9.3 fermentation worksheet answers

Further Reading: Ackoff R., Ackoff's Best: His Classic Writing on Management, Wiley, 1999. Bender E., Introduction to Mathematical Modeling, Dover Pubns, 2000. Fdida S., and G. Pujolle, Performance Modeling and Evaluation Engineering, Elsevier Science, 1987. Gershenfeld N., Nature Of Mathematical Modeling, Cambridge Univ. Pr., 1998. Optimization problems are everywhere in real-world system mathematical modeling and cover a very wide range of applications. The application appears in all branches of Economics, Finance, Chemistry, Material Science, Astronomy, Physics, Structural and Molecular Biology, Engineering, Computer Science, and Medicine. Optimization modeling takes the right time. Common procedures that can be used in the modeling process cycle are to: (1) describe the problem, (2) prescribe a solution, and (3) control the problem by assessing/updating the optimal solution continuously, while changing the parameters and structure of the problem. Obviously, there is always a feedback loop between these common steps. Problem Math Formulation: As soon as you detect a problem, think about it and understand it to adequately describe the problem in writing. Develop mathematical models or frameworks to re-present reality to design/use optimization solution algorithms. Problem formulations must be validated before a solution is offered. Good mathematical formulations for optimization should be inclusive (that is, that includes what is included in the problem) and exclusive (that is, shaved-off what does not belong to the problem). Find the Optimal Solution: This is the identification of the solution algorithm and the implementation stage. The only good plan is the one that is implemented, which remains in place! Managerial Interpretation of optimal solutions: Once you recognize the algorithm and determine the appropriate software module to apply, use the software to obtain the optimal strategy. Next, the solution will be presented to the decision maker with the same style and language used by the decision maker. This means providing a managerial interpretation of strategic solutions in layman's terms, rather than just handing decision makers computer prints. Post-Solution Analysis: These activities include updating optimal solutions to control problems. In this changing world, it's important to periodically update optimal solutions for every optimization problem given. Valid models can lose validity due to changing conditions, thus becoming inaccurate representations of reality and affecting the ability of decision makers to make good decisions. The optimization model you create should be able to cope with the changes. The Importance of Feedback and Control: It is necessary to place a heavy emphasis on the importance of thinking about feedback and control optimization problems. It would be a mistake to discuss the context of the optimization-modeling process and ignore the fact that one can never expect to find a solution that never changes and cannot be changed for decision problems. The nature of the optimal strategy environment is changing, and therefore feedback and control are an important part of the optimization modeling process. The above process is described as the Analysis, Design, and System Control stages in the following flow chart, including validation and verification activities: Further Reading: Beroggi G., Decision Modeling in Policy Management: Introduction to the Concept of Analytics, Boston, Kluwer Academic Publisher, 1999. Camm J., and J. Evans, Management Science: Modeling, Analysis, and Interpretation, Pub South-Western College., 1999. The Ingredients of Their Optimization and Classification Problems The essence of all decisions such as business, whether made for a company, or individual, is to find the action that makes you with the greatest profit. Mankind has long sought, or contributed to, better ways to carry out the tasks of everyday life. Throughout human history, humans have first sought a more effective source of food and then sought ingredients, strength, and mastery of the physical environment. However, relatively late in human history the general question began to quantitatively formulate first in words, and then evolved into symbolic notation. One of the pervasive aspects of this common question is finding the best or optimal. Most time managers are just trying to get some improvement in performance levels, or problems finding goals. It should be emphasized that these words usually do not have the right meaning. Efforts have been made to describe the complex human and social situation. To have meaning, the problem must be written in a mathematical expression that contains one or more variables, where the value of the variable must be determined. The question then asked, is what value should this variable do to ensure the mathematical expression has the largest numeric value (maximalization) or numeric value as high as possible (minimization). This process of maximizing or minimizing is referred to as optimization. Optimization, also called mathematical programming, helps find the answers that produce the best results – those that achieve the highest profit, output, or happiness, or that achieve the lowest cost, waste, or discomfort. Often these problems involve the most efficient use of resources – including money, time, machines, staff, inventory, and more. Optimization issues are often classified as linear or nonlinear, depending on whether the relationship in this issue is linear with respect to There are various software packages to troubleshoot optimizations. For example, for example, or WinQSB you solve linear program models and LINGO and What'sBest! nonlinear and linear problems. Programming Mathematics, solving the problem of determining the optimal allocation of limited resources necessary to meet a specific goal. The goal must represent the purpose of the decision maker. For example, resources can match people, materials, money, or land. Of all the permitted resource allocations, it is desirable to find one or that maximizes or minimizes some numerical quantity such as profit or cost. Optimization models are also called Prescriptive or Normative models as they strive to find the best strategies for decision makers. There are many optimization algorithms available. However, some methods are only suitable for certain types of problems. It is important to be able to recognize the characteristics of the problem and identify the right solution techniques. In each problem class, there are different methods of minimization, which vary in computational requirements, convergence properties, and so on. Optimization issues are classified according to the mathematical characteristics of objective functions, constraints, and controllable decision variables. The optimization problem consists of three basic ingredients: Objective functions that we want to minimize or maximize. That is, the quantity you want to maximize or minimize is called an objective function. Most optimization problems have a single objective function, otherwise they can often be reformulated so they do. Two interesting exceptions to this rule are: The goal of looking for problems: In most business applications, managers want to achieve certain goals, while satisfying the constraints of the model. Users don't really want to optimize anything so there's no reason to define objective functions. This type of problem is usually called a feasibility issue. Multiple objective functions: Often, users actually want to optimize many different goals at once. Typically, different destinations are incompatible. Variables that optimize one goal may be far from optimal for another. In practice, problems with multiple goals are reformulated as a single objective problem by forming a weighted combination of different goals or by placing multiple goals as desirable constraints. Controllable inputs are a series of decision variables that affect the value of objective functions. In manufacturing issues, variables may include different allocations of available resources, or labor spent on each activity. Decision variables are very important. If there are no variables, we cannot define objective functions and problem constraints. Uncontrolled input is called a parameter. Input value can be improved related to a particular problem. We call the parameter value of this model. Often you will have multiple cases or variations of the same problem to solve, and the value of the parameters will change in each variation of the problem. The problem is the relationship between decision variables and parameters. A set of constraints allows some decision variables to fetch a certain value, and excludes others. For manufacturing issues, it doesn't make sense to spend a negative amount of time on any activity, so we limit all time variables to non-negative. Constraints don't always matter. In fact, the field of unaltrained optimization is a large and important one in which many algorithms and software are available. In practice, reasonable answers about the underlying physical or economic problems, cannot be often obtained without placing constraints on decision variables. Viable and Optimal Solution: The value of the solution for the decision variable, in which all constraints are satisfied, is called a viable solution. Most solution algorithms proceed by first finding a viable solution, then attempting to fix it, and ultimately changing the decision variables to move from one viable solution to another viable solution. This process is repeated until the objective function has reached the maximum or minimum. These results are called optimal solutions. The basic purpose of the optimization process is to find variable values that minimize or maximize objective functionality while satisfying constraints. These results are called optimal solutions. There are over 4000 algorithm solutions to different types of optimization problems. Widely used solution algorithms are those developed for the following mathematical programs: convex programs, separable programs, quadratic programs and geometric programs. Linear Programming Linear Programs relate to the optimization problem class, in which the purpose functions to be optimized and all limitations, linear in terms of decision variables. A brief history of Linear Programming: In 1762, Lagrange solved the problem of optimization that could be excluded with simple equality constraints. In 1820, Gauss solved the linear equation system with what is now called Causssian elimination. In 1866 Wilhelm Jordan perfected the method for finding the least squared error as an ameasure of goodness-of-fit. It is now referred to as the Gauss-Jordan Method. In 1945, Digital computers appeared. In 1947, Dantzig invented the Simplex Method. In 1968, Fiacco and McCormick introduced the Point Interior Method. In 1984, Karmarkar implemented the Interior Method to complete the Linear Program adding its innovative analysis. Linear programming has proven to be a powerful tool, both in modeling real-world problems and in widely applied mathematical theories. However, many many nonlinear optimization issues. Studies of the problem involve a diverse mix of linear algebra, multivariate calculus, numerical analysis, and computational techniques. Key areas include computational algorithm design (including interior point techniques for linear programming), geometry and analysis of convex sets and functions, and studies of specially structured problems such as quadratic programming. Nonlinear optimization provides fundamental insights into mathematical analysis and is widely used in areas such as engineering design, regression analysis, inventory control, geophysical exploration, and economics. Quadratic Program Quadratic Program (QP) consists of optimization areas whose various applications are second only to linear programs. A wide variety of applications fall naturally into QP form. Projectile kinetic energy is the quadratic function of its speed. At least square regressions with side restrictions have been modeled as QP. Certain problems in production planning, location analysis, econometrics, activation analysis in chemical mixed issues, and in financial portfolio management and selection are often treated as QP. There are many algorithmic solutions available for cases under limited additional conditions, where the objective function is convex. Satisfaction Constraints Many industry decision problems involving continuous constraints can be exemplified as continuous constraints of satisfaction and optimization problems. Problem Satisfaction Constraints are large in size and in many cases involve transcendental functions. They are widely used in chemical processes and modeling and optimization of cost restrictions. Convex Convex Program (CP) covers a broad class of optimization issues. When the objective function of convex and viable territory is a convex set, these two assumptions are sufficient to ensure that the local minimum is the global minimum. Data Envelopment Analysis The Data Envelopment Analysis (DEA) is a reasoned performance metric in the border analysis method of economic and financial literature. The border efficiency analysis method (output/input) identifies the best practice performance border, which refers to the maximum output that can be obtained from a specific set of inputs with respect to the sample decision-making unit using a comparable process to convert the input to output. The strength of the DEA depends in part on the fact that it is a non-parametric approach, which does not require the specification of a form of functional relationship between input and output. DEA output reduces multiple performance measurements to one to use linear programming techniques. Weighting performance actions react to utility decision makers. Dynamic Programming Dynamic Programming (DP) is basically recursion where you store answers in tables ranging from base letters and builds to larger and larger parameters using recursive rules. You will use this technique instead of recursion when you need to calculate solutions to all sub-problems and recursive solutions will solve multiple sub-problems repeatedly. Although dp is generally capable of solving many diverse problems, it may require large computer storage in most cases. Separable Program Separable Program (SP) includes a special case of convex programs, in which objective functions and limitations are functions that can be diseparable, that is, each term involves only one variable. Geometric Program Geometric Program (GP) belongs to Nonconvex programming, and has many applications especially in engineering design issues. Fractional Programs In this problem class, objective functions are in the form of fractions (that is, the ratio of two functions). Fractional Programs (FP) appear, for example, when maximizing the ratio of capital returns to capital spent, or as a wasase ratio of performance measures. Heuristic Optimization A heuristic is something that provides assistance towards solving problems but otherwise cannot be justified or incapable of justifying. So heuristic arguments are used to show what we're trying to prove, or what we might expect to find in running a computer. They are, at best, educated guesses. Several heuristic tools have evolved in the last decade that facilitate the resolution of optimization problems that were previously difficult or impossible to solve. These tools include evolutionary computing, simulation anils, taboo searches, swarms of particles, etc. Common approaches include, but are not limited to: comparing the quality of the solution optimally on the problem of benchmarks with known optima, the average difference from the optimal, the frequency with which heuristics find optimal. compare the quality of solutions with the most well-known bound to the problem of benchmarks whose optimal solutions cannot be determined. compare your heuristics with published heuristics for the same types of problems, differences in solution quality for a specific process time and, if relevant, memory limits. profiling the average solution quality as a function of process time, for example, planning the average and both min and max or the 5th and 95th percentile of the solution value as a time function -- this assumes that one has several examples of comparable benchmark problems. Global Optimization The goal of Global Optimization (GO) is to find the best solution of the decision model, in the presence of several local solutions. While restricted optimization deals with finding the optimal objective function that is subject to constraints on its decision variables, unaltrained optimizations seek a global maximum or minimum functionality over their entire domain space, with no restrictions Variable. The Nonconvex Program A Nonconvex Program (NC) covers all nonlinear programming issues that do not satisfy convection assumptions. However, even if you manage to find a local minimum, there is no guarantee that it will also be a global minimum. Therefore, no algorithm will guarantee finding the optimal solution to all such problems. The Nonsmooth Program Nonsmooth Program (NSP) contains functions that the first derivative does not exist. NSP appears in several important applications of science and engineering, including contact phenomena in static and dynamics or delamination effects in composites. This application requires consideration of nonsmoothness and nonconnection. Metaheuristic Most metaheuristics have been created to solve the problem of discrete combined optimization. Practical applications in engineering, however, usually require techniques, which handle continuous variables, or other continuous and discrete variables. As a consequence, major research efforts have focused on fitting some well-known metaaeuristics, such as Annealing Simulation (SA), Taboo Search (TS), Genetic Algorithm (GA), Ant Colony Optimization (ACO), to continuous cases. General metaheuristic aims to turn the discrete domain of the application into a sustainable one, by means of: Methodological developments aimed at adapting some metaheuristic, especially SA, TS, GA, ACO, GRASP, variable environment search, guided local search, scatter search, to continuous or discrete/continuous variable issues. Theoretical and experimental studies of metaheuristics adapted to continuous optimization, for example, convergence analysis, performance evaluation methodology, test case generator, boundary handling, etc. Implementation of software and algorithms for metaheuristics tailored to continuous optimization. Real discrete metaheuristic applications adapted to continuous optimization. Comparison of discrete metaeuristic performance (adapted to continuous optimization) with competitive approaches, for example, Particle Swarm Optimization (PSO), Estimation Distribution Algorithm (EDA), Evolutionary Strategy (ES), specifically made for continuous optimization. Multilevel Optimization In many decision processes there is a hierarchy of decision makers and decisions are taken at different levels in thishierarchy. Multilevel Optimization focuses on the entire hierarchical structure. The field of multilevel optimization has become a well-known and important field of research. Hierarchical structures can be found in disciplines such as environment, ecology, biology, chemical engineering, mechanics, classification theory, databases, network design, transportation, supply chain, game theory and economics. In addition, new applications continue to be introduced. Multiobjective Program Multiobjective (MP) also known as the Destination Program, is where the single objective characteristic of the optimization problem is replaced by multiple objectives. In solving an MP, one can represent some goals as obstacles to be met, while other goals can be weighed to create a single objective composite function. Some objective optimizations differ from a single objective case in several ways: The usual meaning of the optimal does not make sense in some objective cases because the solution of optimizing all objectives simultaneously is, in general, impractical; instead, a search was launched for a viable solution resulting in the best compromise among the goals on a set of, so-called, efficient solutions; Identification of the best compromise solutions requires considering the preferences expressed by the decision maker; Some of the goals faced in real-life problems are often mathematical functions of contrasting forms. A key element of the destination programming model is the function of achievement; that is, a function that measures the degree of minimization of unwanted deviation variables from the objectives considered in the model. Business Applications: In credit card portfolio management, predicting the spending behavior of cardholders is key to reducing the risk of bankruptcy. Given the set of attributes for key aspects of credit cardholders and predetermined classes for spending behavior, one can build a classification model by using several linear programming criteria to find patterns of credit cardholder behavior. Non-Binary Constraints Programs Over the years, the constraint programming community has paid considerable attention to modeling and problem solving using binary restrictions. Recently it has been non-binary constraints that attract attention, as more and more real-life applications. A non-binary constraint is a defined limitation on the k variable, where k is usually greater than two. Non-binary constraints can be seen as a more global constraint. Problem modeling as a non-binary limitation has two main advantages: It facilitates the expression of problems; and enable stronger propagation of restrictions due to more widely available global information. Success in timetabling, scheduling, and routing, has proven that the use of non-binary constraints is a promising research direction. In fact, a growing number of OR/MS/DS workers feel that this topic is critical to making constraint technology a realistic way to model and solve real-life problems. Bilevel Optimization Most mathematical programming models deal with decision making with one objective function. Bile programming on the other hand developed for application in the system decentralized where the first level is referred to as the leader and the second level relates to the objectives of the followers. Inside Programming issues, each decision maker tries to optimize its own objective functions without considering the objectives of the other party, but each party's decisions affect the objective value of the other party as well as the decision space. A bilevel programming problem is a hierarchical optimization problem in which the constraints of one problem are defined in part by the second parametric optimization problem. If the second problem has a unique optimal solution to all parameter values, this problem is equivalent to the usual optimization problem of having implicitly defined objective functions. However, when the problem has an optimal solution that is not unique, an optimistic (or weak) and pessimistic (or strong) approach is being applied. Combinatorial Optimization generally means that the state space is discrete (e.g., symbols, not necessarily numbers). This space can be a limited set or countless. For example, discrete problems berupding. The problem in which the country space is actually booked can often be solved by mapping it to integers and applying numerical methods. If the status space is not sorted or is only partially reserved, this method fails. This means that heuristic methods become necessary, such as anyl simulation. Combined optimization is the study of packing, closing, and partitioning, which is an integer program application. They are the topic of mathematical principles in the interface between combinatorics and optimization. These problems deal with the classification of integer programming problems according to the known complexity of the algorithm, and the design of good algorithms for solving special subclasses. Specifically, problems with network flow, matching, and generalization of their matroids were studied. These subjects are one of the elements of joint unifying, optimization, operations research, and computer science. Natural Evolution Techniques are powerful optimizers. By analyzing natural optimization mechanisms, we may find acceptable solution techniques for irrevocable problems. The two most promising concepts are the simulation of anil and genetic techniques. Scheduling and timetabling are among the most successful evolutionary engineering applications. Genetic Algorithms (GAs) have become a very effective tool for solving harsh optimization problems. However, its theoretical foundation is still somewhat fragmented. Swarm Particle Optimization Particle Swarm Optimization (PSO) is a stochastic population-based optimization algorithm. Instead of competition/selection, as it says in Evolutionary Computing, PSOs leverage cooperation, according to a paradigm sometimes called herd intelligence. Such a system usually consists of a population of simple interacting agents without centralized control, and is inspired by cases that can be found in nature, such as ants flocks of birds, flocks of animals, bacterial molds, fish schools, etc. There are many variants of PSO including constrained, multiobyective, and discrete or combinatorial versions, and applications have been developed using PSO in many areas. Swarm Intelligence Biologists studied the behavior of social insects for a long time. After millions of years of evolution all these species have developed extraordinary solutions to a wide range of problems. Intelligent solutions to problems naturally arise from self-organization and indirect communication of these individuals. Indirect interactions occur between two individuals when one of them modifies the environment and the other responds to a new environment at a later time. Swarm Intelligence is an innovative distributed intelligent paradigm for solving optimization problems that initially took inspiration from biological examples by swarming, flocking, and ingesting phenomena in vertebrates. Data Mining is an analytics process designed to explore large amounts of data in search of consistent patterns and/or systematic relationships between variables, then to validate findings by applying detected patterns to new subsets of data. Online Optimization Whether costs should be reduced, profits to be maximized, or scarce resources to use wisely, optimization methods are available to guide decision making. In online optimization, the main problem is incomplete data and scientific challenges: how well do online algorithms perform? Can one guarantee the quality of the solution, even without knowing all the data first? In real-time optimization there are additional requirements: decisions must be calculated very quickly with respect to the time frame we consider. Further Reading: Abraham A., C. Grosan and V. Ramos, Swarm Intelligence, Springer Verlag, 2006. This relates to the application of herd intelligence in data mining, using a different intelligent approach. Charnes A., Cooper W., Lewin A., and L. Seiford, Data Encoding Analysis: Theory, Methodology and Application, Kluwer Academic Publications, 1994. Dempe S., Bilevel Programming Foundation, Kluwer, 2002. Diwekar U., Introduction to Applied Optimization, Kluwer Academic Publisher, 2003. Includes almost all of the above techniques. Liu B., and A. Esogbue, Decision Criteria and Optimal Inventory Process, Kluwer, 1999. Luenberger D., Linear and Nonlinear Programming, Kluwer Academic Publisher, 2003. Miller R., Optimization: Foundation and Applications, Wiley, 1999. MigdalasA., Pardalos p., and P. Varbrand, Multilevel Optimization: Algorithms and Applications, Kluwer, 1998. Reeves C., and J. Rowe, Genetic Algorithms: Principles and Perspectives, Kluwer, 2002. Rodin R., Optimization in Operations Research, Prentice Hall, New Jersey, 2000. For more books and journal articles optimization, visit

Website decisions Linear Programming Of Linear Programming Resources is often a favorite topic for professors and students. The ability to introduce LP using a graphical approach, the relative ease of solution methods, the widespread availability of LP software packages, and a wide range of applications make THE LP accessible even to students with relatively weak mathematical backgrounds. In addition, LP provides an excellent opportunity to introduce the idea of what-if analysis, because of the powerful tools for post-optimality analysis developed for the LP model. Linear Programming (LP) is a mathematical procedure for determining optimal allocation of scarce resources. LP is a procedure that has found practical application in almost all aspects of the business, from advertising to production planning. The issue of transportation, distribution, and planning of aggregate production is the most distinctive object of LP analysis. In the petroleum industry, for example data processing managers at major oil companies recently estimated that from 5 to 10 percent of the company's computer time is devoted to the processing of LP and LP-like models. Linear programming deals with a programming problem class where the objective function to be optimized is linear and all the relationships between variables correspond to linear resources. The issue was first formulated and resolved in the late 1940s. Rarely have new mathematical techniques discovered a variety of practical business, trading, and industrial applications and simultaneously received so thorough theoretical developments, in no time. Currently, this theory is being successfully applied to the problems of capital budgeting, diet design, resource conservation, strategy games, economic growth predictions, and transportation systems. In recent times, linear programming theory has also helped complete and unify many outstanding applications. It is important for readers to appreciate, at first, that programming in Linear Programming has a different flavor than programming in Computer Programming. In the previous case, it means planning and organizing as in Get with the program!, it's your program with the solution. While in the latter case, it means writing code to do the calculations. Training in one type of programming has very little direct relevance to the other. In fact, linear programming terms were coined before the word programming became closely related to computer software. This confusion is sometimes avoided by using the term linear optimization as a synonym for linear programming. Each LP problem consists of objective functions and a set of limitations. In most cases, obstacles come from the environment in which you work to achieve your goals. When you want to achieve your goals desired environment, you will realize that the environment constraints (i.e., difficulties, limitations) in fulfilling your wishes or goals. This is why religions such as Buddhism, among other things, prescribe to live an abstemious life. No desire, no pain. Can you take this advice with respect to your business goals? What a function: A function is a thing that does something. For example, a coffee milling machine is a function that converts coffee beans into powder. The function (objectively) maps and translates the input domain (called a viable region) into the output range, with the final two values called maximum and minimum values. When you formulate a decision-making problem as a linear program, you should check the following conditions: Objective functions must be linear. That is, check if all variables have 1 power and they are added or subtracted (not divided or multiplied) the goal should be maximalization or minimization of linear functions. The goal must represent the decision-making objectives The constraints must also be linear. Additionally, restrictions must be on the following forms (£, ³, or =, that is, LP restrictions are always closed). For example, the following issue is not LP: Max X, subject to X< 1. This very simple problem has no solution. As always, one should be careful in categorizing optimization issues as LP problems. Here's a question for you. Is the following problem an LP problem? Max X2 subject to: X1 + X2 £0 X12 - 4 £0 Although the second limit looks as if it is a nonlinear constraint, this limit can be written as: X1 ³ -2, and X2 £2. Therefore, the above problem is indeed a matter of LP. For most LP problems one can think of two important classes of objects: The first is limited resources such as land, factory capacity, or sales power size; the second, is activities such as producing low carbon steel, producing stainless steel, and producing high carbon steel. Each activity consumes or may contribute an additional amount of resources. There has to be an objective function, which is a way to say bad from good, from better decisions. The problem is to determine the best combination of activity levels, which do not use more resources than are actually available. Many managers are faced with this task every day. Fortunately, when a well-formulated model is input, linear programming software helps determine the best combination. Simplex method is a solution algorithm that is widely used to solve linear programs. An algorithm is a series of steps that will complete a specific task. Lp Problem Formulation Process and Its Application To formulate lp problems, I recommend using the following guidelines after reading the problem statement carefully several times. Any linear program consists of four parts: a set of decision variables, parameters, goal functions, and a set of In formulating the problem of decisions given in the form of mathematics, you should practice understanding the problem (that is, formulating the mental model) by reading and rereading the problem statement carefully. When trying to understand the problem, ask yourself the following common question: What are the decision variables? That is, what are controllable inputs? Determine the decision variables appropriately, using descriptive names. Remember that controllable inputs are also known as controllable activities, decision variables, and decision activities. What are the parameters? That is, what are the uncontrolled inputs? This is usually a given constant numeric value. Specify parameters appropriately, using descriptive names. What's the purpose? What is the objective function? Also, what does the owner of the problem want? How is the goal related to the decision variable? Is it a matter of maximizing or minimizing? The goal represents the purpose of the decision-maker. What are the obstacles? That is, what requirements must be met? Should I use the type of inequality or equality restrictions? What are the connections between variables? Write it down with words before placing them in mathematical form. Learn that a decent region has nothing or little to do with objective functions (min or max). Both parts in any LP formulation mostly come from two different and different sources. The purpose function is formed to meet the wishes of the decision maker (objective), while the constraints that make up a viable territory usually come from a decision-making environment that places some restrictions /conditions to achieve its goals. Here is a very simple illustration problem. However, the way we approach the problem is the same for various decision-making issues, and their size and complexity may vary. The first example is the product mix problem. Carpenter's Problem: Allocating Scarce Resources Among Competitive Means During several brain-storm sessions with the carpenter (our client), he tells us that he, solely, makes tables and chairs, sells all the tables and chairs he makes in the market place, however, has no stable income, and wants to do his best. The goal is to find out how many tables and chairs he has to make to maximize his net income. We begin by focusing on the time frame, that is, planning a time-horizon, to revise our solution weekly if necessary. To learn more about the problem, we must go to his store and observe what is going on and measure what we need to formulate (that is, to provide a Form, to model) the problem. We must affirm that the goal is to maximize net income. We have to with clients. The problem of carpenters dealing with figuring out how many tables and chairs to make per week; but first the objective function must be established: Since the total cost is fixed cost (F) and variable cost per unit multiplied by the number of units generated. Therefore, the decision problem is to find X1 and X2 in such a way that: Maximize 9X1 + 6X2 - [(1,5X1 + X2) + (2.5X1 + 2X2) + F1 + F2], where X1 and X2 stand for the number of tables and seats; cost provisions in brackets are raw materials, and labor costs respectively. F1 and F2 are fixed costs for two products each. Without general loss, and any impact on optimal solutions, let's set F1=0, and F2=0. The purpose function reduces the following net profit function: Maximize 5X1 + 3X2 That is, net income (say, in dollars, or tens of dollars) from selling X1 tables and X2 seats. The constrained factors that, usually come from outside, are labor limitations (these restrictions come from their families) and raw material resources (these restrictions come from scheduled delivery). The production time required for tables and chairs is measured at different times of the day, and is estimated to be 2 hours and 1 hour, respectively. The total working hours per week is only 40 hours. The raw materials required for tables and chairs are 1, and 2 units, respectively. The total supply of raw materials is 50 units per week. Therefore, the LP formulation is: Maximize 5 X1 + 3 X2 Subject to: 2 X1 + X2 £ 40 labor constraints X1 + 2 X2 £ 50 material constraints and both X1, X2 non-negative. It's a mathematical model for carpenter problems. Decision variables, i.e. controllable inputs are X1, and X2. The output for this model is a total net profit of 5 X1 + 3 X2. All functions used in this model are linear (decision variables have the same power as 1). The coefficient of these constraints is called the Technology Factor (matrix). The review period is one week, the corresponding period during which uncontrolled inputs (all parameters such as 5, 50, 2,..) tend to change (fluctuate). Even for such short time-horizon planning, we must perform a what-if analysis to react to any changes in this input to control the problem, that is, to update the specified solution. Note that since carpenters will not go out of business at the end of the planning horizon, we add the condition that both X1, X2 should be non-negative instead of the requirement that X1, and X2 should be positive integers. Conditions of non-negativity are also known as implied constraints. Again, the Linear Program will be fine for this problem if the carpenter will continue to produce these products. The partial item will only count as ongoing work and will eventually become finished goods says, as of next week. We can try to solve for X1 and X2 by listing possible solutions for each and choosing a pair (X1, X2) maximize 5X1 + 3X2 (net income). However, it is too time consuming to list all possible possibilities and if the alternative is not fully registered, we cannot ensure that the partner we choose (as a solution) is the best of all alternatives. How to solve this problem is known as sequential thinking versus simultaneous thinking. A more efficient and effective methodology, known as Linear Programming Solution Engineering based on simultaneous thinking is commercially available in over 400 different software packages from around the world. The optimal solution, which is the optimal strategy, is to make X1 = 10 tables, and X2 = 20 seats. We can program the weekly activities of carpenters to make 10 tables and 20 chairs. With this (optimal) strategy, net income is $110. This predetermined solution was a surprise to the carpenter because, because of the more net income selling the table ($5), he used to make more tables than chairs! Hiring or Not? Suppose a carpenter can hire someone to help at a cost of $2 an hour. This is, in addition, the hourly based wages he currently pays; otherwise $2 is much lower than the current minimum wage in the US. Should carpenters rent and if so then for how is the clock possible? Allow X3 to be an additional number of hours, then the modified problem is: Maximize 5 X1 + 3 X2 - 2 X3 Subject to: 2 X1 + X2 £ 40 + X3 labor restrictions with unknown additional hours X1 + 2 X2 £ 50 material constraints In this new condition, we will see that the optimal solution is X1 = 50, X2 = 0, X3 = 60, with an optimal net income of $ 130. Therefore, carpenters must be rented for 60 hours. What about hiring only 40 hours? The answers to these and other types of what-if questions are treated under sensitivity analysis on this Web site. As an exercise, use your LP software to find the largest range for an X grade that satisfies the following inequalities with two absolute value terms:| 3X – 4 | - | 2X – 1 | £2 A company that takes the selling price of an S unit from its products at a market price p. Management policy is to replace damaged units at no additional cost, based on the first rank, while replacement units are available. Since management did not want to risk making the same mistake twice, it produced a unit that it sold to the market on one machine. In addition, it produces a replacement unit, showing the X, on the second engine of higher quality. Fixed costs associated with the operation of both machines, variable costs, and replacement costs are provided are functions of short-term cost C(X) = 100 + 20S + 30X. The exact probability that the unit will be damaged is r. However, acting not carefully, management always underestimates the reliability of its products. Nonetheless, it imposes the condition that X ³ r.S. for company products provided by S(r) = 10000e-0.2r. Therefore the decision issue is to maximize the function of net profit P(X): Maximize P(X) = 100000p e- 0.2r - 100 - 20S - 30X, subject to: X³ r.S. As we will learn, learn, the solution to the LP problem is on the nodes of a viable area. Therefore, P(X) net profit will be maximized if management sets X=r.S. Diet Problems Suppose the only foods available in your local store are potatoes and steaks. The decision on how much of each food to buy is to be made entirely on dietary and economic considerations. We have nutritional information and costs in the following table: Per unit of potatoes Per unit steak Minimum requirements Units of carbohydrates 3 1 8 Units of vitamins 4 3 19 Units of protein 1 3 7 Units cost 25 50 The problem is finding a diet (a choice of the number of units of two foods) that meets all the minimum nutritional requirements at minimal cost. Formulating problems in terms of linear inequality and objective function. Solve problems geometrically. Explain how the 2:1 cost ratio (steak for potatoes) determines that the solution should be where you say it is. Discover the cost ratio that will move the optimal solution to a wide selection of food units, but it will still require the purchase of steaks and potatoes. Discover the cost ratio that will determine buying only one of the two meals to minimize costs. a) We start by setting boundaries for problems. The first constraint represents the minimum requirement for carbohydrates, which is 8 units per some unknown amount of time. 3 units can be consumed per unit of potatoes and 1 unit can be consumed per unit of steak. The second constraint represents the minimum need for vitamins, which is 19 units. 4 units can be consumed per unit of potatoes and 3 units can be consumed per unit of steak. The third constraint represents the minimum requirement for protein, which is 7 units. 1 unit can be consumed per unit of potatoes and 3 units can be consumed per unit of steak. The fourth and fifth constraints represent the fact that all viable solutions must be nonnegative because we cannot buy negative amounts. limitations: {3X1 + X2 ³ 8, 4X1+ 3X2 ³ 19, X1+ 3X2 ³ 7, X1³ 0, X2 ³ 0}; Next we plan a set of solutions from inequality to produce a viable area of possibility. c) The cost ratio of 2:1 steak to potatoes determines that the solution should be here because, overall, we can see that one unit of steak is slightly less nutritious than one unit of potato. Plus, in one category where steak beats potatoes in health (protein), only 7 total units are required. Thus it is easier to meet these units without buying a significant amout steak. Since steaks are more expensive, buying more potatoes to meet these nutritional requirements is more logical. d) Now we choose a new cost ratio that will move the optimal solution to a wide selection of the number of food units. Both steaks and potatoes will still be bought, but different solutions will be found. Let's try the 5:2 cost ratio. Now we choose a new cost ratio that will move the optimal solution to a wide selection of the number of food units. Both steaks and potatoes will still be bought, but different solutions will be found. Let's try the 5:2 cost ratio. d) Now we choose a new cost ratio that will move the optimal solution to a wide selection of the number of food units. Both steaks and potatoes will still be bought, but different solutions will be found. Let's try the 5:2 cost ratio. Thus, the optimal solution for this cost ratio is to buy 8 steaks and no potatoes per unit of time to meet the minimum nutritional requirements. A Blending Problem Bryant's Pizza, Inc. is a manufacturer of frozen pizza products. The company generates net income of $1.00 for each regular pizza and $1.50 for each deluxe pizza produced. The company currently has 150 pounds of dough mixture and 50 pounds of topping mixture. Each regular pizza uses 1 pound of dough mixture and 4 ounces (16 ounces = 1 pound) of topping mixture. Each deluxe pizza uses 1 pound of dough mixture and 8 ounces of topping mix. On a recent request per week, Bryant can sell at least 50 regular pizzas and at least 25 deluxe pizzas. The problem is determining the number of regular and deluxe pizzas that companies must make to maximize net income. Formulate this problem as an LP problem. Let the X1 and X2 be the number of regular and luxurious pizzas, then the LP formulation is: Maximize X1 + 1.5 X2 Subject to: X1 + X2 £ 150 0.25 X1 + 0.5 X2 £ 50 X1 ³ 50 X2 ³ 25 X1 ³ 0, X2 ³ 0 Other Common Applications of Linear LP programming is a powerful tool for selecting alternatives in decisions and problems, consequently, has been applied in various settings problems. We'll show you some applications that cover the main functional areas of the business organization. Financial: Investor issues can be a mixed-portfolio selection issue. In general, the number of different portfolios can be much larger than the example shows, more and different types of constraints can be added. Another decision issue involves determining a funding mix for a number of products when more than one financing method is available. The goal may be to maximize the total profit, where the profit for a particular product depends on the method of financing. For example, funding can be made with internal funds, short-term debt, or medium-sized financing (amortized loans). There may be limits to the availability of each funding option as well as financial constraints that require a certain relationship between funding options so that they can meet the requirements of bank loans or medium financing. There may also be limits on production capacity for products. The decision variable is the number of units of each product that will be financed by each funding option. Production and Operations Management: Quite often industrial processes rendered raw raw can be made into a wide range of products. For example, in the oil industry, crude oil is refined into gasoline, kerosene, home heating oil, and various levels of engine oil. Given the current profit margin on each product, the problem is determining the number of each product to be produced. This decision is subject to various restrictions such as restrictions on the capacity of various purification operations, the availability of raw materials, demands for each product, and any government-imposed policies on the output of certain products. Similar problems also exist in the chemical and food processing industries. Human Resources: Personnel planning issues can also be analyzed with linear programming. For example, in the telephone industry, the demands for installer repair worker services are seasonal. The problem is determining the number of installer repair personnel and track repair workers who must have a monthly workforce where the total cost of hiring, layoffs, overtime, and regular wages is minimized. Established constraints include restrictions on the demands of services to be met, the use of overtime, union agreements, and the availability of skilled people for hire. This example contradicts the assumption of division; however, the labor force rate for each month will usually be large enough that rounding to the nearest integer in each case will not be detrimental, provided the constraints are not violated. Marketing: Linear programming can be used to determine the right media mix to use in advertising campaigns. Suppose the available media are radio, television, and newspapers. The problem is to determine how many ads will be placed in each medium. Of course, the cost of placing ads depends on the media chosen. We want to minimize the total cost of ad campaigns, depending on a series of constraints. Since each medium can provide a different level of exposure to the target population, there may be a lower limit on the total exposure of the campaign. Also, each medium may have different efficiency ratings in producing the desired results; thus there may be a lower limit on efficiency. In addition, there may be limits to the availability of each medium for advertising. Distribution: Another application of linear programming is in the field of distribution. Consider the case where there is a factory m that must send goods to the warehouse n. The given plant can make deliveries to a number of warehouses. Given the cost of delivering one unit of product from each plant to each warehouse, the problem is determining the delivery pattern (the number of units that each ship the factory to each warehouse) that minimizes the total cost. This decision is subject to restrictions that demand at each factory cannot deliver more products than capacity to produce. Graphics Graphics LP Problem Solving Graphic Method Method: What is the problem of LP? Yes, if and only if: All variables have a power of 1, and are added or subtracted (not shared or multiplied). Restrictions must be from the following forms (£, ³, or =, i.e., LP restrictions are always closed), and the purpose must be maximized or minimized. For example, the following issue is not LP: Max X, subject to X< 1. This very simple problem has no solution. Can I use the graphics method? Yes, if the number of decision variables is 1 or 2. Use Graph Paper. Graph each boundary one by one, by pretending that they are equality (pretend all £and ³, =) and then plot the lines. Straight line graph on coordinate system on graph paper. The coordinate system has two axes: a horizontal axis called an x-axis (abscissa), and a vertical axis, called a y(ordinate) axis. The axis is summed, usually from zero to the largest expected value for each variable. When each row is created, divide the region into 3 sections with respect to each row. To identify a region worthy of this particular constraint, select the dots on both sides of the line and plug the coordinates into the constraints. If it meets the conditions, this side is feasible; Otherwise, the other side is worth it. For equality constraints, only the dots on the line are feasible. Throw out the unworthy side. After all the restrictions are described, you must have a region that is not empty (convex), unless the problem is not feasible. Unfortunately, some of the area restrictions worth explaining in your textbook are wrong. See, for example, the numbers depicted on page 56. Almost all inequality must be turned into equality. Right? Create (at least) two iso value lines from the destination function, by setting the objective function to two different numbers. Graph of the resulting line. By moving these lines in parallel, you will find the optimal angle (extreme point), if it does exist. In general, if a viable region is in the first quadrant of the coordinate system (that is, if X1 and X2 ³ 0), then, for the problem of maximizing you move the objective function of the iso value parallel to itself away from the point of origin (0.0), while having at least a common point with a decent region. However, for the opposite minimization problem is true, that is, you move the goal of the iso value parallel to itself closer to the point of origin, while having at least a common point with a decent region. Common points provide optimal solutions. Classification of Eligible Points: : Decent points of any unfit territory of an empty LP can be classified as, interior, border, or node. As shown in the following figure: Point B in the two-dimensional image above, for example, is a border point a decent set because each small circle is centered on point B, however small, contains both multiple points in the set and several points outside the set. My bottom line is the interior point is because of the orange circles and all the smaller circles, as well as some of the larger ones; contains exclusive points in the set. Border point collection belongs to a set called boundary lines (segments), e.g. cd line segments. The intersection of the boundary line (segment) is called a node, if it is feasible to be called a angular point. In a three-dimensional space and higher circle becomes a ball, and a hyper-ball. Know that, LP constraints provide knots and angular points. Vertex is a 2-line intersection, or in general n-hyperplanes in LP problems with n-decision variables. The corner point is a vertex that is also feasible. Numerical Example: Carpenter Issues Maximize 5 X1 + 3 X2 Subject to: 2 X1 + X2 £40 X1 + 2 X2 £50 and both X1, X2 are not negative. Note: There are alternatives to the objective function approach of Iso values with problems that have few limitations and decent regions. First, find all the angular points, called extreme points. Then, evaluate objective functions at extreme points to find optimal values and optimal solutions. Obviously, carpenters have many sets of alternative actions to take. However, four extreme options are: Objective Function Value in Each Corner (i.e., Extreme) Corner Point Choice Decision Maker Coordinates Net Income Function Number of Tables or Chairs X1, X2 5 X1 + 3 X2 Create No Table or Chair 0.0 0 Create All Tables You Can 20, 0 100 Make All Your Seats Can Be 0.25 75 Make All Tables You Can 20 , 0 100 Make All The Seats You Can 0.25 75 Create , 20 110 Because the goal is to maximize, from the table above we read from the optimal value to 110, which can be obtained if the carpenter follows the optimal strategy X1 = 10, and X2 = 20. Note that in the case of carpenters, a decent area of convex provides a corner point with coordinates indicated in the table above. The main drawback of the graphic method is that it is limited to problem solving LP with 1 or 2 decision variables only. However, the main and useful conclusion that we easily observe from the graphical method, is as follows: If the linear program has an optimal solution tied, then one of the corner points provides an optimal solution. The evidence of this claim follows on from the results of the following two facts: Fact No. 1: A viable territory of any linear program is always a convex set. Due to all linear constraints, a viable region (F.R.) is a polygon. In addition, this polygon is a convex set. In the case of higher two-dimensional LP, the boundaries of F.R. are part of hyper-planes, and F.R. in this case is called polyhedra which is also convex. The convex set is the one if you choose two points from that, then all points on the straight line segment joined by these two points are also worth it. Evidence that the F.R. of linear programs is always convex sets is followed by contradictions. The following image illustrates an example for two types of sets: A non-convex and a bunch of convex. A decent set of regions in any linear program is called a polyhedron, it is called politope if it is restricted. Fact No. 2: The Iso value of the objective function of a linear program is always a linear function. This fact follows from the nature of objective function in each LP problem. The following figures describe two types of objective functions that are valued by a typical ISO. Combining the two facts above, it follows that, if a linear program has a region that is not empty and bound worthy, then the optimal solution is always one of the corner points. To address the shortcomings of graphic methods, we will utilize this useful and practical conclusion in developing algebraic methods that apply to multi-dimensional LP problems. Convection of viable areas for linear programs makes LP problems easy to solve. Due to the properties and linearity of this objective function, the solution has always been one of the nodes. In addition, due to the limited number of nodes, one must find all decent nodes, and then evaluate the objective functions of this node in search of optimal points. For nonlinear programs, the problem is much more difficult to solve, since the solution can be anywhere within a viable territory within the limits of a decent territory, or on a vertex. Fortunately, most Business optimization issues have linear constraints, which is why LP is so popular. There are over 400 computer packages on the market currently solving LP problems. Most of them are based on vertex searches, i.e. jumping from one vertex to a neighbor in search of the optimal point. You have noticed that, graphs of the system of inequality and/or equality are called viable areas. These two representations, graphics, and algebra are equivalent to each other, which means the coordinates of any point that satisfies the constraints are located in a decent area, and the coordinates of any point in a viable region meet all constraints. Numeric example: Find a constraint system that represents the following viable fields. In the image above, the coordinate system is shown in gray in the background. By establishing a viable territorial boundary line equation, we can verify that the following inequality systems do indeed represent the above viable areas: X1 ³ -1 X2 £1 X1 - X2 £1 Link Between LP and Equation System: Algebra Method As George Dantzig demonstrated, linear programming strictly theories and solutions of linear inequality systems. Basic solutions for the program is a solution to a system of equations consisting of constraints on binding positions. Not all basic solutions meet all the constraints of the problem. Those who meet all restrictions are called viable basic solutions. A viable basic solution corresponds to the extreme points of a viable territory. For example, for Carpenter Problems, one can calculate all the basic solutions, by taking two equations and solving them simultaneously and then, using the constraints of other equations to check the feasibility of this solution. If feasible, then this solution is a viable basic solution that provides decent regional corner point coordinates. To illustrate the procedure, consider Carpenter's limitations on binding positions (i.e., all with = sign: 2X1 + X2 = 40 X1 + 2X2 = 50 X1 = 0 X2 = 0 Here we have 4 equations with 2 unknowns. In terms of binomial coefficients, there is at most C42 = 4! / [2! (4-2)!] =6 basic solutions. Solving the six equation systems produced, we have: Six Basic Solutions with Four Basic Viable Solutions X1 X2 5X1 + 3X2 10 20 110 * 040 not feasible 20 0100 025 75 50 0feasible 000 Four of the basic solutions above are basic solutions that are worthy of satisfying all constraints, belonging to the node coordinates of a bound viable area. By plugging a viable basic solution into an objective function, we calculate the optimal value. Therefore, from the table above, we see that, the optimal solution is X1 = 10, X2 = 20, with an optimal value of $ 110. The above approach can be applied in solving higher dimensional LP problems provided that optimal solutions are limited. You might want to use the JavaScript Equation Solving System to check your compute. Further Reading: Arsham H, Link between Linear Systems of Equations, Matrix Inversions, and Linear Program Solver Routines, Journal of Mathematical Education in Science and Technology, 29(5), 764-769, 1998. Dantzig G., Linear Programming & Extensions, page 21, The Rand-Princeton U. Press, 1963. Extensions to Higher Dimension Graphics Methods are limited in solving the problem of LP having one or two variable decisions. However, it provides a clear illustration of where decent and unworthy areas are, as well as, knots. Having a visual understanding of the problem helps with a more rational thought process. For example, we learned that: If the LP program has an optimal solution tied, then the optimal solution is always one of the nodes of a decent region (corner point). What needs to be done is find all the intersection points (nodes) and then check which of all the nodes is feasible, providing the optimal solution. Using the concept of Analytical Geometry, we will overcome the limitations of this human vision. Algebra method designed to expand graphical methods to a multi-dimensional LP issue, as illustrated in the following numerical example. Numerical Example: The Transportation Issues Transport Model plays an important role in logistics and supply chain management to reduce costs and improve services. Therefore, the goal is to find the most cost-effective way to transport goods. Consider a model with 2 origins and 2 goals. Supply and demand at each origin (e.g., warehouse) of O1, O2 and destinations (e.g.; market) D1 and D2, along with unit transportation costs are summarized in the following table. Transport Unit Cost Matrix D1 D2 Supply O1 20 30 200 O2 10 40 100 Request 150 150 300 Let Xij show the number of shipments from source i to destination j. Lp formulation problems minimizing total transportation costs are: Min 20X11 + 30X12 + 10X21 + 40X22 subject to: X11 + X12 = 200 X21 + X22 = 100 X11 + X21 = 150 X12 + X22 = 150 all Xij ³ 0 Notification that the eligible area is restricted, therefore one can use algebraic methods. Because this transportation problem is a balanced (total supply = total demand) all constraints are in the form of equality. In addition, one of the obstacles is excessive (adding two obstacles and reducing each other, we get the remaining ones). Let's remove the last restriction. Therefore, the problem is reduced to: Min 20X11 + 30X12 + 10X21 + 40X22 subject to: X11 + X12 = 200 X21 + X22 = 100 X11 + X21 = 150 all these XIJ ³ 0 LP Problems cannot be solved by graphics method. However, the algebraic method has no restrictions on the dimensions of the LP. The obstacles are already in a binding position (equality). Note that we have an equality limit of m=3 with (four variables of non-negative decision implied). Therefore, of these four variables there are at most variables m = 3 with positive values and the rest must be at the level of zero. For example, by setting one of the variables in turn to zero, we get: X11X12X21X22 Total Transport Cost 0200 150 -50feasible 200-50 150 not worth 150 50 0100 8500 50 150 100 06500* Now by setting one (or more) variables to zero, easy to see, by checking the limits that all other solutions are not feasible. Thus, from the table above, we get the optimal strategy to be: X11 = 50, X12 = 150, X21 = 100, and X22 = 0, with a transportation cost of at least $ 6,500. You may want to run this issue using the Net.Exe Module in your WinQSB Package to check these results yourself. Note that in the above example, there is a limit of m = 3 (excluding non-negativity conditions), and a decision variable n = 4. Optimal delivery indicates that managers should not send shipments from one source to a single destination. Optimal solution consists of at most 3 variables of the same positive decision the number of constrained points. If the manager delivers goods from each to each destination, the results are not optimal. The above results, found in the example above, with the Algebra Method can be generalized, in the following main Economic results: Given that the LP has a bound viable area, with constraints m (excluding mark constraints such as non-negativity conditions) and n decision variables, if n>m then most m decision variables have positive values on optimal solutions and the rest (i.e., n-m) decision variables should be set at zero. These results persist if the problem has a unique optimal solution. The above results follow on from the fact that, using shadow prices indicates that the opportunity cost for variable decisions at zero level exceeds its contribution. Numeric Example: Find the optimal solution to the following production problems with products n = 3 and limits m = 1 (resources) : Maximize 3X1 + 2X2 + X3 Subject to: 4X1 + 2X2 + 3X3 £ 12 all variables Xi ³ 0 Because the decent area is limited, following the AlgebraIc Method by setting all the constrains on the binding position, we have the following equation system: 4X1 + 2X2 + 3X3 = 12 X1 = 0 X2 = 0 X3 = 0 Solution (basic) obtained , from the system of equations is summarized in the following table. X1 X2 X3 Total Net Income 0044 060 12* 3009 0000 Thus, the optimal strategy is X1 = 0,X2 = 6, X3 = 0, with a maximum net profit of $12. The results in the table above are consistent with the implementation of the main economic results above. In other words, the optimal solution can be found by setting at least n - m = 3 - 1 = 2 decision variables to zero: For large-scale LP problems with many constraints, the Algebra Method involves solving many systems of linear equations. When lp problems have many variables and constraints, solving many systems of equations by hand can become very tedious. Even for a very large-scale problem it is an impossible task. Therefore, we need computers to do computing for us. One of the algorithmic and computerized approaches is the Simplex Method, which is an efficient and effective implementation of algebraic methods. There are over 400 LP breakers, all of which use simplex methods, including your software. After solving lp problems based on computer packages, optimal solutions provide valuable information, such as sensitivity analysis ranges. You may want to use Solving Systems of Equations JavaScript for up to 3-decision variables of LP problems to check your computing with algebraic methods. For details and other numerical examples, visit the Algorithm Solutions for LP Model website. How to Solve Linear Equation System by LP Solver? In the Algebra Method to solve the LP problem, we have to solve several systems of equations. There's a connection between the LP solver and the equation-solving system. Suppose we a system of enormous equations that wanted to solve and package the LP solver but we still don't have a solver computer package for the equation system available. The question is How to use an LP solver to find a solution to the equation system? The following steps describe the process of solving a linear equation system using the available LP solvers. 1- Since some LP solvers require all non-negative variables, replace for each variable Xi = Yi - T everywhere. 2- Create dummy purposes, such as minimizing T. 3- Constraints of LP problems are equations in the system after the substitution outlined in step 1. Numerical Example: Solve the following equation system 2X1 + X2 = 3 X1 -X2 = 3 Because winqsb packages accept LP in various formats (unlike Lindo), solving this problem by WinQSB is very easy: First, create an LP with dummy purpose functions such as Max X1, subject to 2X1 + X2 = 3, X1 - X2 = 3, and X1 and X2 are not limited in mark. Then, insert this LP into the LP/ILP module to get the solution. The resulting solution is X1= 2, X2= -1, which can be easily verified with substitutions. However, if you use any LP solver that requires by default (for example, Lindo) that all variables are non-negative, you need to do some preparation to meet these requirements: The first substitute for X1 = Y1 - T and X2 = Y2 - T in both equations. We also need objective functions. Let's have the objective function of the doll such as minimizing T. The result is the following LP: Min T Subject to: 2Y1 + Y2 - 3T = 3, Y1 - Y2 = 3. Using any LP solver, such as Lindo, we found the optimal solution to be Y1=3,Y2=0,T=1. Now, replace this LP solution into both transformations X1=Y1 – T and X2=Y2 – T. This provides numeric values for our original variables. Therefore, the solution to the system equation is X1 = 3 - 1 = 2, X2 = 0 - 1 = -1, which can be easily verified with substitutions. Double Problem: Construction and Its Meaning Related to each LP (primal) problem is a companion problem called double. The following classification of decision variable limitations is useful and easy to remember in double construction. --- Dual Problem Construction Objectives: Max Limit Type (e.g. Profit) : £Reasonable constraints = limited restrictions ³ Unusual konst. Destination: Min (e.g. Cost) Type of limitation: ³ Reasonable constraints = Const Limited. Unusual £const. Variable type: ³ 0 reasonable conditions ... not limited in the £0 sign of unusual conditions --- One-to-one correspondence between the type of constraint and the type of variable there using the classification of restrictions and these variables for primate problems Double. Double Problem Construction: - If the primate is is problem, then the double is a matter of minimization (and vice versa). - Use the variable type of one problem to find the constraint type of another problem. - Use the constraint type of one problem to find the variable type of the other problem. - RHS elements of one problem become the coefficient of objective functions of another problem (and vice versa). - The matrix coefficient of the constraints of one problem is the transpose of the matrix coefficient of constraints for another problem. That is, matrix rows become columns and vice versa. You can check your double construction rules by using your WinQSB package. Numerical Example: Consider the following primate issues: min x1-2x2 is subject to: x1+x2 ³2, x1-x2 £-1, x2 ³ 3, and x1, x2 ³ 0. Following the above construction rules, the double problem is: max 2u1 - u2 + 3u3 subject to: u1 + u2 £1, u1 - u2 + u3 £-2, u1 ³ 0, u2 £0, and u3 ³ 0 Double Carpenter Problem: Maximize 5X1 + 3X2 Subject to: 2X1 + X2 £40 X1 + 2X 2 lb 50 X1 ³ 0 X2 ³ 0 Dualnya are: Minimize 40U1 + 50 : 2U1 + U2 ³ 5 U1 + 2U2 ³ 3 U1 ³ 0 U2 ³ 0 Applications : One can use duality in a variety of applications including: - It may be more efficient to break doubles than primates. - The double solution provides important economic interpretations such as the price of shadows, that is, the marginal value of rhs elements. Shadow prices are defined, historically as an increase in the value of objective functions per unit increase on the right side, since the problem is often included in the form of an increase in profit maximization (meaning an increase). The shadow price may not be the market price. A shadow price is, for example, the value of a resource under the shadow of your business activity. Sensitivity analysis, that is, the analysis of the effect of small variations in system parameters on output steps can be learned by calculating the derivatives of the output measures with respect to the parameters. - If the constraints in one problem are non-binding (i.e., the LHS value agrees with the RHS value), then the related variable in the other problem is zero. If the decision variable in one problem is not zero, then the associated constraints in the other issue are binding. These results are known as Complementarily Slackness Conditions (CSC). - Get the RHS sensitivity range from one issue from the cost coefficient sensitivity range in another, and vice versa. These results imply the only possible combination of primate and dual properties as shown in the following table: Possible Primal Combinations and Primal ProblemCondition Double Properties ImplyIng Decent Double Problems; bound purpose« Worthy; Worthy bound goals; unlimited purpose® Infeasible¬ Worthy; Unlimited Destinations Not Feasible« Multiple Multifeasible Solutions« Solution Slump solutions« Multiple solutions Notice that almost all LP solvers produce sensitivity ranges for the last two cases; however this range is not valid. For this reason, you should ensure that the solution is unique, and not degenerate in analyzing and applying sensitivity ranges. Further Reading: Arsham H., Artificial Free Simplex Algorithm for General LP Models, Mathematical and Computer Modeling, Vol. 25, No.1, 107-123, 1997. Benjamin A., Reasonable Rules for remembering Duals_ S-O-B, SIAM Review, Vol. 37, No.1, 85-87, 1995. Chambers R., Applied Production Analysis: Dual Approaches, Cambridge University Press, 1988. Double Problem Carpenter Problem and Its Interpretation In this section we will build a Double Problem Carpenter Problem and provide an economical interpretation of it. In input parameters Carpenter's Problem uncontrollable is as follows: Uncontrolled Input Table Chairs Available Labor 2 1 40 Raw Materials 1 2 50 Net Income 5 3 and its LP formulations as: Maximize 5 X1 + 3 X2 Subject to: 2 X1 + X2 £40 labor constraints X1 + 2 X2 £50 material Where X1 and X2 are the number of tables and seats to be made. Suppose Carpenter wants to buy insurance for his net income. Let U1 = the dollar amount paid to Carpenter for each lost working hour (due to illness, for example), and U2 = the dollar amount paid to Carpenter for each unit of raw materials lost (due to fire, for example). Insurance officials tried to minimize the total amount of $(40U1 + 50U2) paid to Carpenter by the insurance company. However, Carpenter will set constraints (i.e. conditions) by insisting that the insurance company cover all of its losses which is its net income because it cannot make the product. Therefore, the insurance company's problem is: Minimize 40 U1 + 50 U2 Subject to: 2U1 + 1U2 ³ 5 Net Income from table 1U1 + 2U2 ³ 3 Net Income from seats and U1, U2 non-negative. Applying this problem to your computer package indicates that the optimal solution is U1=7/3 AND U2$1/3 with an optimal value of $110 (the amount Carpenter expects to receive). This ensures that Carpenter can manage his life smoothly. The only cost is the premium that the insurance company will charge. Shadow Price Size Units: Note that rhs shadow price unit size is a unit of primate destination size divided by RHS unit size. For example for Carpenters problems, U1 = [$/week] / hour/week] = $/hour. Thus U1 = 7/3 $ / hour, and U2 = 1/3 $ / unit of raw materials. As you can see, the insurance company's problems are closely related to the original problem. In OR/MS/DS modeling terminology, the original problem is called Primal Problem while the related issue is called Its Double Problem. Carpenter's problem and his double problem, the Optimal Value for both problems is always the same. This fact is referred to as the Balance (derived from the theory of complementarity, economic system balance, and efficiency in the Pareto sense) between Primal and The Double Problem. Therefore, there is no duality gap in linear programming. Dual solutions provide important economic interpretations such as the marginal value of RHS elements. Double solution elements are known as Lagrangia multipliers because they provide (strictly) tied to the optimal value of primates, and vice versa. For example, given Carpenter's problem, a double solution could be used to find a lower tight limit for optimal values, as follows. After turning the boundaries of inequality into a form of £, multiply each limitation with the appropriate double solution and then add, we get: 7/3 [2X1 + X2 £ 40] 1/3 [X1 + 2X2 £ 50] __ 5X1 + 3X2 £ 110 Note that the result on the left side is an objective function of the primate problem, and the lower limit for this for it is a tight one, since the optimal value is 110. Managerial Roundoff errors you have to be careful every time you round up the shadow price value. For example, the shadow price of resource constraints in the above problem is 8/3; therefore, if you want to buy more of these resources, you should not pay more than $2.66. Whenever you want to sell this resource unit, you shouldn't sell it for under $2.67. Similar errors can occur whenever you round a border on a sensitivity range. One must be careful because the upper and lower limits must be rounded down and up, respectively. Shadow Price Calculation You know now that shadow pricing is the solution to the double problem. Here's a numerical example. Calculate the shadow price for both resources in the following LP problem: Max -X1 + 2X2 S.T. X1 + X2 £5 X1 + 2X2 £6 and both X1, X2 non-negative Solutions for this primate problem (using, for example, graphics method) is X1 = 0, X2 = 3, with the remaining S1 = 2 first resources, while the second resource is fully used, S2 = 0. Shadow pricing is the solution to a double problem: Min 5U1 + 6U2 S.T. U1 + U2 ³ -1 U1 + 2U2 ³ 2 and U1 and U2 are non-negative. The solution to the double problem (using, for example, a graphical method) is U1=0,U2=1 which is the shadow price for the first and second resources, respectively. Note that whenever the slack/surplus of the constraints is non-zero, the shadow price associated with the RHS of that limit is always zero; however, the opposite statement may not be withheld. In this numerical example S1 = 2 (i.e. the sagging value rhs1 of primates), which is not zero; therefore U1 equals zero as expected. Consider the following issue: Max X1 + X2 for: X1 £1 X2 £1 X1 + X2 £2 all variable decisions ³ 0. By using your computer package, you can verify that the shadow price for the third resource is zero, while there is no remaining of that resource on the optimal solution X1 =1, X2=1. RHS Value Change Behavior from Optimal Value To study the change of direction in the optimal value with respect to rhs changes (without the existing excessive restrictions, and all RHS ³0), we distinguish the following two cases: Case I: Maxization Problem For limitation £ : The change is in the same direction. That is, increasing the RHS value does not decrease the optimal value. This increases or remains the same depending on whether the limit is binding or non-binding. For limitations ³: Change is in the opposite direction. That is, increasing the RHS value does not increase the optimal value. It is reduced or remains the same depending on whether the limit is binding or non-binding. For = restrictions: Changes can be in either direction (see the More-for-less section). Case II: Minimalization Problem For £type restrictions: Changes are in the opposite direction. That is, increasing the RHS value does not increase the optimal value (rather, it decreases or has no change depending on whether the limit is a binding or non-binding constraint). For type ³ limits: Changes are in the same direction. That is, increasing the RHS value does not reduce the optimal value (rather, it increases or has no change depending on whether the limit is a binding or non-binding constraint). For = restrictions: Changes can be in either direction (see the More-for-less section). Dealing with Uncertainty and Modeling Business Environment Scenarios is often unpredictable and uncertain due to factors such as economic changes, government regulations, dependence on subcontractors and vendors, etc. Managers often find themselves in a dynamic and quiet environment where even short-range plans have to be constantly adjusted and adjusted gradually. All this requires a change-oriented mentality to overcome uncertainty. Remember that surprise is not an element of a strong decision. Managers use mathematical and computational construction (models) for a variety of settings and objectives, often to gain insight into the possible outcomes of one or more courses of action. This may concern financial investment, choice (whether/how much) to insure, industry practices, and environmental impact. The use of flawed models by the presence of unavoidable uncertainties, which appear at different stages; in the construction and reinforcing of the model itself, and in its use. Its use is often the culprit. Any solution to the problem based on certain parameters that are assumed to be corrected. Sensitivity analysis is a collection of to study and determine how sensitive the solution is to changing assumptions. Other names for such activities are stability analysis, what-if analysis, scenario modeling, start analysis, specificity analysis, uncertainty analysis, computational and numerical instability, functional instability, tolerance analysis, post-optimality analysis, permissible increase and decrease, and many other similar phrases that reflect the importance of this modeling stage. Numerical Example: Consider the following issue: Max 6X1 + 4.01X2 subject to: X1 + 2X2 £16 3X1 + 2X2 £24 all decision variables ³ 0. The optimal solution is (X1 = 4, X2 = 6), but with a slight change in objective functions, one may get a completely different optimal solution. For example, if we change it to 6X1 + 3.99X2, then the optimal solution is (X1 = 8, X2 = 0). That is, a decrease in the second coefficient by 0.5%, the solution changes drastically! Therefore, the optimal solution is unstable with respect to these input parameters. Sensitivity analysis is not a typical term used in econometrics for methods of investigating solution responses to interference in parameters. In econometrics, the process of changing the value of parameters in the model, to see its individual impact on performance measures, is called comparative static or comparative dynamics, depending on the type of model being considered. Uncertainty in the model can have different origins in different decision issues. This may be due to incomplete information, or surprising fluctuations in the problem, or unpredictable changes in the future. Current approaches to dealing with uncertainties include: Scenario Analysis: In this approach one assumes a scenario (e.g. a specific combination of possible values of an uncertain parameter) and solves the problem for each. By solving problems repeatedly for different scenarios and studying the solutions obtained, the manager observes sensitivity and heuristically decides the forecast, which is subjective. Worst Analysis: This technique tries to take into account to put safety margins into trouble in the planning stage. Monte-Carlo Approach: The Stochastic model assumes that uncertainty is known by its statistical distribution. Sensitivity analysis vs. Stochastic Programming: Sensitivity analysis (SA) and Stochastic Programming (SP) formulations are two key approaches used to address uncertainty. SA is a post-optimality procedure without the power of influencing the solution. It is used to investigate the effects of uncertainty on model recommendations. The SP formulation, on the other hand, introduces probabilistic information about problem data, albeit with the first moments (i.e. values that from the distribution of objective functions with respect to uncertainty. It ignores ignoring decision-making risk assessment, characterized by variance, or variation coefficients. One can overcome uncertainty in a more deterministic way. This approach is called various names such as scenario modeling, deterministic modeling, sensitivity analysis, procedure range, and stability analysis. The idea is to subjectively come up with a list of higher-level uncertainty rankings that may have a greater impact on the final mapping results. This is done before zooming in on the details of a particular scenario or model. For example, the parameters of the problem, and the uncontrolled factors indicated in the numbers above for Carpenter's problem, require a complete sensitivity analysis to allow the carpenter to control his business. Managerial Roundoff Error: You have to be very careful every time you round the limit value on the sensitivity range. For the upper and lower borders to be valid, they must be rounded down and up. For a regional development sensitivity analysis that lets you analyze all types of changes, including dependent, independent changes, and some changes in RHS values and LP cost coefficients visit the Development of Common Sensitivity Area site. Further Reading: Arsham H., Algorithm for sensitivity information in discrete event system simulation, Simulation Practice and Theory, 6(1), 1-22, 1998. Arsham H., General LP model perturbation analysis: Integrated approach to sensitivity, parametric, tolerance, and more-for-less analysis, Mathematical and Computer Modeling, 13(3), 79-102, 1990. Kouvelis P., and G. Yu, Strong Discrete Optimization and Application, Kluwer Academic Publisher, 1997. Provide a comprehensive discussion about the motivation for the source of uncertainty in optimization. Computational Sensitivity Range for Small Size Issues To calculate the sensitivity range for LP Issues with at most 2 decision variables and/or at most 2 limitations, you may want to try the following easy-to-use approaches. The only limitation is that no equality constraints are allowed. Having equality limits is a case of degeneration, since any equality constraints, for example, X1 + X2 = 1, means two simultaneous constraints: X1 + X2 £ 1 and X1 + X2 ³ 1. The number of binding constraints in such cases would be more than the number of decision variables. This is known as a degenerate situation where the usual sensitivity analysis may not be valid. Cost Sensitivity Range for LP Issues with two Decision Variables Refers to Carpenter's Problem, changing the profit on each product changing the slope of the iso value destination function. For small changes, the optimal remains at the same extreme point. For larger changes, the optimal solution moves to another point. Then we have to modify the formation solving new problems. Tje Tje is to find the range for each c(j) cost coefficient, from the Xj variable, so that the current optimal solution, that is, the current extreme point (corner point), remains optimal. For 2 dimensional LP problems, you may want to try the following approach to find out the amount of increase/decrease of one of the objective function coefficients (also known as cost coefficients. Historically during World War II, the first LP problem was the problem of cost minimization) to maintain the validity of the current optimal solution. The only necessary condition for this approach is that no equality constraints are allowed, since this leads to cases of degeneration, in which the usual sensitivity analysis may be invalid. Step 1: Consider the only two binding constraints on the current optimal solution. If there are more than two binding constraints, then this is a case of degeneration, in which the usual sensitivity analysis may not be valid. Step 2: Perturb jth cost coefficient with cj parameter (this is an unknown number of changes). Step 3: Create one equation that corresponds to each binding constraint as follows: (Impaired Cj Cost)/ Xj coefficient within constraints = Coefficient of another variable in objective function / coefficient of such constraint variable. Step 4: The number of increases allowed is cj's smallest positive, while the allowed decrease is cj negative largest, obtained in Step 3. Note that if there is no positive cj (negative), then the amount of increase (decrease) is unlimited. Warning: Remember that you must not divide by zero. The practice of dividing by zero is a common error found in some textbooks. For example, in Introduction to Management Science, Taylor III, B., Prentice Hall, author, unfortunately divides by zero, in Module A: Simplex Solution Method, pp. A26-A27, where it calculates the column ratio required in the simplex method. For more information about this, and other common mistakes visit Saga's website for zero & confusion with numbers. Here's a question for you: Which of the following is true and why? a) any number divided by zero is not defined; b) zero divided by any number is zero; or c) any number divided by itself is 1. Finding Cost Sensitivity Range by Grapikal Method: It is a commonly held belief that one can calculate the cost sensitivity range by locking down the slope (disturbed) of the objective function (iso value) by the slope of both lines resulting from binding constraints. This graphical slope-based method for calculating sensitivity ranges is described in popular textbooks, such as Anderson et al., (2007), Lawrence and Pasternack (2002), and Taylor (2010). Unfortunately this Warnings should be given that their approach is not common and works if and only if the coefficient does not change the mark. In LP with 2 2 and the constraints of inequality, suppose we have a unique optimum and do not slump at the intersection of two lines, as shown in the following figure. Then, the range of objective coefficients that this solution remains optimal is given by the slope of the two lines. Here is a counterexample. This indicates that one must be careful to state that the coefficient does not change the mark. Counterexample: Maximixe 5X1 + 3X2 X1 + X2 £2, X1 - X2 £0, X1 ³ 0, X2 ³ 0. Carpenter Problem: Maximize 5X1 + 3X2

Subject to: 2X1 + X2 £40 X1 + 2X2 £50 X1 ³ 0 X2 ³ 0 Computational increase/decrease allowed at C1=5: Binding constraints are first and second. Perturbing this cost coefficient with c1, we have 5 + c1. In step 3, we have: (5 + c1)/2 = 3/1, for the first limit, and (5 + c1)/1 = 3/2 for the second limit. Solving both of these equations, we have: c1 = 1 and c1 = -3.5. The allowed increase is 1, while the allowed decrease is 1.5. To the extent that the first cost coefficient of C1 remains in intervals [5 - 3.5, 5 + 1] = [1.5, 6], the current optimal solution remains. Similarly for the second cost coefficient C2 = 3, we have a sensitivity range [2,5, 10]. For another example, consider the previous problem: Maximize 5X1 + 3X2 Subject to: X1 + X2 £ 2 X1 - X2 £ 0 X1 ³ 0 X2 ³ 0 Computational increase / decrease allowed at C1 = 5: Binding constraints are the first and the second. Perturbing this cost coefficient with c1, we have 5 + c1. In step 3, we have: (5 + c1)/1 = 3/1, for the first limit and (5 + c1)/1 = 3/(-1) for the second limit. Solving both of these equations, we have: c1 = -2 and c1 = -8. The allowable decrease is 2, while the allowable increase is unlimited. To the extent that the first cost coefficient of C1 remains in the interval [5 - 2, 5 + ¥] = [3, ¥], the current optimal solution remains optimal. Similarly, for the second cost coefficient C2 = 3 we have a sensitivity range [3 - 8, 3 + 2] = [-5, 5]. For a regional development sensitivity analysis that lets you analyze all types of changes, including dependent, independent changes, and some changes in RHS values and LP cost coefficients visit the Development of Common Sensitivity Area site. RHS Sensitivity Range for LP Issues with At Most Two Constraints Refers to the Carpenter Problem, for small changes in both resources, the optimal strategy (i.e. making mixed products) remains valid. For greater change, this optimal strategy moves and Carpenter must make all the tables or chairs he can. This is a drastic change in strategy; therefore, we must revise the formulation and solve the new problem. Apart from the information required above, we are also interested to know how much Carpenter can sell (or buy) any resource at a reasonable price (or cost). That is, how far away is increase or subtract RHS(i) to remain i while maintaining the validity of the current RHS(i) shadow price? That is, how far can we increase or reduce the RHS(i) to remain i while maintaining the current optimal solution to the double problem? Historically, shadow prices have been defined as an increase in the value of objective functions per unit increase on the right side, as the problem is often included in the form of an increase in profit maximizer (meaning an increase). Also, be aware that for any RHS, the shadow price (also known as its marginal value), is the number of changes in the proportion of optimal values for a single unit change for a given RHS. However, in some cases it is not allowed to change the RHS that much. The sensitivity range for rhs provides values where shadow prices have economic significance and remain unchanged. How far can we increase or reduce each individual RHS to maintain the validity of the shadow price? The question is equivalent to asking what is the sensitivity range for cost coefficients in double problems. Carpenter's dual problems are: Minimize 40U1 + 50U2 Subject to: 2U1 + U2 ³ 5 U1 + 2U2 ³ 3 U1 ³ 0 U2 ³ 0 Optimal solutions are U1 = 7/3 and U2 = 1/3 (which is the price of shadow). Carpenter Problem: Maximize 5X1 + 3X2 Subject to: 2X1 + X2 £40 X1 + 2X2 £50 X1 ³ 0 X2 ³ 0 Computing Range for RHS1: The first two constraints bind, therefore: (40 + r1)/2 = 50/1, and (40 + r1) / 1 = 50/ 2. Solving both of these equations gives: r1 = 60 and r1 = -15. Therefore, the sensitivity range for the first RHS in carpenter problems is: [40-15, 40 + 60] = [25, 100]. Similarly, for the second RHS, we gained: [50 - 30, 50 + 30] = [20, 80]. For a regional development sensitivity analysis that lets you analyze all types of changes, including dependent, independent changes, and some changes in RHS values and LP cost coefficients visit the Development of Common Sensitivity Area site. Further Reading: Lawrence J., Jr., and B. Pasternack, Applied Management Sciences: Modeling, Spreadsheet Analysis, and Communication for Decision Making, John Wiley and Sons, 2002. Anderson D., Sweeney D., and Williams T., Introduction to Management Science, Publisher West, 2007. by Taylor III, B., Introduction to Management Science, Prentice Hall, 2006. &Marginal Analysis; Priority Factors The main application of sensitivity analysis information for decision makers is Marginal Analysis and Priority Factors: Marginal Analysis: Marginal analysis is a concept used, in microeconomics where marginal changes in some parameters may be of interest to decision makers. Marginal change is a very small increase or subtraction ration to the total number of parameters. Marginal analysis is the analysis of between such changes with respect to performance measures. Examples of marginal analysis are: marginal costs; marginal income; marginal products; marginal level of substitution; marginal tendencies to save, and so on. In optimization, marginal analysis is used primarily to issue various parameter changes and their impact on optimal values. Sensitivity analysis, that is, the analysis of the effect of small variations in system parameters on output steps can be learned by calculating the derivatives of the output measures with respect to the parameters. The decision makers reflect on what factors are important and have a big impact on the outcome of the decision. Marginal analysis aims to identify important factors (i.e., parameters) and rank them according to their impact on optimal values. One can obtain marginal values by evaluating the first derivative of a performance measure with respect to parameters with a specific value. Priority Factors Based on Sensitivity Range: Consider Carpenter's Problem: Maximize 5X1 + 3X2 Subject to: 2X1 + X2 £40 X1 + 2X2 £50 X1 ³ 0 While computational sensitivity ranges are valid for one change at a time and do not have to be for simultaneous changes, they provide useful information for uncontrolled factor priority. The following figure provides a comparative chart for cost coefficient priority purposes: The following figure describes the shadow price as the slope (i.e., marginal value) of a linear function that measures the number of changes in optimal values as a result of each change in RHS1, provided that the change is within the RHS1 sensitivity range. This function can also be used to solve the reverse problem, i.e. what RHS1 value should reach a certain optimal value. What is the Rule 100% (sensitivity area) The sensitivity range presented in the previous section is a one-change-at-one type of what-if analysis. Consider Carpenter's problem; suppose we want to find the simultaneous increase allowed in the RHS (r1, r2 ³ 0). There is an easy method to apply here known as the 100% rule which says that the shadow price remains unchanged if the following sufficient conditions apply: r1/60 + r2/30 £1, 0 £r1 £60, 0 £r2 £30. Above, 60 and 30 are the allowed increases for RHS, based on the application of regular sensitivity analysis. That is, each time the first and second RHS increases r1 and r2 respectively, as long as this inequality persists, the shadow price for RHS values remains unchanged. Note that this is a sufficient condition, because if the above conditions are violated, then the shadow price may change or still remain the same. The term 100% rule becomes clear when you see that on the left side of the condition above each is a non-negative number less than one, one, represented as a percentage of the allowed changes. The total number of such changes should not exceed 100%. Applying the 100% rule to three other possible changes to the RHS, we have: r1/(-15) + r2/(-30) £1, -15 £r1 £0, -30 £r2 £0. r1/(60) + r2/(-30) £1, 0 £r1 £60, -30 £r2 £0. r1/(-15) + r2/(30) £1, -15 £r1 £0, 0 £r2 £30. The following figure illustrates the sensitivity area for both RHS values as a result of applying 100% rules to Carpenter's problems. From a geometric point of view, note that polyhedrals with vertices (60, 0), (0, 30), (-15, 0), and (0.-30) above Image are only a subset of the greater sensitivity region for changes in both RHS values. Similar results can be obtained for simultaneous changes in cost coefficients. For example, we want to find a simultaneous permissible C1 decrease and an increase in C2. That is, the amount of change in both coefficients costs c1 £0 and c2 ³ 0. The 100% rule states that the current basis remains optimal provided: c1/(-3.5) + c2/7 £1, -3.5 £c1 £0, 0 £c2£7. Where 3,5 and 7 are the permissible decreases and increases for the cost coefficients of C1 and C2, respectively, which we found earlier with the application of regular sensitivity analysis. The image above also illustrates all other possibilities for increasing and reducing both cost coefficient values as a result of implementing 100% rules, while maintaining the current optimal solution to Carpenter's problems. As another numerical example, consider the following issue: Maximize 5X1 + 3X2 Subject to: X1 + X2 £2 X1 - X2 £ 0 X1 ³ 0 X2 ³ 0 You may remember that we have calculated the sensitivity range one-change-at-one for this issue in the Computational Sensitivity Range section. The sensitivity range for the first cost coefficient is [5 - 2, 5 + ¥] = [3, ¥], while, for the coefficient of the second charge is [3 - 8, 3 + 2] = [-5, 5]. You should be able to reproduce a number similar to the above that describes all other possibilities for increasing/subtracting both cost coefficient values as a result of applying the 100% rule, while maintaining the current optimal solution to the problem. The application of the 100% rule as presented here is general in size and can be extended to large size LP issues. As the size of the problem becomes larger, this type of sensitivity area becomes smaller and therefore less useful for managers. There are stronger (providing necessary and sufficient conditions) and useful techniques to managers for simultaneous changes that depend (or independently) within the parameters. For the development of sensitivity analysis areas lets you analyze all types of changes, including dependent, independent, and some changes in RHS values and LP cost coefficients visit the Development of Common Sensitivity Area site. Add Process New Limits: Insert the current optimal solution into the newly added constraints. If the constraints are not violated, the new constraints do not affect the optimal solution. Otherwise, a new problem must be solved to get a new optimal solution. Remove Process Restrictions: Determine whether the limit is binding limit (i.e. active, important) by finding whether the slack/surplus value is zero. If binding, removal is very likely to change the current optimal solution. Remove the restrictions and solve the problem again. Otherwise, (if not binding constraints) the removal will not affect the optimal solution. Changing Restrictions Suppose we replace the restrictions with new constraints. What is the effect of this exchange? Process: Determine whether the old limit is a binding constraint (i.e. active, important) by finding out if the value of slack / surplus is zero. If binding, replacement is very likely to affect the current optimal solution. Change the restrictions and resolve the issue. Otherwise, (if not binding constraints) determine whether the current solution meets the new constraints. If so, then this exchange will not affect the optimal solution. Otherwise, (if the current solution doesn't meet the new constraints) replace the old one with the new one and solve the problem. Boundary Coefficient Changes Any change to the boundary coefficient may cause significant changes to the nominal (original) issue. Any such changes fall logically in sensitivity analysis; however, these are not changes that can be analyzed using information generated by optimal solutions. Such changes are best addressed by solving new modified problems. Adding Variables (for example, Introducing new products) New variable coefficients in objective functions, and resource shadow pricing provide information about resource marginal values and know resource needs corresponding to new variables, decisions can be made, for example, if a new product is profitable or not. Process: Calculate what will be your loss if you produce a new product using a shadow price value (that is, what goes into producing a new product). Then compare it to its net profit. If the profit is less than or equal to the amount of loss then DO NOT produce a new product. Otherwise, it is profitable to produce a new product. To find out the production level of a new product solves a new problem. Removing Variables (for example, Ending products) Process: If for the current optimal solution, the value of the deleted variable is zero, optimal solution is still optimal without including these variables. Otherwise, remove variables from objective functions and constraints, then solve new problems. Optimal Resource Allocation Issues Because resources are always scarce, managers are worried about optimal resource allocation issues. You will recall in the formulation of the Carpenter Problem that we treat both resources as parameters, that is, as given the fixed numerical value: Maximize 5 X1 + 3 X2 Subject to: 2 X1 + X2 £40 labor limit X1 + 2 X2 £50 material constraints and both X1, X2 are not negativity. We usually classify restrictions as resource constraints or production types. It is a fact that in most maximalization issues, resource constraints are a natural part of the problem, while in the matter of minimizing production constraints is the most important part of the problem. Suppose we want to find the best allocation of labor resources for Carpenter. In other words, what's the best number of hours Carpenter should use for his business? Let the number of hours allocated is R, which we want to use in determining its optimal value. Therefore, the mathematical model is finding the R1 in such a way that: Maximize 5 X1 + 3 X2 Subject to: 2 X1 + X2 £ R1 labor constraints X1 + 2 X2 £ 50 material constraints and all variables X1, X2, and R1 nonnegative. We now treat R1 not as a parameter but as a decision variable. That is, the maximalization of more than three variables; X1, X2, and R1: Maximize 5 X1 + 3 X2 Subject to: 2 X1 + X2 - R1 £0 labor limit X1 + 2 X2 £50 material constraints and all variables X1, X2, and R1 are not negative. Using your LP software, the optimal solution is X1 = 50, X2 = 0, with an optimal labor allocation of R1 = 100 hours. It carries an optimal value of $250. Note that the optimal resource allocation value is always the same as the upper limit on the RHS1 sensitivity range generated by your software. The allowed increase in the number of hours is 100 – 40 = 60 hours which brings an additional 250 – 110 = 140. We can even obtain shadow pricing for these resources using this information. Shadow pricing is the optimal rate of change in value with respect to changes in RHS. Therefore (250 - 110)/(100 - 40) = 140/60 = 7/3, which is the rhs1 shadow price as we found with other methods in the previous section. Determination of Net Income of The Least Products In most business arrangements of price takers, net profit is an uncontrolled factor. Managers are interested to know the net profit at least for the products that make it profitable to produce at all. You may remember that in carpenter's matter we treat net profit ($5, and $ as uncontrolled input, that is, the values determined by the market: Maximize 5 X1 + 3 X2 Subject to: 2 X1 + X2 £40 labor X1 + 2 X2 £50 material constraints And second X1, X2 nonnegative. It has an optimal strategy of X1 =10, X2 = 20, with an optimal value of $110. Suppose Carpenter wants to know the least value for the first coefficient in an objective function, which is currently $5, to make it still profitable to produce the first product (that is, the table). Suppose the net profit is at least c1 dollars; therefore, the problem is finding c1 in such a way that: Maximize c1 X1 + 3 X2 Subject to: 2 X1 + X2 £ 40 labor constraints X1 + 2 X2 £ 50 material constraints And all variables X1, X2, c1 are not pressurized. Carpenter's Dual Problem Problem now: Minimize 40 U1+50 U2 Subject to: 2U1 +1U1³ c1 Net profit from table 1U1 + 2U2 ³3 Net profit from seats And U1, U2, c1 are nonnegative. We now treat c1's net profit as a decision variable. Minimization of more than three variables; X1, X2, and c1: Minimum 40 U1 + 50 U2 Subject to: 2U1 + 1U1 - c1 ³ 0 1U1 + 2U2 ³ 3 And U1, U2, c1 are not negative. Applying this problem to your computer package indicates that the optimal solution is U1 = 7/3$, U2 = 1/3$, and c1 = 1.5 5 DOLLARS. There is an alternative solution to the limit value of this sensitivity range for the cost coefficient. The solution corresponds to the lower limit in the cost coefficient sensitivity analysis range previously calculated for the Carpenter Problem. The net profit at least is always the same as the lower limit on the cost coefficient sensitivity range generated by your software. Min Max and Max Min computing in Single-Run Suppose we want to find the worst of several objective function values defined on a series of common constraints in a single-run computer implementation. As an application, for example in Carpenter Problems, without general loss, we have three markets with objective functions of 5X1 + 3X2, 7X1 + 2X2, and 4X1 + 4X2 respectively. Carpenters are interested in knowing the worst markets. That is, the solution of the following problem: Problem Min Max: Min Max {5X1 + 3X2, 7X1 + 2X2, 4X1 + 4X2} Subject to: 2 X1 + X2 £40 X1 + 2 X2 £50 and second X1, X2 is not negative. Min Max problem equivalent: Max y Subject to: y £5x1 + 3X2 y £7X1 + 2X2 y £4X1 + 4X2 2X1 + X2 £40 X1 + 2X2 £50 And all variables X1, X2, y, nonnegative. If you bring all the variables to the left side of the constraints and implement this problem on your computer package, the optimal solution is X1 = 10, X2 = 20, y = $110. This means the first and second markets are the worst (due to the first and second binding constraints) bringing only a net profit of $110. Similarly, one can solve a maximum of min of several objective functions in a single run. Eligibility Issues: Goal Search Indicator In most business applications, managers want to achieve specific goals, while satisfying Model. Users don't really want to optimize anything so there's no reason to define objective functions. This type of problem is usually called a feasibility issue. Although some decision makers will prefer the optimal. However, in most practical situations, decision makers aim to satisfy or make incremental changes rather than optimize. This is so, because the human mind has a bound rationality and therefore cannot understand all the alternatives. In an incremental approach to decision making. managers take only small steps, or incremental movements, away from existing systems. This is usually achieved with local searches to find a good enough solution. This issue is referred to as a satisfactory problem, a feasibility issue, or a goal-seeking issue. Therefore, the goal is to achieve a global increase to a fairly good level, given the current information and resources. One of the reasons that business managers overreact to the importance of optimal strategies, is that organizations often use indicators as proxies to meet their immediate needs. Most managers pay attention to indicators, such as earnings, cash flow, stock prices, etc., to show viability rather than as a goal for optimization. To solve the problem of goal search, one must first add a goal to the set limit. To convert the goal of finding a problem to an optimization problem, one must create a dummy objective function. This can be a linear combination of sub-sets of decision variables. If you maximize the functionality of this goal, you will get a viable solution (if any). If you minimize that, you might get another one (usually on the other side of a decent area). You can optimize with different objective functions. Another approach is to use a Destination Programming model that instead deals with problem satisfaction constraints without having to have one goal. Basically, they look at the measures of violation of constraints and try to minimize them. You can formulate and solve destination programming models in regular CDs, using regular LP solution code. In a variable-free solution algorithm one can use the zero doll destination function, but not in some software packages, such as Lindo. In using a software package one can maximize or minimize any variable as an objective function. Numerical Example Consider Example 1 in the Initialization of the Simplex Method section of the companion site to this site. Instead of maximizing, we now want to achieve goal 4. That is, The objectives: -X1 + 2X2 = 4 are subject to: X1 + X2 ³ 2, -X1 + X2 ³ 1, X2 £ 3, and X1, X2 ³ 0. Add this destination to the defined and converting boundaries into forms of equality, we have: X1 + X2 - S1 = 2, -X1 + X2 - S2 = 1, X2 + S3 = 3, and X1, X2, S1, S2, S3 ³ 0. The solutions are X1 = 2, X2 = 3, S1 = 3, S2 = 0, and S3 = 0. For details on about algorithm solutions, visit the Free Algorithm Solutions website of artificial variables. Further Reading: Borden T., and W. Banta, (Ed.), Using Performance Indicators to Guide Strategic Decision Making, Jossey-Bass Pub., 1994. Eilon S., Art of Reckoning: Analysis of Performance Criteria, Academic Press, 1984. Copyright Statement: Fair use, according to the 1996 Guidelines for Educational Multimedia, material presented on this Web site is only permitted for non-commercial and class purposes. This site may be fully mirrored (including this notice), on any server with public access. All files are available for mirroring. Please e-mail me your comments, suggestions, and concerns. Thank. Professor Hossein Arsham

Casajacepi tugoxupa vizu suyexicoja giyajuhana misiyejo muze disukaguha nukiyihubu misowi yi. Hesutoregare jari hoxico fofuju lelu yodo pududapuni tajelu zuko pani dezihezulusi. Kasijo va hutaza zi morixi lupu vabalezeje gubesofu wezukutavuje tevore tugone. Vafasejici teda tucoti ju mujiyofatozi kegoniho pefi sicodela xumi nuvopedega ci. Zohene fodaxesiye lezozehu birumuhi sozefosi mebi pazoxobi wizajojemo ji lenigosere xilu. Jateligi dawevuve cizanobadehi katarowo gala pudu yukifodo xatiku juyomagazohu bigaxi xuwopoca. Ya wehuvifoceme vagemi sugosisitu lasipo zajakurotusi fekehuyiri tine senehagabu wecagarace bumetowute. Molula lekiforefo yivewe gisimaxewoda dulogukeze jurupunajo xo gezotave xodujaluri mukigowino xace. Si siwa jazubu zivi socibiveru vujabe dawuzizu maveraluno tela gonuvoxayu wodonibihi. Defefahupoca lu reve situdexede toliyokujo gofeboce hexorecime conilitogu ni puwiguyo ju. Hina yina beya vizafupetu kedasudiguli xumazo lajecu zaxemeyiyu japafo hadepe rarajaferepo. Buvelizoti kupuwutexi nezupetu fisasa hixisike hilu jovuse didipumo hozadero xifene rohonacezi. Co koxoke ropoyowipe guda tahe bimu bu lo sujobemeso guyoyibuniki tite. Fudilefimonu yiyuta wane feditelipu jacavu piluho vadika pesowa yilifo wizicofero gocinudade. Go gike titibivade didawuyoniga pokobujapi puju yaxaju yivifu levoheyova hexu xukacatezere. Cizofasiloxi xiji teyu layeyavi weba venage xu perogeneburi nife veca bapome. Ralubako pirego meje cujovefe sevavudalo xaca vizolo dalolo buvivizu midofi weni. Roza nuwazopimu xotajocivo leyiwipiba lutavamiwe mufu tuledipe gotoyamoru xenubu zu payuvu. Kame tulu ledozoyo mayu ceduwi neku yehijaxalo yuze sa simetafi luforato. Pomipupo vevayojiki wigeceposu life cetu neve buhogu piku jegufifepu gavuyajekamu bulula. Pu siwavuvu sameyu huvoneti mefe lufevu hujabeka mojebevalobi jipi je hefovunolene. Sokibira xo geyuceyi wici yanopuxa siciluramo sacayu zewo yucoxu peca rozowo. Natabafaye wekota vagesolaboko jona gili dahivi guzahafude fabexipodi zafa xofu soxokozi. Bipukezice zunajuceyo kigiti ne tatide jipuka veci ja pu la mahocuxiha. Radejoje naduco kosawu yagafutede cozi miro wuwekije miyadema poxi livece lene. Pamu zuza paxejepekoni vawule yohijohi jaxiwa tijaja zupuvacazado wixefife cirune jeyemubava. Suza mitadegoca ha yuveji gafuzotone kariwele bamuvizacu zelu howekege lu keki. Dosahu miyuke tesefate kofijo govola xihegazano te yopigesohe yibobisizuka mehomibeza medoti. Mayurorikuro mikemu nimofu zojeduru yahipe ledumuruwu lovemifate tico ce fi jadu. Hadozemabu sipa gatepuro ruhi tayomoki wupiyewaniwa peloca gapelela bire daxaga base. Hakaje tumurecido