# How to solve systems of equations with 3 variables by elimination

Learning results Identify an inconsistent system of equations containing three variables that solve a system of three equations with three variables. Use standard notation to represent the system solution of a dependent equation containing three variables. John received a $12,000 inheritance and invested in a money market fund that was divided into three parts and paid 3% interest per year. In municipal bonds that pay an interest rate of 4% per year. In mutual funds that pay an interest rate of 7% a year. John invested $4,000 more in local funds than municipal bonds. He got $670 in interest in his first year. How much did John invest in each type of fund? (Credit: Wikimedia Commons Elembis) Finding a solution should follow a pattern when you understand the right approach to setting up such a problem. This section solves a problem similar to this one, which includes three equations and three variables. In doing so, a technique similar to that used to solve the system of two equations of two variables is used. But finding a solution to the system of three equations requires a little more organizing and a touch of visual gymnastics. To solve a system of three variables (three) equations that solve a system of three equations with three variables, we remove one variable at a time to achieve a reverse substitution. The solution to the system of the three equations of the three variables [latex]\left (x,y,z\right)\text{}[/latex] is called an ordered triple. To find a solution, you can do the following: Multiplies both sides of the formula by a nonzero constant. Adds a non-0 multiple of one formula to another formula. Graphically, an ordered triple defines the intersection of three planes in space. You can visualize such intersections by imagining the corners of rectangular rooms. Corners are defined by three planes: two adjacent walls and a floor (or ceiling). The point at which the two walls and the floor cross represents the intersection of the three planes. This plane shows the possible solution scenarios for each of the three systems. A system with a single solution, after deletion, will be a solution set consisting of an ordered triple [latex]\left\left (x, y, z\right)\right\right\3[/latex]. Graphically, an ordered triple defines the intersection of three planes in space. A system with an infinite number of solutions will always have a true expression, such as [latex]0=0[/latex] after erasure. Graphically, an infinite number of solutions represent a line or match plane that functions as the intersection of three planes in space. A system without a solution is one that results in conflicting statements after elimination.Geometrically, a system without analysis is represented by three planes that they have nothing in common. (a) The three planes intersect at one point, representing a three-way system with a single solution. (b) Represents a three-way system in which three planes intersect in one line and have an infinite solution. Determine if the ordered triple [Latex]\Left (3,-2,1\Right)[/Latex] is the solution to the system. [Latex]\begin{gatherd}x+y+z=2 \\ 6x - 4y+5z=31\5x+2y+2z=13 \end{gatherd}[/latex] Method: Considering the linear system of the three equations, we solve three unknown equations. Select any pair of equations and solve one variable. Select another pair of equations and solve the same variable. You have created a system of two unknown expressions. Resolve the resulting system by two. Back assigns a known variable to one of the original expressions to resolve the missing variable. Find a solution for the following system: [latex]\begin{align}x - 2y+3z=9&&\text{1)} \\ -x+3y-z=-1 6&&\text{} \\ 2x - 5y+5z=17&\text{} \end{align}[3]/Latex] A solution system for three variables. [Latex]\begin{array}{l}2x+y - 2z=-1\hfill \\ 3x - 3y-z=5\hfill \\ x - 2y-3z=6\hfill \end{array}[/latex] The following video provides a visual representation of three possible outcomes of the solution to the system of equations for the three variables. There is also an example of working with removal to resolve a system. In the issue raised at the beginning of the section, John invested a $12,000 inheritance in three different funds: part of a money market fund that pays 3% interest each year. A portion of municipal bonds that pay 4% each year. The rest are mutual funds that pay 7% each year. John invested $4,000 more in mutual funds than he invested in municipal bonds. The total amount of interest earned in a year was $670. How much did he invest in each type of fund? The equation represents three parallel planes, two parallel planes and one intersecting plane, or three planes that intersect the other two planes but are not in the same position. The process of elimination will result in false statements such as [latex] 3=7 [/latex] or other inconsistencies. Resolve the following systems: Latex\begin{x - 3y+z=4 & Left (1\Right) \\ -x+2y - 5z=3 &\Left (2\Right) \\ 5x - 13y +13z=8 && \Left (3\Right) \end{align}[/Latex] Solves a system of three equations of three variables. [Latex]\begin{array}{l}\text{}x+y+z=2\hfill \\ \text{ }y - 3z=1\hfill \\ 2x+y+5z=0\hfill \end\\array[/latex] Know from a system of three variables that represents the system's solution to a dependent equation containing three variablesThe system of equations involved has an infinite number of solutions. The same is true for the dependent system of the relationship equations of the three variables. If the number of solutions occurs infinitely, you can get from several situations. The three planes are the same, and the solution of one equation is the solution of the other two equations. All three equations may be different, but they intersect on a line with an infinite solution. Alternatively, the two equations are the same and may intersect the third equation on the line. Find solutions to specific systems of three equations of three variables. Latex\begin{align}2x+y - 3z=0 &&\Left (1\Right)\\4x+2y - 6z=0 && \Left (2\Right)\x-y+z=0&&Left (3\Right)\End{Align}[/Latex] No, a generic solution can be described in any variable, but it is common to write [latex] x [/latex] and , if necessary, in terms of [latex] x [/latex] and [latex]y[/latex]. Resolve the following systems: Latex\begin{x+y+z=7 \\ 3x - 2y-z=4 \\ x+6y+5z=24 \end{gatherd}[/latex] The key concept solution set is an ordered triple [latex]\left\left(x,y,z\right)\right\/latex representing the intersection of three planes. A system of three equations of three variables can be solved using a series of steps that force the removal of the variable. This procedure includes reordering formulas, multiplying non-0 constants on both sides of a formula, and adding multiples other than 0 for one formula to another. A system of three equations of three variables helps solve many different types of real-world problems. In the presence of a solution, the system of equations for the three variables is inconsistent. After the removal operation, the results are inconsistent. If the system of equations for the three variables is inconsistent, you get three parallel planes, two parallel planes and one intersecting plane, or three planes that intersect the other two planes but are not in the same position. The system of equations for the three variables depends on the infinite number of solutions. When you perform an erase operation, the result is an ID. A system of equations for three dependent variables can result from three planes, three planes intersecting on a line, or two planes intersecting a third plane on a line. The glossary solution does not get excited by the fact that the second equation contains only two variables before starting the process of solving, before setting up all the ordered pairs or triple sets that satisfy all equations in the system of equations. This happens when there are two or more equations in the system. In fact, it take advantage of the fact that there are only two variables, one of which, \(y\), has a factor of -1. This equation is easily solved to get \(y\) and \[y = 4x + 5\] we can substituteIn the first and third equations, as follows: \[Start {align*}2x - 4\left ( {4x + 5} \right) + 5z & - 33\- 2x + 2\Left ({4x +5} \right) - 3z & = 19\end{align*}] This is a system of two linear equations with two variables (\(x\) and \(z\)). You know how to solve these kinds of systems from the work in the previous section. First, you need to do a bit of system simplification. \[\begin{array}{*{20}{c}}{2x - 16x - 20 + 5z = - 33}\{- 2x + 8x + 10 - 3z = 19}\end{array}\hspace{0.25in} \to \h space{0.25in}\begin{array}{*{20}{c}}{{{{{ - 14x + 5z = - 13}\{6x - 3z = 9}\end{array}}] The simplified version looks the same as the system that was resolved in the previous section. Well, it's pretty much the same. This variable is \(x\) and \(y\) instead of \(x\) and \(z\), but there is no real difference. The task of resolving this is the same. A substitution method or an exclusion method can be used to solve a new system of these two linear equations. If you use the id method, you can easily solve the second equation in \(z\) (you can see why it is easiest to solve the second equation of \(z\) and replace it with the first equation. This allows you to find \(x\) and both \(z\) and \(y\). However, in order to make the point that both methods are often used in the solution of the three linear equations, let's use the exclusion method to solve the system of the two equations. Just multiplie the first equation by 3 and the second expression by 5. Doing this can be \[\begin{align*} -14x+5z & =-13 & \right arrow{\,3}\hspace{0.5in} & -42x+15z=-39 \\ 6x-3z & =9 & \right arrow {\times\\\\\\\\14x +5z&=right arrow\hspace{0.5in} & \downline {\ hspace{0.25in} 30x-15z=45} \\ & & &\hspace{0.5in} -12x=6 \\\\end{align*}\} \(x\ \frac{1}{2}\\) can now be easily resolved. The coefficient of the second equation is small, so let's plug it into that equation and solve \(z\). Here is the work. \[\begin{align*}6\left({-\frac{1}{2}} \right) - 3z & = 9\\ - 3z & = 9\\ - 3z & = 12\\ z & = 4\end{align*}] Finally, you must determine a value of \(y\). This is very simple. Remember that you used substitution in the first step and the following expression in that step: I know the value \[y = 4x + 5\] \(x\), so all I have to do is plug it into this equation and get the \(y\) value. \[y = 4\left({-\frac{1}{2}} \right) + 5 = 3\] If you assign in this step, the expression used in this step contains both \(x\) and \(z\), so you will need both values to get the third variable. Now, to end this example here is the solution: \(x = - \frac{1}{2}\), \(y = 3\), \(z = - 4\)4\).