2d kinematics problems worksheet

Learning Goal: Concept 1: Relative Velocity Concept 2: Vectors in 2D - Perpendicular Vectors Concept 3: Vectors in 2D - Non-Perpendicular Vectors Concept 4: Projectiles (3 cases) I went for a walk one day. We went north 6.0 km to 6.0 km/h and then west 10 km to 5.0 km/h. (This problem is deceptively easy, so be careful. Start each part by reviewing the appropriate physical definition.) Determine... the total distance of the entire journey the total displacement of the entire journey the average speed of the entire journey the average speed of the entire journey the average acceleration of the entire travel distance? no problem. First we walked 6.0 km and then we walked 10 km for a total of 16 km. Distance is a scalar quantity, so individual distances are added just like regular numbers. 16 km travel ing is a little more difficult. Displacement is a vector and vectors have direction, so it is best to diagram this problem (a procedure that is remarkably useful in general). The resulting displacement is the vector sum of the two trips encountered during the journey. Since they are perpendicular to each other, the result is the hypopenuse of a straight triangle. Its extent can be found using Pythagoras' theorem, and its direction can be found using the tangent function. r = √[(6.0 km)2 + (10 km)2] r = 11,6619... km tan ρ = 10 km 6.0 km ρ = 59° r = 11.7 km at 59° north west Speed was 6.0 km/h for the first 6.0 km and 5 km/h for the last 10 km. The naïve solution is to average speeds using the add-and-divide method taught in high school. This method is wrong, not because the method itself is wrong, but because it does not apply to this situation. 6.0 km/h + 5.0 km/h = 5.5 km/h 2 The wrong method of average Weights of the two segments are not equal. The second segment lasted twice as long as the first (after you'll see soon). Return to the definition to resolve this issue. The average speed is the total distance (which we have already found) divided by the total time (which we must find). Because time is a scaler, add times for each stage of the journey to get the total time. Δt1 = Δt1 = Δt1 = Δt2 = Δt2 = Δt2 = Δt = Δt1 + Δt2 Δt = 1.0 h + 2.0 h Δt = 3.0 h The average speed is then... The speed was 6.0 km/h north in the first 6.0 km and 5 km/h west in the last 10 km. The average speed is the total displacement divided by the total time. Both quantities have already been determined. Acceleration in this context is relatively meaningless. It would be better to illustrate acceleration in two dimensions with a different problem (such as the one below). A swimmer heads straight for a river swimming at 1.6 m/s in relation to flat water. It reaches a point downstream of the point directly across the river, which is 80 m wide. Determine... current speed magnitude of the swimmer's resulting speed the resulting speed of the swimmer during the time it takes the swimmer to cross the river Because the distance and speed are directly proportional, it starts as a similar problem triangles. Since the speed and distance are directly proportional, the ratio between the distance downstream and the width of the river is the same as the ratio between the current speed and the speed of the swimmer. x = vx y vy 40 m = vcurrent 80 m 1.6 m/s vcurrent = 0.8 m/s Determination of the resulting speed is a simple application of Pythagoras' theorem. v2 = vx2 + vy2 v = √[(0.8 m/s)2 + (1.6 m/s)2] v = 1.8 m/s Steering angles are often best determined using the tangent function. This issue is no exception. The only thing open to discussion is our choice of angle. I suggest you use the angle between the resulting speed and the displacement vector that points directly across the river, but this is just my preference. Be sure to indicate that the result is on a specific part of this vector for clarity. tan ρ = x = vx y vy tan ρ = 40 m = 0.8 m/s 80 m 1.6 m/s tan ρ = 0.5 ρ = 27° downstream Here becomes interesting. By now you should understand that time is the ratio of travel to speed. This is a vector problem, so direction matters. This is why we should probably use the words displacement and speed instead of distance and speed. The only question is what distance and speed should we use? The simple answer is choose the pair you like the most, just make sure you point in the same direction. It works along any of the component directions... t = 40 m = 80 m 0.8 m/s 1.6 m/s It also operates along the resulting direction... t = √[(40 m)2 + (80 m)2] √[(0.8 m/s)2 + (1.6 m/s)2] There is an interesting edge to this question that deft readers would have noticed when looking at the first report in the chain of three shown above. The time it takes to cross a river by a swimmer swimming directly over is independent of the speed of the river. The only factors that matter are the swimmer's speed and the width of the river. This swimmer will always cross the river in 50 years, regardless of the speed of the river. 1 m/s, 10 m/s, 100 m/s, it doesn't matter. This example is a perfect illustration of an idea to be presented in the next section of this book. The two-dimensional motion can be well described with two independent one-dimensional equations. This idea is essential for the field of analytical geometry. A car enters an intersection of 20 m/s, where it collides with a truck. The impact rotates the car at 90° and gives it a speed of 15 m/s. Determination of the average acceleration of the if it was in contact with the truck for 1.25 s. Finding a change of speed is complicated in this matter by changing direction. A chart is indispensable. Let's say that the initial direction of the machine is 0° (right in the standard position) and that the final speed will be 90° (towards the top of the page in the standard position). The difference between two vectors drawn in this way would then connect the head of the original vector to the head of the final vector. Use The Pythagoras Theorem for magnitude and tangent for direction, as usual. Only after we have done all this can we then connect numbers in definition. Δv = √((20 m/s)2 + (15 m/s)2) = 25 m/s a = Δv = 25 m/s = 20 m/s2 Δt 1.25 s tan ρ = 15 m/s 20 m/s = 143° a = 20 m/s2 to 143° Asteroid 2007 VK184 is as a nearterrestrial object (NEO). It has an orbit that brings it close enough to Earth, often enough that we have to worry. It will make 7 close approaches in the 21st century. The first occurred in 2007, when the asteroid was discovered (thus provisional name 2007 VK184). The last will be in 2048, when there is a low but non-zero probability (< 1%) collision with Earth. This is the only asteroid currently known to pose a threat to Earth in this century. On May 30, 2048 at 22:11 UTC (18:11 New York Time), asteroid 2007 VK184 will travel with an estimated value of 16.9401 km/s in relation to the Sun. If it hits the Earth, the impact speed would be 16,9436 km/s (Source: JPL). Determine the angle of impact between the Earth's speed vectors and the asteroid. Start with a chart. Undress it until its essence. Two sides of this triangle are given (vasteroid and vimpact). None of the angles are known. The third part (vearth) can be determined from basic knowledge. The average speed of the Earth is the distance covered in a single orbit (circumference) divided by the time it takes to complete that orbit (one year). We could do this on a hand-held computer... v = 2α(1.50 × 1011 m) (365.25 × 24 × 60 × 60 s) or use an online computer (which knows the average Distance Earth-Sun with more precision)... v = 2α(1 astronomical unit) (365.25 day) We now have three sides of a triangle and we can find the desired angle using the law of cosine. a2 = b2 + c2 − 2bc cos A where... a = impact speed (16,9436 km/s) b = Earth speed (29,7853 km/s) c = asteroid speed (16,9401 km/s) A = angle of impact (our objective) Solve algebraic, replace numerical values and calculate response. cos A = a2 − b2 − c2 − 2bc cos A = (16.9 km/s)2 − (29.8 km/s)2 − (16.9 km/s)2 (29.8 km/s) (16.9 km/s) In this chapter we generalize the study of movement in a single dimension when moving objects in two In this respect, we discuss two of the most important forms of two-dimensional motion, projectile movement and circular motion. Illustrations Explorations Problems Mechanics TOC Overview TOC TOC