, 3 with a corresponding probability p1 = 0,45, p2 = 0,35, p3 = 0,20. Thus, linear transformation does not affect aDMS entropy. Issue 3.21 :(a) This figure depicts the Huffman code when coding one level at a time: 27 for 33. Codeword Level Probability 1 1 a1 0.3365 00 0 a2 0.3365 0.6635 0 010 a3 0.1635 0.1635 0,327 011 a4 0,1635 1 æ Average number of digits per source level is :¯R = P (ai)ni = 1,9905 (iSource entropy) is: H(X) = − P (ai)logP (ai) = 1,9118 bits/level i(b) Two levels of coding at a time: 28 34. Codeword Levels Probability 1 001 a1 a1 0.11323 0 010 a1 a2 0.11323 0.22646 1 011 a2 a1 0.11323 1 0 100 a2 a2 0.11323 0 0 0.22327 0 0.55987 1011 a1 a3 0.05502 0.11004 1 1010 a1 a4 0.05502 1 0.33331 0 0 00000 a2 a3 0.05502 0.11004 0 00001 a2 a4 0.05502 1 0.22008 0 0 00010 a3 a1 0.05502 0.11004 0.44023 1 1 00011 a3 a2 0.05502 1 0 1100 a4 a1 0.05502 0 0.11004 1101 a4 a2 0.05502 1 0 0.21696 11100 a3 a3 0.02673 0.05346 1 11101 a3 a4 0.02673 1 0.10692 0 1 11110 a4 a3 0.02673 0.05346 11111 a4 a4 0.02673 1 1 æ ¯The average number of binary digits per level pair is R2 = k P (ak)nk = 3.874 bits/pairresulting in an average number¯R = 1,937 bits/level(c) RJ 1 H(X) ≤ < H(X) + J J J → ∞, RJ J → H(X) = 1,9118 bits/level. 29 35. Problem 3.22: First, we need a national probability P(xi), i = 1, 2. For fixed Markov processes, they can be found in general with system solution: P Π = P, Pi = 1 ikur P is a vector of national probability, and Π is a transitional matrix : Π[ij] = P (xj |xi). However, in the case of the two-state Markov source, we can find P (xi) in a simpler way, indicating that the probability of moving from country 1 to country 2 is equal to the probability of a transition from country 2 to country 1 (so that the probability of each country will remain the same). Hence : P (x1 |x2)P (x2) = P (x2 |x1)P (x1) ⇒ 0.3P (x2) = 0.2P (x1) ⇒ P (x1) = 0.6, P (x2) = 0.4Then : P (x1) [−P (x1 |x1) log P (x1 |x1) − P (x2 |x1) log P (x2 |x1)] + H(X) = P (x2) [−P (x1 |x2) log P (x1 |x2) − P (x2 |x2) log P (x2 |x2)] = 0.6 [−0.8 log 0.8 − 0.2 log 0.2] + 0.4 [−0.3 log 0.3 − 0.7 log 0.7] = 0.7857 bits/letterIf the source is a binary DMS with output letter probabilities P (x1) = 0.6, P (x2) = 0.4, itsentropy will be : HDM S (X) = −0.6 log 0.6 − 0.4 log 0.4 = 0.971 bits/letterWe see that the entropy of the Markov source is smaller, since the memory inherent in it reducesthe information content of each output. Problem 3.23 :(a) H(X) = (.05 log2.05 + .1 log2.1 + .1 log2.1 + .1 log2.1 + .1 log2 .1 + .15 log2 , 15 +.05 log2.05 + .25 log2.25 + .3 log2.3) = 2.5282 (b) By quantitative, the new alphabet is B = {−4, 0, 0. 4} and the corresponding symbol probabilitations are calculated P (−4) = P (-5) + P (−3) = .05 + .1 = .15 P (0) = P (−1) + P (0) + P (1) = .1 + .15 + .05 = .3 P (4) = P (3) + P (5) = .25 + .3 = .55 30 36. Thus, H(Q(X)) = 1,4060. As is observed quantitatively reduces the source of entropy. Issue 3.24: The following image depicts the design of the triple Huffman code. 0.22 0 10.18 0 1.50 1 11.17 12.15 2 20.13 0 21.1 1.28 2 22.05 2 Average codeword length is¯R(X) = P (x)nx = .22 + 2(.18 + .17 + .15 + .13 + .10 + .05) x = 1,78 (triple symbols/output)For fair of the average codeword length with the source entropy, we calculate it with logarithms in base 3. Thus, H(X) = − P (x) log3 P (x) = 1,7047 xAs is expected to be H(X) ≤ R(X). ̄Problem 3.25 :P arsing sequence with lempel-ziv coding system rules we get phrases0, 00, 1, 001, 000, 0001, 10, 00010, 0000, 0010, 00000, 101, 00001,000000, 11, 01, 01, 0000000, 110, ... The number of phrases is 18. For each phrase we need 5 bits plus an extra bit represented in the new source of output. 31 37. Dictionary Dictionary Codeword Location Contents 1 00001 0 00000 0 2 00010 00 00001 0 3 00011 1 00000 1 4 00100 001 00010 1 5 00101 000 00010 0 6 00110 0001 00101 1 7 00111 10 00011 0 8 01000 00010 00110 0 9 01001 0000 00101 0 10 01010 0010 00100 0 11 01011 00000 01001 0 12 01100 101 00111 1 13 01101 00001 01001 1 14 01110 000000 01011 0 15 01111 11 00011 1 16 10000 01 00001 1 17 10001 0000000 01110 0 18 10010 110 01111 0Problem 3.26 :(a) ∞ 1 −x 1 x H(X) = − e λ ln(e− λ)dx 0 λ λ 1 ∞ 1 ∞ 1 x x e− λ dx + e− λ dx x = − ln( ) λ 0 λ 0 λ λ 1 ∞ 1 −x = ln λ + e λ xdx λ 0 λ 1 = ln λ + λ = 1 + ln λ λ ∞ 1 −x ∞ 1 x λ e− λ dx = λ. xkur we used the fact 0 λe λe dx = 1 and E[X] = 0(b) ∞1 − |x| 1 |x| H(X) − − e λ ln(e− λ)dx −∞ ∞ <1> 2λ 2λ 1 ∞ 1 |x| 1 ∞ 1 − |x| = − ln() e− λ dx + |x| = − ln () e- λ dx + |x | 2λ -∞ 2λ λ -∞ 2λ 32 38. 1 0 1 x ∞ 1 x = ln(2λ) + −x e λ dx + x e− λ dx λ -∞ 2λ 0 2λ 1 = ln(2λ) + λ + λ + 1 + ln(2λ) 2 λ 2λ(c) 0 x+λ x+λ -x + λ –x + λ H(X) = − 2 ln 2 dx – 2 ln dx −λ λ λ λ λ λ λ λ λ λ λ -x + λ λ = λ = − ln 2 d x + dx λ -λ2 0 λ2 0 x+λ λ -x + λ − ln(x + λ)dx − ln(−x + λ)dx −λ 2 λ2 λ = ln(λ2) − 2 z ln zdzλ 0 λ 2 z 2 ln z z 2 = ln(λ2) − 2 − λ 2 4 0 1 = ln(λ2) − ln(λ) + 2Problem 3.27 :(a) Since(D) = log D and D = λλ , we get R(D)=log(λ/2)=log(2)=1 bit/sample. λ 2 λ(b) Figure R(D) for λ = 0,1, .2 and .3. As can be seen from the show, the increase in the parameter λ increases the required speed for a specific deformation. 7 6 5 4 R(D) 3 2 l=.3 1 l=1 l=2 0 0.05 0.1 0.15 0.2 0.25 0.3 Distortions D 33 39. 3.28. Problem :(a) Gaussian random variable that is zero mean and variance σ 2, speed deformation function 2 is calculated from R(D) = 1 log2 σ . Thus, the upper limit is satisfied with equality. In the lowest 2 Dbound recall that H(X) = 1 log2 (2šeσ 2). Thus, 2 1 1 1 H(X) − log2 (2seed) = log2 (2šeσ 2) - log2 (2sehand) 2 2 2 2 1 2πeσ 2 = log2 = R(D) 2 2seed is observed, the upper and lower limits coincide. (b) Differential entropy of laplacian source with parameter λ is H(X) = 1 + ln[2λ]. The laplacian ∞ 1 − |x| σ2 = x2 e λ dx = 2λ2 −∞ 2λ √Saum, σ 2 = 1, we λ = 1/2 and H(X) = 1+ln(2λ) = 1+ln(2) = 1,3466 nats/symbol = 1,5 bits/symbol. The following illustration shows the sample area of the lower and upper limits of R(D). Laplacian Distribution, unit variance 5 4 3 R(D) 2 1 Upper limit 0 Lower bound -1 0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1 Deformation D(c) Calculation of triangular distribution with 0 x+λ 2 λ -x + λ σ2 = x dx + x2 dx –λ2 0 λ2 1 4 λ 3 0 1 λ = x + x + 2 − x4 + x3λ2 4 3 −λ 4 3 0 λ2 = 6 34 40. √ √Saeed, with σ 2 = 1, we get λ = 6 and H(X) = ln(6)−ln(6) +1/2 = 1.7925 bits /source output. The following illustration shows the sample area of the lower and upper limits of R(D). Triangular distribution, unit deviation 4.5 4 3.5 3 R(D) 2.5 2 1.5 1 Upper limit 0.5 Lower limit 0 -0.5 0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1 Nuisance DProblem 3.29 : A2 σ 2 = E[X 2 (t)] = RX (τ)|τ =0 = 2Situation, X2 A2 SQNR = 3 · X2 A2 SQNR = 3 · 4th X 2 = 3 · 4ν = 3 · 4chi 2 x2 max 2AAr SQNR = 60 dB, we get 3 · 4q 10 log10 = 60 = ⇒ q = 9.6733 2 The smallest integer greater that q is 10. Thus, the required number of quantification levels is 10.Problem 3.30 :(a) ∞ ∞ H(X|G) = − p(x, g) log p(x|g)dxdg −∞ -∞ 35 41. But X, G are independent, so: p(x, g) = p(x)p(g), p(x|g) = p(x). Thus: ∞ ∞ H(X|G) = − -∞ p(g) -∞ p(x) log p(x)dx dg ∞ = − --∞ p(g)H(X)dg = H(X) = 1 log(2šeóx) 2, where the last equality derives from Gaussian pdf X.(b) I(X; y) = H(Y) − H(Y | X)Since Y is the sum of two independent, zero-average Gaussian r.v. with variance : σ 2σx + σn . Thus: H(Y) = 1 log (2πe (σx + σn)). In addition, as y = x + g: y 2 2 2 2 2 (y−x) 2 1 − p(y|x) = pg (y − x) = √ e 2σn 2 2πσnEnce : ∞ ∞ H(Y | X) = − p(x, y) log p(y|x)dxdy −∞ −∞ ∞ ∞ 1 (y − x)2 = − p(x) log e p(y|x) ln √ exp(− 2) dydx −∞ −∞ 2πσn 2σn ∞ ∞ √ (y − x)2 = p(x) log e pg (y − x) ln(2πσn) + 2 dy dx −∞ −∞ 2σn ∞ √ 1 2 = p(x) log e ln(2πσn) + 2 σn dx −∞ 2σn √ 1 ∞ = log(2πσn) + log e p(x)dx 2 −∞ 1 2 = log 2πeσn (= H(G)) 2 ∞ ∞ 2where we have used the fact that : -∞ pg (y − x)dy = 1, −∞ (y − x) pg (y − x)dy = E [G2] = σn . 2No H(Y), H(Y | X) : 1 2 2 1 2 1 σ2 I(X; y) = H(Y) − H(Y | X) = log 2še(σx + σn) − log 2πeσn = log 1 + x 2 2 2 σn 36 42. Problēma 3,31 : Codeword Letter Varbūtība 2 x1 0,25 2 00 x2 0,20 0 1 0,47 01 x3 0,15 0 2 02 x4 0,12 0 10 x5 0,10 2 12 x6 0,08 0 0,28 110 x7 0,05 1 1 1 111 x8 0,05 0,1 2 x9 0 æ¯R = 1,85 trīskāršie simboli/burtsProblem 3,32 : Dots (n1, n2, n3, n4) = (1, 2, 2. panta 2. punkts 3) mums ir : 4 9 2−nk = 2−1 + 2−2 + 2−2 + 2−3 = >1 k =1 8Since Craft nevienlīdzība nav apmierināta, binārais kods ar koda vārda garumu (1, 2, 2, 3), kasapmierina prefiksa nosacījumu nepastāv. Problem : 2n 2n −nk 2 = 2−n = 2n 2−n = 1 k=1 k=1 37 37