Calculus and vectors

This course is based on his previous experience with functions and his understanding of developing rates of change. You will solve problems involving geometric and algebraic representations of vectors and representations of lines and planes in three-dimensional space; broaden their understanding of change rates to include derivatives of polynomial, sinusoidal, exponential, rational and radical functions; and apply these concepts and skills to real-world relationship modeling. You will also refine your use of the mathematical processes needed for success in senior mathematics. This course is intended for students who choose to pursue careers in areas such as science, engineering, economics and some business areas, including those students who will be required to take a university-level calculus course, linear algebra or physics. Note: Advanced Functions (MHF4U) must be taken before or simultaneously with Calculation and Vectors (MCV4U). In this unit, students will examine the values of the average rate of variation over a range to approximate the instantaneous rate of change at one point. The boundary concept will be formally defined, and students will use a graph of a function and the properties of the boundaries to evaluate boundaries of a variety of functions. The concept of limit, as a value addressed, will be reinforced by examining how Greek mathematicians developed the formula for the area of a circle. The two fundamental problems of the calculation will be defined. Students will use the threshold concept along with the average rate of change to approximate the instantaneous rate of change of a function at one point. Students will learn the formal definition of a limit and the three conditions necessary for a boundary to exist. Students will evaluate the limit of multiple functions to a given value of x by looking at the y-value(s) in a chart that are covered on the left and right side. Students will learn 7 bound properties and apply these properties to algebraically evaluate boundaries of various functions. In past explorations of functions and their graphs, students will have noticed that from start to finish the graphs of some functions are made of an uninterrupted curve, while others include breaks within their domain. This module will use limits to define the three conditions that must be met for a function to be continuous throughout your domain. In addition, students will learn the various types of discontinuity and the algebraic method of finding the location of a discontinuity. Special focus will be given to the evaluation of limits of polynomian and rational functions. Students will identify the removable discontinuity of a rational function before simplifying the expression, and then apply boundary properties to evaluate the boundary. Special focus will be given to the assessment of the limits of contain radicals. Students will methods of rationalization of numerators and denominators, as well as the mastery and range of radical functions. Students will use the domain of a role to identify whether the boundary exists before applying rationalization to evaluate function boundaries that contain radicals. Convergent and divergent sequences will be defined and students will observe large values of these sequences to determine whether the limit exists in infinity. This module will connect boundaries at infinity with an algebraic method to determine the location of horizontal asytotes. This unit will enter the formal definition of the derivative. Students will examine charts and use the derivative definition to verify the rules for determining derivatives: constant function rule, power rule, constant multiple rule, sum and difference rules, product rule, chain rule, and quotient rule. They will apply these rules to differentiate polynomaria, rational, radical and composite functions. Students will connect the value of the derivative to a certain value of x with the slope of the tangent line at a point in a curve, and they will use that slope and point to determine the equation of the tangent line. Students will use the definition of the derivative to differentiate polynomian and rational functions, as well as functions that contain radicals. When examining a variety of functions, students will identify the value(s)/range(s) of x for which a function is no different. Through the chart exam, students will foresee possible differentiating rules and then verify those rules by applying the definition of the derivative to the general statements. The differentiation rules explored in this module are the constant function rule, the power rule, the constant multiple rule, and the sum and difference rules. Students will connect the value of the derivative to a specific value of x with the slope of the tangent line to a curve at a specific point. Students will use the derivative value with the slope point equation of a line to determine the equation of a tangent line. Students will use the derivative definition to develop the differentiation rule for the product of two functions. Students will then differentiate the product from two or more functions by applying the product rule. Students will identify the internal and external roles that make up a composite function, and then apply the chain rule to differentiate. Students will use the derivative definition to develop the chain rule Students will develop the quotient rule by applying the product rule and the chain rule to a quotient of two general functions. Students will then apply the quotient rule to differentiate rational functions and the quotient from two functions. In this unit, applications of the definition of the derivative are explored. We define higher-order derivatives of a learn to sketch the derivative of a function from the function graph, and see how instantaneous rates of change calculations can be used to solve real-world problems in life sciences and social sciences. In this module, we will define the second derivative of a function, obtained by differentiating the derivative of the function, and, more generally, the derivative n^{th} of a function for any positive integer n. In this module, we will learn how to draw the graphics of the first and second derived from a polynomial function f, given the graph of f. Applications of the derivative as an instantaneous rate of change are explored in the fields of geometry, life sciences and social sciences. Applications of the first and second derivatives are explored for problems involving movement. In this unit, we developed an algorithm to sketch a curve given the algebraic equation of the curve. We discuss the extreme value and the mean value theorems, and examine the notion of a turning point, an absolute extreme, an increase or decrease interval, concavity, and an inflection point. The notion of a turning point and an absolute extreme of a function are defined. We learned how to use the first derivative to find turning points and the extreme values of a function in a closed range. The average value theorem, which connects the average rate of variation and the instantaneous rate of change, is declared and explored using examples. In this module, we explore function turning points and introduce the first derived test. More precisely, we will learn to use the first derivative to find the intervals of increase and decrease of a given function. The second sign derived from a function can give information about the shape of the chart. The concave terms up, concave down and inflection point are defined, and the second derivative test is introduced. Using the tools acquired throughout our function study, we developed an algorithm to sketch a curve given the curve equation. The functions studied include polynomies, rational functions and functions involving radicals. Now that we are familiar with how to calculate derivatives, we will use them in this unit to solve real-world problems in optimization and also as a way to determine related rates. We will also introduce Newton's method as a way to approximate roots to equations. We will express a particular problem in mathematical language, determining a function that must be maximized or minimized. Finding extreme values of this function allows us to solve problems such as maximizing the area or minimizing time. The problems in this module will be similar to those of the previous module, with the focus shift to maximize revenue and profit or minimize costs. Marginal costs, marginal profits and the demand function will be studied. Some can be described by a relationship as $x^2+y^2=9$ where y is not explicitly given in terms of x. In these cases, the implicit differentiation method can be used to determine the derivative of one variable relative to another. In many real-world situations, a change in quantity causes a change in another quantity or occurs along with a change in another quantity. That is, the two rates of change are related. Linearization is the method of using a tangent line to approximate the function that is tangent to near the tangency point. This module will also introduce an algorithm called newton's method to find approximate roots of equations. This unit begins with an introduction to euler's number, and. In addition to developing the derivatives of exponential, logarithmic and trigonometric functions, we will also expand our algebraic and equation resolution abilities with these three types of function. Let's set \(and\) to match the value of a fundamental limit. The relationship between $f(x)=e^x$ and other exponential functions and their connection to the natural logarithm will be discussed. We will also investigate the derivative of $f(x)=e^x$. We will use a maple investigation to help establish the derivative of $f(x)=\ln(x)$ and then use our derived rules to differentiate more complex functions involving natural logarithm. In this module, we will develop the derivative of any exponential function with a positive constant basis. In this module, we will develop the derivative of any logarithmic function based on positive constant. Through research, we developed the derivatives of the sine and cossine functions, and then, using two fundamental trigonometric limits and an identity, we proved our conjectures. In this module, we develop and use the derivatives of each of the functions: $f(x)=\tan(x)$, $f(x)=\csc(x)$, $f(x)=\sec(x)$ and $f(x)=\cot(x)$. In this unit, several applications of exponential, logarithmic and trigonometric functions are explored. Familiar topics will be revisited, including change rates, curve sketching, optimization, and related rates. We have seen how calculus, more specifically the derivative, can be used to study rates of change in physical quantities. In this module, we will explore such applications, where modeling equations involve exponential, logarithmic and trigonometric functions. The L'Hospital rule is a tool for assessing limits of indeterminate quotients that cannot be evaluated using boundary laws. This rule is particularly useful for evaluating quotient limits involving exponential and logarithmic functions. In this module, we will revisit the algorithm for curve sketches and apply it to curves whose equations involve exponential, logarithmic and trigonometric functions. Exponential, logarithmic, Functions arise naturally in many real-world applications of calculation. We revisit the theme of optimization with a special focus on problems involving functions of this type. In this module, we extend our study of related rates. In particular, we study the rates of change in quantities that are related through formulas involving exponential, logarithmic or trigonometric functions. This unit introduces the second branch of calculation, called integral calculation, which is used to find areas. The notion of antiderivative, from the differential calculation, and the definitive integral are defined and connected using the fundamental theorem of the calculation. Indefinite integral is introduced and methods to simplify the integration process are explored, including: integration rules resulting from known differentiation rules, useful properties of integrals, replacement method, and part integration. This module defines the notion of an antiderivative of a function and explores antiderivatives of integer powers of x. This module introduces the problem of calculating the total distance traveled over a period of time when the speed varies. This leads to the question of estimating areas of aircraft regions using rectangular approximations. Sigma notation is a compact way to write large sums of similar terms. Riemann's sons will be defined, using this notation, as a method to estimate net areas of plan regions. The definitive integral of a given function over a given range is defined as the limit of Riemann sums. This module will introduce the integration process and terminology. In this module, we will explore examples where defined integrals can be evaluated through a liquid area interpretation, without considering Riemann's somas. This module features some basic properties of defined integrals that will help simplify the integration process. Properties include integration order, zero rule, additivity, constant multiple rule, and sum and difference rules. The module ends with a holding on the fundamental theorem of the calculation. The fundamental theorem of the calculation connects the two branches of calculus: differential calculus and integral calculus. As a result of this theorem, we will gain a powerful tool for evaluating definitive integrals using antiderivatives, without considering Riemann sums or liquid areas. This module explores the antiderivatives of many familiar functions and defines the undefined integral of a function. We will see that each differentiation rule gives rise to a corresponding rule for undefined integration. In this module, we evaluate definitive integrals using a table of indefinite integrations known in conjunction with the fundamental theorem. This module introduces one of the two main integration methods: the replacement system. This method arises from the chain rule to and allows us to simplify integrands using a change of variables. This module introduces the second main method of integration: part integration. This method is derived from the product rule for differentiation and allows us to move from integrand in question to a new, hopefully simpler, integrand. In this unit, we will explore some full calculation applications. We will use defined integrals to calculate the net variation of a quantity, volumes of three-dimensional solids, average values of functions, and curve lengths. The end of this unit is dedicated to the theme of differential equations, including a discussion of direction fields, outline of solutions, separable equations, and exponential growth and decay. In this module, the fundamental theorem of the calculation is reformulated in terms of net variation. We will use this result to solve problems involving distance and displacement. In this module, we use defined integrals to calculate the area of regions delimited by continuous curves. Volumes of three-dimensional solids can often be calculated using a definitive integral. We explore familiar formulas, such as the formula for the volume of a sphere, and calculate volumes of more exotic solids. In this module, we define the average value of a function and the length of a curve in a closed range. We will see that the integrals defined are central to the calculation of each of these quantities. Often, mathematical modeling results in the study of an equation involving the rate of change of an amount. This is called the differential equation. We explore some known issues of this nature and introduce the terminology needed for this topic. It is often impossible to find an explicit formula for a solution to a specific differential equation. In this module, we learn how to sketch solutions to a differential equation without actually solving the given equation, and use these sketches to obtain quantitative information about the solutions. Most differential equations require graphical or numerical approaches when solving. In this module, we explore a certain family of differential equations —called separable equations—that can often be solved explicitly using indefinite integration. Many physical quantities increase or decrease at a rate proportional to the quantity of the quantity that is present. This property is known as the law of natural growth. In this module, we examine the family of differential equations, and their solutions, which arise in this context. This unit introduces the concept of vector as being a mathematical object with magnitude and direction. The mathematical operations on geometric vectors developed will culminate in the modeling and resolution of problems involving the physical amounts of force and speed. This module introduces the of a vector as a targeted line segment. O O of equal vectors, opposite vectors, angle between vectors, scalar multiplication and unit vectors will be taught. This module investigates the properties of adding vectors, subtraction, and scalar multiplication. The law of the triangle and the law of the parallelogram will be taught. How much force does it take to pull a wagon? What is balance and when do forces act on an object to produce it? How does the wind affect the speed and direction of an airplane? What other speed issues can be solved using vectors? This unit introduces vectors into a Cartesian coordinate system. The new model allows us to perform vector operations and investigate interesting geometric and physical applications. This module connects the geometric model of a vector to the algebraic model. Components, position vectors, three-dimensional model, and steering angles (or steering cosines) will be taught. This module investigates the properties of the addition of vectors, subtraction, and scalar multiplication of algebraic vectors. We will extend our operations in vectors to include the product point (or scalar product). The properties of the point product will be taught and used to find the angle between two vectors. We will extend our operations in vectors to include the cross product (or vector product). Cross product properties, triple scalar product, triple vector product, right hand rule and crossproduct magnitude will be investigated. Vector projections will be taught and used to solve geometric and physical applications. We'll discuss the concepts of work and the triple-scalar product, and apply your definitions. This unit extends our knowledge of line equations to new forms involving vectors. We will consider these lines in two and three dimensions, as well as determine intersections and distances between the lines. Let's look at two new ways to equation a line in the plane and consider the role vectors play in these new descriptions. Although the scalar equation or Cartesian equation of a line in R2 must seem familiar, we will consider the role of vectors, specifically the normal vector, when describing a line in this form. We extend the parametric and vector equations of the two to three dimensions lines. Symmetric equations of a line in R3 will also be introduced. Do two lines on the plane always intersect? How about two lines in three dimensions? We will consider possible cases for the intersections of lines in $\mathbb{R}^2$ and $\mathbb{R}^3$ and determine the point of intersection where it exists. Let's revisit the projection of one vector in another to help us determine the distance between a point and a line in $\mathbb{R}^2$. Will this same approach allow us to find the distance between a point and a line in $\mathbb{R}^3$? This unit introduces the forms of the equations of and extends our techniques for the resolution of systems of linear equations (such as the equations of the planes). Line operations on arrays will be introduced to help find such algebraic solutions, which will then be interpreted geometrically. This module extends our knowledge of the equations of the lines to the vector and parametric equations of the planes in $\mathbb{R}^3$. Let's extend our knowledge of a normal vector to help describe the equation of a scalar-shaped plane. We will also derive a formula for the distance between a point and a plane in $\mathbb{R}^3$ and then use this work to help determine the distance between the slope lines. Does a particular line and an airplane in $\mathbb{R}^3$ always intersect? If they do, how can we determine where they intersect? How can we sketch a plane? What are the possible ways that two planes in $\mathbb{R}^3$ can cross paths? We will solve the typical two equations in three unknowns to determine whether the two planes intersect. Are you able to draw the different ways that three planes can cross? Algebraically, we will introduce the matrix, Gaussian elimination and line-level form as tools used to determine if and where planes intersect. An investigation will help connect the algebraic solution to the equation system to the geometry of the planes. This module extends our work with matrices for gauss-jordan elimination and reduced line tier shape. We continue to explore algebra and geometry determined by the different ways that three planes can cross. Intersect.