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17.4 similarity in right triangles worksheet answers

Transcript for UNIT 7 Similarity and Better Triangles OF CONTENT CONTENT COMMON CORE G-SRT. A.1a G-SRT. A.2 G-SRT. A.2 G-SRT. A.3 JOINT CORE BUSINESS G-SRT. B.4 G-GPE. B.6 G-SRT. B.5 G-SRT. B.4 COMMON CORE G-SRT. C.6 G-SRT. C.6 G-SRT. C.8 G-SRT. C.8 F-TF. C.8 823A Module 7 MODULE 16 Similarity and Transformation Hour 16.1 Hour 16.2 Hour 16.3 Hour 16.4 Enlargements . The supporting figures are similar using changes . Corresponding parts of similar numbers . AA Similarity of triangles . MODULE 17 Lesson on the use of similar triangles 17.1 Hour 17.2 Hour 17.3 Hour 17.4 Triangle Proportionality Theorem . Segment division in a given ratio . Use of proportional relationships . Similarity in the triangles on the right. . MODULE 18 Lesson with correct triangular 18.1 Hour 18.2 Hour 18.2 Hour 18.4 Hour 18.5 Ratio of tangy . Sine and co-compatibility relationship . . Special better triangles... ... and I do not know what to do. . Solving problems with trigonometry Using Pythagorean identity. . 827 837 851 861 . . . . 881 891 903 913 . . . . 931 941 953 967 981 UNIT 7 Unit Pacing Guide 45-Minute Class Module 16 DAY 1 DAY 2 DAY 3 DAY 4 DAY 5 Lesson 16.1 Lesson 16.1 Lesson 16.2 Lesson 16.3 Lesson 16.4 DAY 6 Module Review and assessment readiness module 17 DAY 1 DAY 2 Day 3 DAY 4 DAY 5 Hour 17.1 Hour 17.2 Lesson 17.3 Lesson 17.3 Lesson 17.4 Day 1 DAY 2 DAY 3 DAY 4 Day 5 Lesson 18.1 Hour 18.1 Hour 18.1 Hour 18.2 Lesson 18.2 Lesson 18.3 Day 6 DAY 7 8 DAY 8 DAY 10 Hour 18.4 Lesson 18.4 Lesson 18.5 Module Review and Evaluation Readiness Unit Review and Assessment Readiness DAY 6 Module Review and Evaluation Readiness Module 18 90-minute classes module 16 DAY 1 DAY 1 DAY 1 2 Day 2 Lesson 16.1 Lesson 16.2 Lesson 16.2 Lesson 16.3 Lesson 16.4 Module For Review and Evaluation of Preparedness Module 17 DAY 1 DAY 2 DAY 3 Lesson 17.1 Lesson 17.2 Lesson 17.3 Lesson 17.4 Module Review and Evaluation Preparedness Day 1 DAY 3 DAY 3 DAY 4 5 Day 5 Lesson 18.1 Lesson 18.2 Lesson 18.3 Lesson 18.4 Module Review and Evaluation Readiness Lesson 18.4 Lesson 18.5 Unit Review and Readiness Assessment Module 18 Unit 7 823B Program Resources DEAL PLAN AND EXPLORE HMH Teacher App Access Complete Set of Teacher Resources and Online Online Devices. Schedule and manage classes, to-dos, and activities. Real-World Videos Involved students with interesting and relevant applications of mathematical content Module. Explore activities students interactively explore new concepts using different tools and approaches. ePlanner Just plan your classes, create and view tasks and access all program resources with your online, customizable planning tool. Professional Development Videos Authors Juli Dixon and Matt Larson model successful teaching practices and strategies in actual classroom settings. QR codes scan your smart phone to jump directly into your print book with online videos and other resources. DO NOT EDIT --Changes must be made to file info CorrectionKey=NL-A; CA-A Teacher's Edition Supports students with asking strategies, teaching tips, differentiated teaching materials, additional activities, and more. NO EDIT--Changes must be made to file info DODO NOT EDIT--Changes must be made to File Info CorrectionKey=NL-A; CA-A CorrectionKey=NL-A; CA-A 22.2 Name Name Onosceles and Equilateral Triangles NO EDIT--CHANGES MUST BE MADE THROUGH FILE INFO DODO NO EDIT-CHANGES MUST BE MADE FILE INFO CORRECTIONKey = NL-A; CA-A CorrectionKey=NL-A; The CA-A class straight edge draw segment draw a line line segment bcbc . . CCUse the straight edge 22.2 Equiles are osceles and equilateral 22.2 Triangles Common Basic Mathematics Standards when exploring the Onosceles Triangles integrate TECHNOLOGY Resource Resource Locker G-CO. C.10 Discover discover thetheoreme around the triangles. Vertex angle Vertex angle MP.3 Logic corners are the base of the lateral main corners. The corners that are the base of the asasas are the bottom corners. DEAL with the work space in advance. angle. in the intended space. Use a straight line to draw the angle. AA Do Do Do Your Job Differently Every Time. is a difference in size each time. Mark your angle ∠A as shown in the illustration. Mark your angle ∠A as shown in the illustration. Reflective © Houghton Mifflin Harcourt Publishing Company Make a Guess Looking at your results, what conjecture made about the base angles, 2. 2. Make a guess when looking at your results, what kind of assumption can be made about base angles, ∠C? ∠B∠B and ∠C? The base corners are conatomic. The base corners are kongaad. With the help of a compass, place the point top intersecting with a compass, places the point on top of the top and draws a kaarka that intersects bbUsing Explain Explain 1 1 ProofEsoscele oscele Võrdhaarne Triangle triangle thesis sides at the corner. Mark corner points B and sides. Mark points B and C.C. andItsConverse Converse and A In the Explore, made the assumption under the corners of the osceles triangle congruent. In Explore, you made the assumption that the base corners of the osceles triangle are in sync. This assumption proved that he could This assumption can be proved in such a way that it can be said as aastheore. C C Equine triangle Theorem Isosceles Triangle Theorem If the two sides of the triangle congruent, then corners on opposite sides When the two sides of the aoftriangle are compatible, then the two corners of the opposing side are in sync. Congruent. This theorem is sometimes called Base Corners Theorem marked as Base Corners This theorem is sometimes called Base Corners TheTheOrem and can also be noted when base corners are osceles triangle congruent. the equine triangle is a conglomerate. Module 22 22 must have EDIT-changes A; CA-No CorrectionKey = NL- made through lesson lesson 2 2 1097 1097 File info corners that have base side are the base corners. In this activity, you construct equilateral triangles and examine other possible characteristics/properties of these special triangles. How do you know triangles built onosceles triangles? 1. 1. How do you know that the triangles you built are triangles? ——. The compass represents the length of the length ∠A; therefore abab ≅≅ ACAC . The compass stands for equal ∠ A on both sides; therefore check the students' constants. Check the student's constants. B B opposite the vertex corner, there is a base. How could you draw equilateral triangles without using a compass? Possible answer: Draw ∠A and drawing point B on one ∠A. Then _ use the ruler AB and figure point C to measure on the other side ∠A so that AC = AB. Repeat steps A through D at least more times to save the data to the table. Make sure that steps A to D are correct at least twice more, and then save the results in the table. Make sure ∠A∠A EERepeat module module 22 22 1098 1098 lesson lesson 2 2 Date EXPLAIN 1 Proving the Equilateral Triangle Theorem and the Converse Essential COMMON CORE IN1_MNLESE389762_U8M22L2 IN1_MNLESE389762_U8M22L2 10971097 Question: G-CO. C.10 Relationships Special What are the Triangles? and the equilateral Prove theorems triangle is a congruent angle of the side of the triangle sides formed opposite at least called feet are the peak among the osceles Resource Locker HARDCOVER PAGES 1097,1110 HARDCOVER PAGES 1097,1110 PROFESSIONAL DEVELOPMENT SPECIALIST about triangles. The exploration of Explore An is osceles sides of the corners and feet vertex angle on the Onosceles two congruent triangles side. Feet Vertex angle triangle. Base angle. Base of the base corners. the main corners. other potential base and explore what are triangles are osceles then build special triangles. Angle. their effect to make properties / properti Use straightedge space in advance. Figure. work Do your as shown in ∠A, Tick your corners Check students 19/19/14 12:10 19/19 12:10 P.M. Watch hardcover student edition page student edition page of this lesson. for this lesson. IN1_MNLESE389762_U8M22L2 IN1_MNLESE389762_U8M22L2 10981098 LearningProgressions Progressions Learning this Lesson, students will add their prior knowledge of onosceles and equilateral In this lesson, students will add their prior knowledge of onosceles and equilateral 4/19/14 12:10 4/19/14 12:10 PM Do your room down. Use a straight line to draw the angle. Mark your angle ∠A as shown in the illustration. Connect vocabulary ask volunteers to define the onosceles triangle and have students give real world examples of them. If possible, show the class a baseball cap or other flag in the form of an equine triangle. Tell students that they will prove theorem of equilateral triangles and study their characteristics in this lesson. Klass al and Equilater 22.2 Equine Triangles Name Feet The angle formed from the feet is the vertex angle. What must be true about the triangles you build to be equid triangles? They must have two congases. m∠B m∠B m∠C m∠C Vertex angle Congases are called triangle feet. INTERVIEW STRATEGIES Possible response to Triangle m∠A 70°; m∠B ∠55°; m∠C 55°. Possible response to triangle 1: 1: m∠A == 70°; m∠B == ∠55°; m∠C == 55°. In this activity, constant triangles explore other possibilities in this activity, you build equilateral triangles and explore other possible features/properties of these special triangles. the characteristics/characteristics of these special triangles. © Houghton Mifflin Harcourt Publishing Company Triangle 4 4 © Houghton Mifflin Harcourt Publishing Company View Engage section online. Discuss the photo by explaining that the instrument is a sextant and that a long time ago it was used to measure the rise of the sun and stars, allowing you to calculate its position on the Earth's surface. Then, check the hour performance task. Triangle Triangle 3 3 © Houghton Mifflin Harcourt Publishing Company PREVIEW: LESSON TASK Triangle 2 2 m∠mA∠A Base Corners Bin Corners Opposite Vertex Angle is the base. Laterally opposite the vertex angle is the base. Explain to your partner what you can derive about a triangle if it has two sides of the same length. In the equid area, there are opposite sides. In the equilateral triangle, all sides and corners are compatible, and each corner is 60°. The triangle triangle 1 1 formed is the vertex angle. The angle formed from the legs is the top angle. Language objective Important question: What are the special relationship angles and sides of osceles triangles? Feet Feet congruent sides are called triangle. The legs of the triangle are called. Studying common core DDOnoscele triangles is an equilateral triangle with at least a synchronizing sides. AnAn is an osceles triangle is an ais triangle with at least two simultaneous sides. By studying mathematical practices, the Onosceles Triangles onosceles triangle is a triangle with at least two synchronized sides. Students have the opportunity to complete peer-to-peer triangle activities either in the book or online. protractor measured at each angle. In the Table Table in the Tk column, mark the protractor to measure each corner. Under the table, mark the dimensions for the triangle in the Triangle column. 1. Resource cabinet G-CO. C.10 Prove theorem about triangles. Explore C C Student is expected: COMMON CORE COMMON CORE EXPLORE A B B Important question: What special relationships between corners and sides are osceles Important question: What is the special relationship between corners and sides of osceles and equilateral triangles? and equilateral triangles? Date Important question: What are the special relationships between the angles and sides of osceles and the equilateral triangles? NO EDIT--Changes must be made to file info DODO NOT EDIT--Changes must be made to File Info CorrectionKey=NL-A; CA-A CorrectionKey=NL-A; CA-A __ Date Class 22.2 Equilateral and equilateral triangles DONOT NOTEDIT--Changes EDIT--Changes must be made through File Fileinfo info DO CorrectionKey = NL-A; CA-A CorrectionKey=NL-A; CA-A DOES NOT CHANGE --Changes must be made to File InfoKey=NL-A; CA-Lesson Name Base Base Corners PROFESSIONAL DEVELOPMENT TEACH Evaluation and Intervention in Mathematics On-Site Video Tutorials, featuring program authors Dr. Edward Burger and Martha Sandoval-Martinez, comes with each example of a textbook and give students step-by-step instructions and explanations of key math concepts. Interactive Teacher Edition Customize and present course materials for collaborative and integrated shaping assessment. C1 Lesson 19.2 Accuracy and Accuracy Evaluate 1 Lesson XX.X Reference Hour Linear, Exponential and Rectangular Models 19.2 Precision and Accuracy Teacher Support 1 EXPLAIN Concept 1 Explain Personal Math Trainer offers online practice, homework, ratings and intervention. Track students' progress through reports and notifications. Create and adapt specific lessons or assignments that are aligned with common basic standards. • Internship - With dynamic objects and tasks, students receive unlimited internships on basic concepts, supported by guided examples, step-by-step solutions, and video tutorials. • Evaluations – Choose course tasks or customize your based course content, Basic standards, difficulty levels and more. • Homework – Students can do online homework with a variety of problems, including the ability to enter phrases, equations and graphs. Let the system automatically go through homework so you can focus where your students need the most help! • Intervention – Let Personal Math Trainer automatically provide a targeted, personalized intervention path for your students. 2 3 4 17 definition 2 question 3 Precision ComplEtINg th E SquArE wIth AVOID common errors Some students may not pay attention to whether b is positive or negative, since c is positive regardless of sign b. Can the student change the sign b to some problems and compare factory forms in both expressions. questioning strategies for perfect square trinomia, is the last term always positive? Explain. es, the perfect square trinomial can be either (a + b)2 or (a – b)2, which can be counted as (a + b)2 = 2 + 2ab = b 2 and (a – b)2 = a 2 + 2ab = b 2. Either way, the last term is positive. to reflect the results of the The character (b) does not affect the character c, because c = (b __ 2) 2 and the non-zero number in the square is always positive. Therefore, c is always positive. c = (b __ 2) 2 and a different number from zero to c = (b __ 2) 2 and non-zero number 5 6 7 View Step by Step 8 9 10 11 - 17 Video Tutor Personal Math Trainer Textbook X2 Animated Math Solve Rectangular Factoring. 7x + 44x = 7x − 10 As you have seen, measurements are made with certain accuracy. Therefore, the x = reported value does not necessarily reflect the actual value of the measurement. For example, measuring 5 centimeters, which is , the control given to the nearest full unit can actually range from 0.5 units, from 4.5 centimeters, to 0.5 units but not more than 0.5 units, by 5.5 cm. The actual length l is within the range of possible values: Save and close centimeters. Similarly, the length of the nearest tenth may actually range from 0,05 units, which is less than the value given, to 0,05 units above, but not. So the length reported at 4.5 cm could actually be as low as 4.45 cm or as high as nearly 4.55 cm. ? ! Turn it to Work Look Back Focus higher-level thinking raise the bar for homework and practice, which includes higher-level thinking and mathematical practices for each lesson. Differentiated learning tools Support all learners with differentiated learning tools, including • Leveled practice and problem solving • Reading strategies • Success for English learners • Challenge Calculate minimum and widest possible area. Round your response to the nearest square centimetres. The width and length of the rectangle shall be 8 cm and 19,5 cm respectively. Prepare students for success in high-stakes tests for integrated math 2 custom for each module and item, find the actual range of length and width values in the rectangle. Minimum width = 7.5 cm and maximum width < 8.5 cm My answer Find the range of values of the actual length and width of the rectangle. Minimum length = 19,45 cm and maximum length < 19.55 Name __ Date __ LESSON class __ __ Accuracy and significant numbers 6-1 Success for English learners Linear functions Reteach Linear functions Reteach Linear measurement accuracy the smallest unit of time or the actual length and width of the rectangle. function is a straig ht line. fraction of the unit used. Ax + Author + C = 0 is the standard form equat ion linear functi • A, B and C is turned on. Problem 1 Minimum area = Minimum width × number of the minimum length. A and B are not both zeros. • Variables x and y Select advanced measurement. = 7,5 cm × 19,45 cm of exhibitors 1 is not multiplied with no denom 42,3 g of 42,27 g of intors, exhibitors or radical characters. nearest tenth of the nearest. the closest examples These are not a hundredth. linear functions: 2 + 4 = 6 variables x2 = 9 exponent x ≥ 1 xy = 8 and y multiplied by 42,3 g or 42,27 g in total 6 = 3 Since the hundredth of a gram is less than a tenth of a gram, 42,27 g x denominator x is more accurate. 2y = 8 y Exhibitor Problem 2 Above exercise, the location of the uncertainty linear y = 5 y square root measurements resulted in different amounts of uncertainty calculated choose advanced measurement: 36 inches or 3 feet. Measurement. Explain how to solve this problem. Find out whether each function is linear or not. 14 = 2 x 2. 3xy = 27 3. 14 = 28 4.6x 2 = 12 x ___ Reflect ____ ___ __ _____ Graph y = C is always horiz ontal line. The graph is always a vertical line. x = C _____ is item 7 Send to notebook ___ The object is weighed on three different scales. The results are shown in the Find out table. Which scale is most accurate? Explain your answer. Measurement ______ When you decide which measurement is more accurate, what should you consider with the formula? Scale Tailor ratings and response to interventions to meet the needs of all your classes and students, including • Leveled modular quizs • Leveled unit tests • Unit performance tasks • Placement, diagnostics, and quarterly benchmark tests Your turn y=1 T x=2 y = -3 x=3 823D Math Background COMMON CORE G-SRT. A.2 LESSONS 16.1 and 16.2 Transformation is a function that changes the position, size or shape of the drawing. In the process, the emphasis is on reflections that are most closely related to conjugation and similarity: reflections, translations, rotations and enlargements. However, it is important to understand that there are many other changes. Perhaps the easiest transformation is the transformation that maps each point itself. It's called an identity conversion. Another simple transformation is one that maps each point of origin. Expands COMMON CORE G-SRT. A.1 LESSON 16.1 Enlargement is a transformation that changes the size of a number, but not its shape. As such, enlargement is an example of a transformation which is not an isome (unless the expansion scale factor is 1). There is exactly one fixed point in each enlargement that is the centre of enlargement. Although enlargements do not retain distance, they retain a number of other characteristics. For example, enlargements maintain an angle measure. In other words, beneath the dilation, the angle of the pre-image is consistent with the corresponding angle of the image. Enlargement also maintains parallel lines. This is two rows that are parallel to the preimage is mapped to two parallel rows in the image. If two numbers are in sync, then there is an isometry that charts one number onto another. If the two digits are similar, one can be mapped to the other by a combination of dilation and isomemise. Enlargement is enlargement with a scale factor greater than 1. Reduction is an expansion with a scale factor greater than 0 but less than 1. It is also possible to extend the definition of dilation to allow a scale factor of 0 (in this case the entire pre-image 823E unit 7 has collapsed to one point, the centre of enlargement) and negative scale factors. Enlargement with a scale factor of k , where k > 0 is equivalent to a dilatation scale factor k followed by a turn in relation to the centre of 180° expansion. Similarity to COMMON CORE G-SRT. A.2 LESSONS 16.2 to 16.4 Recall that two digits can be defined as one time if there is an isoethics — a set of reflections, translations, and/or revolutions — that map one number to another. Similarity can also be defined as changes. In particular, the two digits are similar if one can be obtained by means of a combination of dilation and one or more isomeethics. Expansion will change the numbers by expanding or reducing them. Finally, it is worth noting that the similarity is equivalence; it is, the similarity is reflexive, symmetrical and transitive. For F 1, F 2 and F 3, F 1 ± F 1 (reflexivity); if F 1 ± F 2, F 2 ± F 1 (symmetry); and if F 1 ± F 2 and F 2 ± F 3, F 1 ± F 3 (transit). Triangle Proportionality COMMON CORE G-SRT. B.4. 17.1 The theorem of the proportionality of the triangle provides that, where the line is parallel to the side of the triangle and cuts the other both parties, it divides those parties proportionally. In ‹ › _ − AE AF indicator, EF ∥ BC. Therefore __ = __ . EB FC A E B F C PROFESSIONAL DEVELOPMENT Certificate using similarity and facts about proportions shall proceed as follows: Since these are the appropriate angles, ∠AEF ≅ ∠ABC. As ∠A ≅ ∠A, △AEF €△ABC by AA Similarity. Similarity to the right triangles of AE AF Thus __ = __ , so mutuals are also equal, AB AC 1816, French mathematician August Leopold Crelle noted: It is really wonderful that as simple an indicator as a triangle has such inexhaustible properties. It is possible to develop and prove the theoremes that are increasingly elegant and unexpected, such as the rather surprising fact that the three heights of any triangle are concurrent. AC AB __ = __ . Add aE AF Segment with postulate AB = AE + EB and AE + EB AF + AC = AF + FC. Replacement ___ = ______ or AE AF FC FC EB EB 1 + __ = 1 + __ . Therefore, __ = __ is equal to the required part of AF AE AF AE. Proportional relations common core G-SRT. B.5 LESSONS 17.2 and 17.3 Part is an equation that says that the two relationships are equal. Students should understand that the steps to resolve a part such as __3x = __58 are the same as solving any other type of equation; after that, the equation must be changed to simpler equivalent equations in order to distinguish the variable from one side of the equation. A possible first step __3x the solution to the data = __58 is to clear the fractions by multiplying both sides of the equation by 3 to 8 or 24. This results in an equivalent equation of 8x = 15 that can be solved by dividing both parties by 8. Students may already be familiar with the shortcut that results from the execution of the initial multiplication of the overall proportion of the Cross Product property. This property says that if __ba = __dc where b ≠ 0 and d ≠ 0, then ad = bc. To see why the cross-product characteristic is true, start by multiplying the __ba the __dc bd. a _ c _ = , b d a ( )_c (bd)_ = bd , b d bda _ bdc _ = b d Multiplication Multiplication Of Real Estate Equality Multiply. da = bc Simplify. ad = bc Mult's commutative property. COMMON CORE G-SRT. B.4 LESSON 17.4 This lesson contains the following important theorems: • The hypotenus height of the right triangle forms two triangles that resemble each other and the original triangle. • The height of the hypotenus of the right triangle is the geometric mean of the length of the two segments of the hypotenus. • The length of the right triangle leg is the geometrical average of the hypotenus and the hypotenus length of the right triangle. • Geometric mean of the adjoining hypotensive and segment length of this leg. • Geometric mean of the adjoining hypotensive and segment length of this leg. Theorem Pythagorean Theorem, one of the best-known relationships in mathematics, can track back almost as much as the recorded history itself. Most early occurrences of theorem occur in the form of Pythagorean triple (sets three zero integers a, b and c such as 2 + b 2 = c 2). One example is a clay tablet known as Plimpton 322, which was written in Babylonia about 1800 B.C.E.; it contains 15 lines of numbers based on Pythagorean tripled. Such archaeological findings show that the relationship was known long before the Greek mathematician Pythagoras (approx. 582 B.C E–507 B.C.E.); It is thanks later to the work of Greek and Roman historians that the theorem has come to bear the Pythagoras name. Regardless of its beginning, the theorem continues to inspire both professional and amateur mathematicians because of its elegance and adaptability to various methods. In fact, Pythagorean proposal by Elisha Scott Loomis presents more than 350 different evidence of Pythagorean Theorem! Unit 7 823F UNIT 7 UNIT 7 Similarity and Right Triangles MODULE Similarity and Right Triangles in Mathematics Careers Unit Activity Preview 16 Similarity and Conversions MODULE 17 Using Similar triangles MODULE 18 Trigonometry Correct Triangles After completing this unit, students complete the mathematics career task using similarity to make calculation special effects engineer. Critical skills include modelling real-world situations and using similar indicators to find missing measurements. © Houghton Mifflin Harcourt Publishing Company • Photo authors: ©TriStar Pictures & Touchstone Pictures/Everett Collection, Inc. For more on career mathematics as well as various math recognition subjects, visit the American Mathematical Society at . The mathematics of career special effects engineers make movies come to life. With the use of mathematics and some creative camera angles, special effects engineers can make great things seem small and vice versa. If you are interested in a career special effects engineer, you should examine these mathematical themes: • Algebra • Geometry • Trigonometry Research other careers that require engineering to understand real scenarios. See career activity at the end of this unit. Unit 7 811 MONITORING YOUR LEARNING IN2_MNLESE389847_U7UO.indd 811 4/12/14 13:44 13:44 Before that in the entity After that, if students understand: • parallel lines, intersections and angular relationships • cross lines and bisectors • slopes and equations of parallel and cross-facing lines • parallel, rectangular, diamond and square properties • theorems for parallelograms • Special triangles Students will find out: • similarity and enlargement • similarity • corresponding parts of similar numbers • prove similar triangles • Theority of the triangular proportionality • segment sharing in a given relationship • geometric tools theores • Use of tangents, sine and cosine Students learn: • mid-and-infused corners • chords, secants, tangents and arches • segment lengths in circles • corners formed in circular sections • angst and circle area formulas • Sector area 823 Unit 7 Reading Start -Up Vocabulary Use words ✔ basic idea to complete the web. Write the revised words on squares and add definitions. Translation Translation Translation All point indicator the same distance in the same direction. Images formed from rigid transformations are consistent with their preimages. Rotation of reflection Over the line maps each point to its image so that the line forms a point and the bisector perpendicular to that image. If the dot is on the line, then the picture is as well. The rotation moves around each point in the middle so that the distance from the middle does not change and all corners formed the point and its image are the same dimensions. Fill in the sentences with preview words. 2. 3. The shape of the transformation/enlargement image of the A(n) similarity is the same shape as the pre-image. The scale factor indicates the ratio of the lengths between the respective sides of the two similar numbers. geometric mean . x a = __ Proportions __ x b , x called Active Reading Review Words ✔ betweenness (intermediación) ✔ colinearalidad) ✔ congruent (congruente) ✔ hypotenuse (hipotenusa) ✔ feet (cateto s✔ orient (orientación) ✔ parallel (paralelo) ✔ reflection (reflejo) ✔ rotation (rotación) ✔ transformation (transformación) ✔ translation (traslación) Do students stop working alone or with others. VISUALIZE VOCABULARY The word webgraphic helps students review the vocabulary associated with the changes. If time allows, brainstorm other connections between review words. The dilatación dilatación (dilatación) dilatación (dilatación) indirect measurement (medición indirecta) scale factor (factor de escala)) Similarity transformation (transformación de semejanza) sysy (seno) tangens (tangente) trigonometric ratio (razón trigonométrica) ARU VOCABULARY Use the following explanations to help students learn preview words. Enlargement or similarity transformation is a conversion that changes the size of the drawing, but not its shape. The image of enlargement is a pre-image shape. The center of enlargement is the intersection of the lines that connect each point of the image to the corresponding point in the pre-image. Indirect measurement is measurement method using similar figures. A coefficient that is used to change one number to a similar number for each dimension is a factor of scale. © Houghton Mifflin Harcourt Publishing House understand the vocabulary 1. Reading Start Up Vocabulary ACTIVE READING Students can use these reading and note-taking strategies to help them organize and understand new concepts and vocabulary. Encourage students to ask questions about any reference to a new vocabulary and related concepts that seem unclear. Emphasize the importance of understanding accurate mathematical language so that problems are accurately ed. Double-Door Fold Create a double-door valve before starting the device. Write the properties of congruency under one valve. Fill in the second flap with the corresponding similarity features to make it easier to compare the two themes. Unit 7 IN2_MNLESE389847_U7UO.indd 812 812 4/12/14 1:43 ADDITIONAL RESOURCES Differentiated Instruction • Reading Strategies Unit 7 824 MODULE 16 Similarity and Transformations Similarity and Transformations Essential Question: How can you use similarity IMPORTANT QUESTION: and change to solve real world problems? Answer: Similarity and transformations are useful tools for any real world of things that are the same shape, but of different sizes. 16 MODULE LESSON 16.1 Dilations LESSON 16.2 Proving Figures Are Similar To Using Conversions LESSON 16.3 Corresponding Parts Similar Figures This version is Algebra 1 and Geometry Only Professional Development VIDEO LESSON 16 4 AA Similar Triangles professional development video professional development my.hrw.com © Houghton Mifflin Harcourt Publishing Company • Image Authors: ©Izzy Schwartz/Digital Vision/Getty Images Author Juli Dixon models successful teaching practices in an actual high school classroom. REAL WORLD VIDEO Find out how properties of similarity and changes can be used to create scale models of large, real-world structures such as monuments. MODULE PERFORMANCE TASK PREVIEW Modeling the Washington Monument in this module, you will be challenged to create a plan scale model for the Washington Monument. How to use similarity and enlargements to help you produce an accurate model? Let's find out. Module 16 DIGITAL TEACHER EDITION IN2_MNLESE389847_U7M16MO 825 Access a full set of educational materials when and where you need them: • Access content online or offline • Customize lessons to share with your class • Communicate with your students in real time • View student scores and instantly guide your instructions where and where you need most 825 module 16 825 PERSONAL TRAINER Assessment and intervention Set automatically sorted homework, quizzes, tests and interventions. Prepare your students with updated, common basic practice tests. 4/18/14 7:47 PM YOU READY? You ready? Complete these exercises to view the skills you need in this module. Evaluate transformation readiness characteristics Example 1 Stretch △ABC with paragraphs A,1, 2, B(3, 2) and C(3, -1) horizontally and vertically with a factor of 4. x, y) → (4x, 4y) Use the assessment on this page to determine whether students need the necessary or strategic intervention to meet the prerequisites for the module. • Online Homework • Tips and Help • Extra Practice Write transformation rule. A′(4, 8), B(12, 8), C′(12, –4) Use the conversion to write each redesigned point. Describe the conversion. 1. Stretch △DEF points D(-2, 1), E(–1, –1) and F(-2, -2) horizontally and vertically with a factor of -3. D′(6, -3), E′(3, 3), F′(6, 6) 2. Is there a stretch of rigid motion? ASSESSMENT AND INTERVENTION No 3. Is it true that △DEF ≅ △D′E′F′? No. 3 2 1 Personal Math Trainer automatically creates a standard-based, personalized intervention task for your students that is tailored to the individual needs of each student! Similar figures Example 2 △ABC conversion to points A(3, 4), B(-1, 6) and C(0,1), shifting it to 2 units to the right and 1 unit up. (x, y) → (x + 2, y + 1) Write a conversion rule. A′(5) (5), B′(1, 7), C′(2, 2) Write down each reworking point. 5.4) Write the rule used to convert △ ABC. y x + 1, _ (x, y) → _ -4 (2 2 Describe the transformation event shown in the illustration. The answers may vary. Sample: Compress △ABC Vertically and IN2_MNLESE389847_U7M16MO 826 Tier 1 Lesson Intervention Worksheets Reteach 16.1 Reteach 16.2 Reteach 16.3 Reteach 16.4 B 2 -4 -2 horizontally with a factor of 2 and then shift it to the right to 1 unit and down to 4 units. Module 16 y A 0-2-4 C 2 A' x 4 B′ C' © Houghton Mifflin Harcourt Publishing Company Describe the transformation shown in the graph. 4. Additional resources for tier 1, level 2, tertiary skills See the table below for a complete list of the intervention resources available for this module. Response to intervention resources also includes: • Second-tier skill tests for each module • Second-level flights follow-up tests for each skill 826 Response to intervention Tier 2 Strategic intervention skills intervention worksheets 35 Dilatation characteristics 41 Similar drawings 29 Geometric drawings Differentiated guide 18/04/14 7:47 PM Tier 3 Intensive Intervention Worksheets available online Building block skills 36, 50, 80, 86, 95, 99, 103, 104, 112 Challenge Worksheets Expand Math Lesson Activities TE Module 16 826 LESSON 16.1 Name Enlargements Class Date 16.1 Dilations Essential Question: How does enlargement change the figure? Common Core Math Standards Student is expected: COMMON CORE G-SRT. A.1a, G-SRT. A.1.b experimentally check the properties of dilatations given center and scale factor ..... Also G-CO. A.2 △ABC and its image following questions after enlargement: MP.5 Use tools B Language purpose B' Work with partner to identify expansion and scale factor. A ENGAGE See the engage section online. Discuss the photo. Invite students to describe the relationship between the forms created by the hands and the shapes displayed on the wall. Then, check the hour performance task. © Houghton Mifflin Harcourt Publishing Company PREVIEW: LESSON PERFORMANCE TASK A' C B Use the ruler to measure the following lengths. Measure the nearest tenth of an inch. Important question: How does enlargement change the figure? The expansion changes the size of the number without changing its shape, so that the image resembles the image, but not the conglomerate. The Expansion Resource Cabinet is a transformation that can resize a polygon but leaves the shape unchanged. The enlargement centre has an expansion centre and a scale factor which together determine the position and size of the image of the drawing after enlargement. Mathematical practices COMMON BASIC Dilations characteristics Explore 1 C AB = 6,0 cm A'B' = 3,0 cm AC = 4,0 cm A'C' = 2,0 cm EC = 3,0 cm B'C' = 1,5 cm C' Use protractor to measure the corresponding angles. m∠A = 22° m∠A' = 22° m∠B = 33° m∠B' = 33° m∠C = 125° m∠C' = 125° Meet the following ratios _ 33° m∠C = 125° m∠C' = 125° Meet the following ratios _ 33° m∠C = 125° m∠C' = 125° Meet the following ratios _ 33° m∠C = 125° m∠C' = 125° 3,0 A'B' = _ = 1 _ AB 2 6.0 _ _ 2,0 A'C' = _ = 1 _ AC 2 4,0 1,5 B'C' = _ = 1 _ BC 2 3,0 Reflect 1. What do you notice about the corresponding sides of the numbers? What do you notice about the relevant angles? The ratio of the lengths of the respective sides is equal. The corresponding angles are consistent because the corresponding angles are the same. 2. Discussion What are the similarities between reflections, translations, rotations and enlargement? What difference does it make? Reflections, translations, rotations and enlargements all maintain an angle measure. Reflections, translations and rotations maintain all distances, but enlargements do not maintain distance. Module 16 must be EDIT--Chan EI key=NL-A; CA-Correction Lesson 1 827 gh File info made throu Date Class name ons 16.1 Dilati IN2_MNLESE389847_U7M16L1 827 figure? transformation center and enlargement given dilations How properties Question: entally Essential Verify experim G-SRT. A.1.b Dilyls .2 COMMON G-SRT. A.1a, CORE ..... Also G-CO. Perties scale factor in the shape of the ating Pro n but leaves ine 1 Investig er determ the size of polygo togeth Explore, which can change the factor rmation that n and scale n is transfo is the center of dilatio after expansion. Dilatio ns. Expansion of the figure questio unchanged. the image is next n and size n answe positio after dilatio △A'B'C' B and its image Use △ABC Resource Locker HARDCOVER PAGES 827,836 See hard student page numbers for this lesson. B' C A y g Compan Publishin Harcour t © Houghto n Mifflin _ Lesson 16,1 m∠A = 22° m∠B = 33° m∠C = 12 5° m∠A' = 22° m∠B' = 33° m∠C' = 1 125° _ 1 1,5 b'C' = _ = 2 _ BC 3,0? What the drawing ng side corners is correspondi sponding about equal. What did you notice that you've noticed the corners? sides are what to do the same. sponding korres s is a notice of lengths korre ding angle correspon ns, ratio dations, rotatio meas because ions, transla ctions, en reflect kongruent ure. By coming, there's a distance between the similarities of the angle of the rve. ons all prese ssion What is it? should be stored and dilati 2. Do not you know if it is good What's ons no s, rotations and dilatio nce but dilati lesson 1 s, translation rve dista Reflection ions all prese s and rotat translation 827 Module 16 827 _ re ctor to measu Use protra corners. 1. 1. 6L1 827 47_U7M1 ESE3898 IN2_MNL ◑ to the nearest measu Use ruler lengths. Measu eter. tenth of a cent 3,0 cm A'B' = cm AB = 6,0 = 2,0 cm A'C' 4,0 cm AC = 1,5 cm B'C' = cm BC = 3,0 ing ratios, followed by 1 2,0 Complete A'C' = _ 2 _ 1 _ 1 1 3.0 AC 4.0 A'B' = 2 _ AB 6.0 > C' A' 18/04/14 7:51 PM 18/04/14 7:53 Pm Explore 2 Dilating Line Segment EXPLORE 1 Expansion Line segment (pre-image) in line segment, the length of which is the product scale factor and the length of the pre-image. ‹ › – Use the following steps to apply dilation 3 times to the middle o-point, to ac current. O exploring the characteristics of dilations C to integrate technology B Students have the opportunity to do expansion activities either in the book or online. QUESTIONING STRATEGIES A To find Point A, pull it. B le fast a. draw ray from O to B. Place B′ its ray so that the distance from O to B′ is three times farther from O to B.C to find point C′, draw ray O to C. Place C′ on this ray so that the distance from O to C′ is three times the distance from O to C. D Draw a line through A', B', and C'. E _ _ _ _ _ — measure AB, AC and BC. Dimensions A'B, A'C and B'C'. Make assumptions about the lengths of the enlarged segments. Does it look like dilations maintain an angle? Yes, do enlargements seem to be in the distance? no What happens if you expand the scale factor 1? The picture and the image coincide. EXPLORE 2 The length of the enlarged segment is the initial length of the scale factor. 3. After expanding with a scale factor k, 4 cm of segment image length shall be applied up. Can a picture ever be shorter than a preview? The image is 4k centimeters long. Yes, if k is between 0 and 1, the image is displayed than the preview. 4. What can you say about the segment image under enlargement? Does your answer depend on the location of the segment? Explain the segment m image is parallel to the m. The only exception is when line m passes through the © Houghton Mifflin Harcourt Publishing Company Reflective Dilating Line Segment to integrate mathematical practices to focus on modeling MP.4 Use an overhead projector to project onto the board and then track the shape. Move the projector closer and follow the shape again to show the decrease. Take it further and follow it again to show expansion. in the middle of the dilation. In this case, the enlargement is on line m. Module 16 828 Lesson 1 questioning strategies for professional development IN2_MNLESE389847_U7M16L1 828 Math background dilations are one of the main changes that students are learning. Unlike previous conversions (reflections, translations and rotations), enlargements are not rigid movements. It is that they do not retain both the shape and size of the figure. However, enlargements do not maintain the shape of the figure. Thus, any enlargement is either enlargement or reduction. The image that results from the expansion is not generally a congruent pre-image, so the expansion is not isometry. The exception is enlargement with a scale factor of -1, which is equal to a 180° turn per centre of enlargement. 18/04/14 7:53 Is it ever possible that the image point drops exactly pre-image? Explain. yes, I don't know. if the scale factor is 1 Dilations 828 Explain 1 Explain 1. The relationship between the length of the corresponding sides of the image and the pre-image is called the scale factor. Application of dilation properties Dilation properties Dilation properties • Dilations retain the angle measure. • Dilations maintain a change. INTEGRATE MATHEMATICAL PRACTICES Focus on Math Connections MP.1 Can students review characteristics • Dilations to maintain colinearism. • Enlargement directions maintain orientation. • Dilations map a line segment (pre-image) to another line segment, the length of which is a product of scale factor and pre-image length. • The expansion directions map a line that does not pass through the centre of enlargement to the parallel line, and leave the line passing through the centre unchanged. similar indicators. Help them see that enlargement is a transformation that changes the size of the drawing, but not the shape. The image and the preliminary image of the number under enlargement are similar. Example 1 determine whether the coordinate-level conversion is an expansion. If there is, then give the scale factor. Maintains an angle measure: yes 6 Maintains the variation: yes D' A' 4 Maintains the colinearity: yes y A D 2 Maintains orientation: no Ratio of the respective sides: : 1 B B' C' -6 -4 -2 0 2 Cx 4 6 8 Is this conversion an expansion? No, it doesn't maintain orientation. ◑ maintains angle measurements (Y/N) © Houghton Mifflin Harcourt Publishing Company maintains an altercation (Y/N) Maintains the linearity (Y/N) of the scale factor Does this transformation have an expansion? Module 16 yes y 4 C' yes 2 yes yes x -4 829 -2 A 2 Yes C A' 0 -2 B B' -4 Lesson 1 collaborative learning IN2_MNLESE389847_U7M16L1 829 Peer-to-Peer Activity Can't get students into pairs. Have one student draw a picture and picture, and then the other will determine whether the picture is expanding and explain why or why not. If it's expansion, they should find a scale factor. Then invite students to switch roles. 829 lesson 16.1 18.18.2014 7:53 Your turn questioning strategies to determine whether changes are enlargements. 5. If the transformation retains the angle size, variation and colinearity, is the conversion an extension? Explain. Not necessarily; rigid transformations, such as translations, preserve these things. y 4 2-4 0-2 D The transformation retains the same angular dimensions, alternating, comlinearity and orientation, but not A' C' C D' B' B 2 -2 A 4 x 3. This is therefore enlargement. The scale factor is __ . 2 E' -4 6. AVOID COMMON ERRORS C 4 A Transformation maintains a change and x B -6 A' -4 B' C' Explain 2 0 -2 -2 Some students may think that any larger or smaller number is enlargement. Use examples that indicate that the shape and angles of the image and the image must be the same and the length of the sides proportional. • 2-8 distance. The ratios of the corresponding lengths are all the ratios of the measured, angled or corresponding sides. It doesn't maintain orientation. This is not enlargement. EXPLAIN 2 -4 Determination of the center and scale of dilation Determination the center and scale of dilation If you have a drawing and scale after enlargement, you will find the center of enlargement by drawing lines that combine the corresponding tops. These lines intersect at the heart of enlargement. the center of enlargement and the scale factor for the expansion of triangles. ‹ › − − ‹ › ‹ › – Draw AA', BB' and CC'. The point at which the lines intersect is the centre of enlargement. Mark the intersection O. Measure to find the scale factor. OA = 25 mm OB = 13 mm OC = 19 mm OA′ = 50 mm OB′ = 26 mm OC′ = 38 mm A' B' C' A Scale factor is 2 to 1. Integrate mathematical practices Focus on critical thinking MP.3 Discuss the reason why expansion does not © Houghton Mifflin Harcourt Publishing Company Example 2 influence the line through the center of expansion, but do not affect the line not through the centre. Remind students that the picture of the point line through the center of enlargement is on the line and moves only along it. B C O Module 16 830 Lesson Differentiated mentoring IN2_MNLESE389847_U7M16L1 830 Critical Thinking 18/04/14 7:53 Students may ask how to build an expansion image indicator if the scale factor is not an integer. Although it is not possible to use a compass and a straight line to create an image for each scale factor, certain scale factors can be used to copy the segment using a combination of construction, segment dual sector construction and segment division construction for a known relationship. If you wish, invite students to use a combination of these structures to draw an image of the triangle after enlargement with the dilation center O and a scale factor of 1.75. Enlargement 830 ‹ › − − − › › › – Draw AA', BB' and CC'. Measure from each point to the nearest millimetre at intersection O.B POLLING STRATEGIES How can you use this scale factor to predict whether the image is an expansion? Scale factor >1 increases expansion. OA′ = 30 mm OB′ = 19 mm OC = 52 mm B' Scale factor is O A' A 26 mm OC′ = QUESTIONING STRATEGIES C' B OB = 38 mm SPECIFIED 1 to 2 . Mirror How can enlargement produce an image in which the preview and image are two different conjugated digits? The image under dilation with a scale factor of -1 is in line with the pre-image and rotated to 180°. 7. What is the enlargement centre in your 5, which is the centre of enlargement? Explain how you can say it without drawing lines. Origin; several dots and their pictures are located on the axes, which intersect with origin. For points that are not on the axes, you can use the slopes to check whether the lines connecting the point and the picture pass through origin. Your turn 8. SUMMARIZE THE LESSON Specify the center of enlargement and the expansion scale factor. Measurements may vary, but the scale factor should not. © Houghton Mifflin Harcourt Publishing Company How can you determine whether the image figure is due to expansion? How could you set a scale factor? If the sides of the image are proportional to the sides of the pre-image and the corresponding angles are conjugated, the image shall be enlargement. If the numbers are in the coordinate belt, you can divide the coordinates of the points in the image by the coordinates of the corresponding points in the pre-image to find the scale factor. C OA = 60 mm OA' = 18 mm cm, OA = 54 mm Dilation scale factor is 1 to 3 A' Operate 9. B C . C' B' How is the length of the image of the line segment O under the expansion associated with the length of this pre-image? The ratio between the length of the image and the length of the pre-image is a factor of scale. 10. Discussion What is the indicator to expand using a scale factor of 1? In the event of this enlargement, will the position of the image of the the preview? Explain. The expansion by the scale factor 1 leaves the number unchanged. It shall remain in the same position, regardless of which point is used as the centre of enlargement. Module 16 831 Lesson 1 LANGUAGE SUPPORT IN2_MNLESE389847_U7M16L1 831 Connect Verbal dilation to get larger or smaller using, for example, the human eye. Students expand in response to changes in light; in the dark, they get bigger to let more light, and in bright light, students become smaller to reduce the light entering the eye. Explain to students that when people talk about student expansion, they use the mathematical term correctly. 831 Lesson 16.1 18.2014/14 19:53 11. Important question check-in in general how does enlargement change the figure? Enlargement changes the size of the drawing without changing the shape. Dilations retain price angle Measures and direction, but not length. Evaluate: Homework and internship 1. • Online Homework • Tips and Help • Extra Practice To Consider The Concept of Expansion. Enlargement is a transformation that can change the polygon size, but leaves the shape unchanged. How are the ratios of the measures of the respective parties involved in enlargement? DETERMINATION INSTRUCTIONS The ratios of the lengths of the respective sides are equal. Tell me if one figure appears to be an expansion of another number Explain. 2. 3. No, it is not enlargement, because the ratio of the lengths of the respective sides is not equal. Some sides are about the same length and others are not. It seems to be an expansion that maintains the angle of measure, variation, colinearity and orientation. It also seems that the ratio of the lengths of the respective sides is equal. 4. 1 ? Explain. Is △ of the extension of the ABC is equal to _ 2 6 4 A' A 5. B' B Exercise IN2_MNLESE389847_U7M16L1 832 6 8 B 28 Exercises 1–5 Explore 2 Dilating Line Segment Exercises 6-11

Example 1 Application Features Dilations Exercises 12-13 Example 2 Finding Center and Scale Dilation Exercises 14-15 Similar? Why or why not? - yes, I don't know. when the numbers are in sync, there is a jab of rigid movements that maps one to the other. Rigid movements are also similar changes, so the figures must also be similar. Lesson 1 832 Depth Knowledge (D.O.K.) x 4 A _ Module 16 2 35 Square A is an expansion square B. What is the scale factor? 1 5 35 a. _ = The answer is (c). The ratio is 7 4 28 4 b. _ 5 5 c. _ 4 d. 7 25 e. _ 16 Explore the characteristics of dilation to integrate mathematical practices to focus on Math Connections MP.1 If the two digits are compatible, they are also C 2 0 Practice C' © Houghton Mifflin Harcourt Publishing Company No, the scale factor is 2. The centre of enlargement is (0,0). The dimensions from the centre of enlargement to A' are twice the size of the centre of enlargement to A. The Commission has is greater than the pre-image so that the scale factor must be greater than 1. y Concepts and skills COMMON CORE Mathematical practices 1-5 2 Skills / Concepts MP.2 Recital 6-15 2 Skills / Concepts MP.5 Using Tools 16-17 2 Skills / Concepts MP.3 Logic 18 2 Skills / Concepts MP.4 Modeling 19 3 Strategic Thinking MP.1 Problem Solving 20 3 Strategic Thinking MP.1 Solving problems 18/04/14 2 Skills 832 6. AVOID COMMON ERRORS _ Apply AC expansion with a scale factor of 2 and mid 6. 7. _ 1 Apply geelac with scale factor _ 3 and mid point O. O Some students may find it difficult to determine scale factors. In particular, they may have difficulty distinguishing enlargement by scale factor k, which is a scale factor with a scale factor _ 1 k. Remind students to always compare the picture to the preview. Also, remind students that if the image is an increase in the pre-image, the scale factor must be greater than 1. O C C' B A' B' C B' A' B. C'A What happens when the triangle expands, using one of the peaks as the centre of expansion? 9th Draw an image of WXYZ. The centre of enlargement is O and the scale factor is 2. The sides of the triangles adjacent to the centre of enlargement are colinear. The third side of the preview and the picture is parallel. The peak used as the centre of expansion is in the same place in both triangles. X' Y' X Y O Z W Z' 10. Image △ABC . The centre of enlargement is C and the scale factor is 1,5. 11. Compare enlargements with rigid movements. How are they alike? How are they different? Rigid movements maintain the angle measure, between them and the collinearity. Dilations retain all but distance. The expansion of the line segment (preliminary image) is the second line segment, the length of which is the basis of the scale factor and the length of the pre-image. B' B C A A C' Determine whether the transformation of Figure A to figure B at the coordinate level is enlargement. Check the ratio of the respective side lengths for enlargement. 12. 13. y 6 y 8 4 6 B 4 B 2 A 2 x A 2 4 6 x 0 2 4 6 8 10 Yes, this is enlargement. The relationship corresponding to this is equal. No, this is not enlargement. The relevant 2 - side length is _ . 2 1 Module 16 IN2_MNLESE389847_U7M16L1 833 833 Lesson 16.1 833 Lesson 1 1/18/04/14 7:53 PM Specify dilation of the mean and scale factor expansion. 14. 15. E' A' F D C' E' B' A F' D' B C O Scale factor is 1-2 . The scale factor is 3 to 1 . 16. You work in a photography shop. The customer has a picture that's 4.5 inches long. The customer wants a reduced copy image to match the space of the 1.8 inch long postcard. What scale factor should you use to reduce the image to the right size? 1,8 _ 2 _ = 5 4,5 in 2 A scale factor should be used. 5 _y Length the preliminary image ∣5-2∣ = 3 units. 14 C'(15, 12) B'(6, 12) The height of the preliminary image is ∣4 − 2∣ = 2 units. 10 The height of the ∣12−6∣ = 6 units. 8 6 4 2 Image length ∣15 - 6∣ = 9 units. B(2, 4) C(5, 4) Module 16 IN2_MNLESE389847_U7M16L1 834 6 3 lengths is _ = _. The scale factor is 3-1. 1 2 D(5,2) A(2,2) 2 3 9 The height ratio is _ _ 1 3 D (15, 6) A(6, 6) x 4 6 8 10 12 14 16 18 834 © Houghton Mifflin Harcourt Publishing Company • Image Credits: ©Digital Vision/Getty Images 17. Computer Graphics Artist uses a computer program to enlarge the theme, as shown. What scale factor did the artist use? ©Houghton Mifflin Harcourt enlargement? Lesson 1 18/04/14 7:53 Dilations 834 18. Explain the error, what mistakes did the student make when he tried to discover the center of enlargement and scale factor? Specify an expansion center. JOURNAL Can students write an explanation of how, given the triangle and its image under enlargement, you might want to use a ruler to find the sting factor of the expansion factor. P P' O O R' Q' R Lines were built ‹ › - wrongly. Po should pass through ‹ › − points P and P' make up PP'. The lines must pass through the top and the corresponding peak to meet at the center of the expansion O. Q F' y H.O.T. Focus on higher-level thinking in the 19th century. Draw△DEF with peaks D (3, 1) E (3,5) F (0.5). 12 years. Determine the △ perimeter and area of the DEF. 10 The perimeter is 12 units, the area is 6 square units 8 b. Draws the image △DEF after enlargement with a scale factor of 3, with the center of the enlargement being the origin (0,0). Determine the perimeter and area of the image. 6 E F 4 The perimeter is 36 units, the area is 54 units c. E' 14 D 2 perimeter △D'E'F' How is the scale factor related to ratios _ perimeter △DEF region △D'E'F' and _ ? x Area D △DEF 0 2 4 6 perimeter △D'E'F' 36 3 area △D'E'F' 54 9 _ _ = _ = scale factor; _ = _ = square perimeter of △DEF 12 area △DEF 1 6 1 © Houghton Mifflin Harcourt Publishing Company 20. Draw △WXY peaks (4, 0), (4, 8) and (-2, 8). A. Dilate △WXY using factor _14 and origin in the middle. Then expand your image using a scale factor of 2 and the origin of the middle. Draws the final picture. b. Use the scale factors in part (a) to determine the scale factor that you can use to dispense △WXY, the origin of which is the center of the final image in one step. 1 1 ×2= _ = the scale factor multiply the scale factors _ 4 2 multiply the pre-image to draw the final image in one step. c. 8 y Y X 8 6 X Y 2 X' 2x -2 0 2W 4W Do you get the same final image when you switch part (a) in the order of dilations? Explain your reasoning. 1 1 1 =_ this gives you the same scale factor Yes. If multiplied by 2 × _ 4 2 because multiplication is a commutation. Module 16 835 835 Lesson 16.1 835 Lesson 1 1/18/04/14 7:53 PM Lesson Performance Task integrate mathematical practices focus patterns mp.8 rectangle measuring 3 inches 6 inches is you have hung a sheet on the wall and lit sticker. Now you move your hands between the like and the page, and you create a picture of the creation of the story on the page for the great fun of your audience. Compare and stand what you do with what happens when you draw a triangle expansion on the coordinate plane. Indicate how enlargement and hand puppets are similar and different. Discuss the measures that are preserved in hand-puppet projections and those that are not. Some terms you might want to discuss: projected on the wall. The image of the rectangle on the wall is an area of 200 square inches. What are the dimensions of the wall rectangle? Explain. 10 x 20 years ; sample answer: Since the length of the smaller rectangle is twice as wide, the length of the larger rectangle must also be twice the length. The problem becomes one to find two digits in the ratio of 2:1 product to 200. Solutions 10 in. and 20 in. can be found using a think-and-check problem-solving strategy. • before image • image • image • expansion kes • scale factor • transformation • input • output points that students can do: • the expansion of the coordinate level takes place in two dimensions, shadow cam in three dimensions. INTEGRATE MATHEMATICAL PRACTICES Focus on Math Connections MP.1 Small rectangle is suspended halfway • If the shadow doll is projected to the surface, the angle dimensions are preserved. The lengths have not been preserved. The depth of the hands is not retained, but is converted into a two-dimensional image. © Houghton Mifflin Harcourt Publishing Company • Image Credits: ©Digital Vision/Getty Images • Pre-image or your hands. The picture is a shadow. The centre of enlargement is the light source. • The scale factor of the shadow doll projection is the ratio of the measurement to the fan at a level parallel to the wall at a level parallel to the wall. Module 16 836 between the flashlight and the wall. It's parallel to the wall. Looking from the side, what number and special segment of this situation resembles? How could you use your knowledge of the segment to find unknown dimensions? Sample answer: The light of the flashlight, the side of the rectangle and the side of the shadow form a triangle with a central segment drawn. According to the theorem of the central segment, the side of the rectangle is half the length of the side of the shadow. Lesson 1 extension activity IN2_MNLESE389847_U7M16L1 836 Teams of three or four need a flashlight, index card and a way to support a card fixed away from the wall and parallel to it (one crew member can keep the card, keeping it as parallel as possible). One student holds the second measures the distance of the flashlight from the rectangle and the third measures the dimensions of the projected rectangle. Teams should make a measurement of the flashlight at different distances to the rectangle. They can then examine their data and assume the relationship between the dimensions of the object and the dimensions of that image when it is projected on the wall. 18/04/14 7:53 Scores Rubric 2 Points: The student's response is accurate and complete execution of tasks or tasks. 1 point: The student's response contains the characteristics of the appropriate response, but it is invalid. 0 points: The student's response does not include the characteristics of the relevant response. Enlargements 836 LESSON 16.2 Name Proving Figures Are Similar In Using Changes Important Question: How Can Similarities Be Used to Show Two Figures Are Similar? A similarity conversion is a conversion in which the image has the same shape as the pre-image. Changes in similarity include reflections, translations, rotations and enlargements. The two plane numbers are similar if and only if one number can be mapped to another through one or more similarity conversions. G-SRT. A.2 In view of the two digits, use the definition of similarity of similarity changes to determine whether they are similar ... So is G-C. A.1 The network shows the map of the city park. Use the observing paper to confirm that the park elements are similar. Mathematical practice trace patio EFHG. Rotate the paper so that the terrace EFHG is mapped onto the patio LMON. Describe the conversion. What does this patio confirm? y MP.6 Precision A Verify the same figures. What does the figure JKLM similar? If not explain. Language Support Explain that the text gives the essential components needed to prove the figure is similar to the other. -8 View Engage section online. Discuss the photo. Invite students to identify the game they are playing and the board where it will be played. Then, check the hour performance task. -6 -4N -2 H 0 2 x G 4 6 8 -2 L -4 O © Houghton Mifflin Harcourt Publishing Company PREVIEW: LESSON PERFORMANCE TASK E 2 DEAL IF SIMILARITY CHANGES CAN BE SHOWN TO MAP ONE INDICATOR TO ANOTHER, the figures are similar. F 6 Language purpose Important question: How can similarity changes be used to show the two digits are similar? Source Locker Confirming Similarity Explore student is expected: COMMON CORE Date 16.2 Proving figures are similar to those used in the Common Core Math Standards COMMON CORE class -6 M rotation of 180° around origin; Fountains are alike. B Trace statues ABCDEF and JKLMNO. Fold the paper so that the shape of abcdef is mapped onto the statue JKLMNO. Describe the conversion. What does that confirm about the statues? y K M -6 -4 N 4 D 2 L B 4 A C D O -2 2 x 6 F Reflection over y axis; The numbers are similar. be done edit--Chan NO Key=NL-A; CA-A − everything is Lesson 2 837 gh File made throu Date Class r s Are 16.2 837 gh File ing Figure 16.2 Pro Conversions Using Name IN2_MNLESE389847_U7M16L2 837 HARDCOVER PAGES 837,850 Resource s on Locker two drawings to show ions use ns transformat transformation similarity? Discover... So is G-C. A.1 G-SRT. A.2-view. Similar sawing that conver is ions pre-im as a shape of image which is two planes ns. n Examine rmatio and dilatio rmations. ion is transfo tions, rotation, rity transfo r. transla transformation more simila is simila h one or similarity ns include thoughts, elements of other throug that park transformatio mapped similarity can be . Tracking paper in one drawing Use the LMON patio in the city park. mapped onto a map of EFHG's grid showing patios? to the patio see the hardcover student edition page numbers for this lesson. so m about paper it confirms EFHG. Turn on what trace patio ormation. y transf Describe f 6 ⚲ 4 H E 2 -8 -6 -4N -2 O M y g Compan x G 0 2 4 6 8 -2 L -4 -6 similar. ains are ; The origin of the fount is mapped 180° aroun ABCDEF s? so the shape m about the shape of the paper it constants NO. Fold and JKLM What orientation. s ABCDEF Trace shape ite transl NO. Are shape JKLMN y B 4 K C A figures are J 2 y-axis; X D total M L A reflection 6 4 2 0 -2 statue. -6 4 E F -2 ⚲ © Houghton Mifflin Harcour t Publishin Rotation N Lesson 2 0 837 Module 16 16.2 837 47_U7M1 ESE389 M2_MNL 837 Lesson 16.2 18/04/14 7:58 PM 18/04/14 7:58 PM 0 IN2 2 Explain 1 Do you know this garden? Confirming similarity 2 E x -8 -6 -4 -2 0 D R 0 2 -2 4 F 6 8 T S y 4 2 COMMON CORE gross settlement total. 2 integrate technology into gardens are similar. Students have the opportunity to do transformation activities either in the book or online. Let's reflect the 1st Look back on all the steps. Were any pictures pre-pictures? If so, what similarities were made to these figures? What does this tell you about the relationship between similar and synchronized numbers? The two figures in phases A and B are consistent. What types of changes result in similar figures? similarity. these figures were used for rotations and reflections, which are rigid movements. So the synchronized figures are similar figures. 2. What kind of advance, what are the ones that provide the numbers in sync? Which transformations can lead to similar but not conjugates? rigid movements; enlargement If two are similar, can you conclude that the relevant angles are in sync? Why or why not? Yes, the corresponding corners are congruent because rigid movements and expansions maintain the ang-corners. If the figures are similar, please explain 1 You can mark enlargements using the coordinates sign (x, y) → (kx, ky), where k is the scale factor and the center of enlargement is origin. If 0 < k < 1, expansion is a reduction. If k > 1, expansion is expansion. ⚲ △RST and △XYZ X Map △RST onto △XYZ, there must be some factor k that extends △RST. y R x -5 0 T Z Pre-enlargement R(0, 1) X(0, 3) S 5 Y Module 16 S(1) T(-1, -1) Y(3, -3) Z(-3, -3) 838 if the figures are similar to © Houghton Mifflin Harcourt Publishing Company Determine whether the two digits are similar using similar changes. Example 1 EXPLAINING 1 QUESTIONING STRATEGIES Is expanding the only changes that lead to an image that is similar to a pre-image? Explain. No, rigid movements, such as translations, lead to numbers that are in sync and thus similar. Lesson 2 PROFESSIONAL DEVELOPMENT IN2_MNLESE389847_U7M16L2 838 18/04/14 19:58 Math Background Two conditions must be met to say that the two polygons are similar. The two polygons are similar if and only if their respective angles are compatible and their respective sides are proportional. Similarity is the equivalence ratio; it is, the similarity is reflexive, symmetrical and transitive. Figures E, B and C, A ~ (reflexivity); if ~ B, then B ~ (symmetry); and if ~ B and B ~ C, then ~ C (transit). The synchronized figures are always similar, but the similar figures are not necessarily in sync. The similarity conversion that maps each sine (1) is an expansion of the middle origin, which is represented by coordinate characters (x, y) → (3x, 3y). Enlargement is a similarity transformation, so △RST resembles △XYZ. INTEGRATE MATHEMATICAL PRACTICES Focus on Math Connections MP.1 Can students discuss different approaches to B PQRS and WXYZ similarity of the similarity, including an intuitive approach (the same shape, may be of different sizes); meet the conditions for alternating angles and proportional sides; and a transformative approach– if you can be obtained through changes in the numbers of similar. To map PQRS onto WXYZ, there must be some factor k that increases PQRS. y X(5, 9) Y(12, 9) 8 6 W(5, 5) Pre-picture dis(2, 2) W(5, 5) R(6, 4) Y(12, 9) Z(12,5) X(5, 9) Q(2, 4) R(6, 4) P(2, 2) S(6, 2) Q(2,4) Q(2, 4) S(6, 2) Z(12, 5) X 0 2 4 6 8 10 12 X5 Y(5, 9) Z(12, 5) 14 Find each distance: PQ = 2, QR = 4, WX = 4 and XY = 7. If kPQ = WX, then k = 2. But 2QR = ≠ XY. The K value cannot be identified that maps PQRS to WXYZ. So, the figures are/are not similar. It's your turn whether the two digits are similar to the similarity changes. 3. LMNO and GHJK y G Houghton Mifflin Harcourt Publishing Company K(2, 12) L(2, 12) J(8, 12) 10 8 6 O(1, 6) 4 N(4, 6) 2 M(4, 2) x 4 6 8 10 H(4, 2) G(2, 4) L(1, 2) 0 2 H(8, 4) M(4, 2) 4 x 6 8 10 Yes; you can use the dilation scale factor 2 and the origin to map LMNO to GHJK. Module 16 839 Lesson 2 Collaborative learning IN2_MNLESE389847_U7M16L2 839 Small group activity Do students work in small groups to design and exchange logos; the logo should repeat one design in several similar numbers. Have them write a description attached to the logo that documents the change. Lesson 16.2 18/04/14 7:58 4. △JKL and △MNP CDEF and TUVF 5. J y C K -5 L 0 M T N F Finding the order of similarity conversions x x 5 U 0 -5 P D No, angles are different. △JKL and △MNP are not similar. Explain 2 Explain 2 y V 5 E integrate mathematical practices in Focus on Math Connections MP.1 Help students see that figures are Yes, the dilation center is located at point F. Scale factor 2. CDEF is similar to TUVF. Finding a sequence of similarity conversions is the relationship of similarity. For the two digits to be similar, there must be some sort of similarity to a different number. Sometimes one similarity is conceived in the sequence of similarity changes. Sometimes, you need to identify more than one conversion to describe the mapping. Example 2 ⚲ Find a similarity conversions list that maps the first figure to the second drawing. Write a note for each conversion coordinate. ABDC to EFHG y A 5 B D C F x E -5 0 5 G H © Houghton Mifflin Harcourt Publishing Company Since EFHG is_ is less than ABDC, the scale of k must be between 0 and 1. AB length is 4 and EF length 2; therefore, the scale factor _12 is Write new coordinates after expansion. Transition rights 2 units and less than 3 units will stop mapping. ( ) Coordinates after enlargement 1.3 A' _ 2 5, 3 B' _ 2 C'(1, 1) D'(1, 1) Coordinates after translation (x + 2, y - 3) 5 E _ 0(2) 0 9 F _ (0 2) G(1, -2) H(3, -2) The post-translation coordinates are the same as the coordinates of EFGH so that you can map ABDC to EFHG by expansion (x, y) → _12 x, _12 y), followed by translation (x, y) → (x + 2, y - 3). (Module 16) 840 Lesson 2 TO DIFFERENTIATE INSTRUCTION IN2_MNLESE389847_U7M16L2 840 18/04/14 7:57 Modeling To help students understand the concepts of this lesson, they have to draw figures in and beyond images Plane. Then let them cut out the numbers and see when they can be done to make them match and when they can't. The supporting figures are similar to using changes from 840 △JKL △PQR B questioning strategies y How to use scale factor dilatation to find coordinates for points in the image indicator? Multiplies the coordinates of the points in the preview by the scale factor. R 8 6 4-8 AVOID COMMON MISTAKES Students may think that all rectangles are similar. Challenge them to draw two rectangles that have no proportional sides. -6 -4 J 0 -2 L P 4 6 8 You can map △JKL △PQR with reflection over x axis followed by enlargement followed by 90° counterclockwise rotation of origin. • Reflection: (x, y) → (x, y) Enlargement: (x, y) → (3x, 3y) 90 counterclockwise rotation: ( ) y, x, y → ( ) Reflective 6. Using example 3A in the figure, describe one enlargement that maps ABDC to EFHG. By combining the corresponding peaks, you can identify (4, -6) as the centering dilation scale factor _12 that maps ABDC EFHG. 7. Use the figure in example 3B to describe the different order of conversions that △JKL △PQR. Mirror △JKL over the y axis. Then expand to the center of the origin and scale factor 3. Then © Houghton Mifflin Harcourt Publishing Company turned 90° clockwise around origin. Your turn Each pair of similar numbers, find a given 3 similarity conversions chart that maps one indicator to another. To describe the conversion, use the coordinates rule. 8. PQRS to TUVW Q 6 R U V -8 -4 W 2 T 0 Reflection: (x, y) → (-x, y) 5 P 2 Followed ... x 2 Module 16 4 6 PQRS can be mapped with TUVW followed by expansion. 841 Dilation: (x, y) → (_13 x, _13 y) Lesson 2 LANGUAGE SUPPORT IN2_MNLESE389847_U7M16L2 841 Connect Vocabulary Make sure students are not confused by the similarities between daily English and maths. In everyday life, things can only be called similar based on how they appear. But in mathematics, we need to look at scale factors on the respective sides and the angle measure in two similar numbers. 841 Lesson 16.2 18/04/14 7:57 △ABC to △DEF EXPLAIN 3 y F 8 You can map △ABC △DEF around the beginning of 180° followed by dilation followed by translation. 6 Rotation: (x, y) → (-x, -y) 4 D 2 E Then... x -8 -6 B -4 -2 A 0 2 4 6 Proving all circles are similar 3 to 8 (integrate mathematical practices focus on critical thinking MP.3 Help students understand that each enlargement is) 3 _ dilation: (x, y) → _x, _y 2 2 2 Followed ... Translation: (x, y) (x + 5, y) -2 -4 inversely proportional and scale factors are mutual. C 10 Could you use an enlargement proof that the two circles are similar, unknown length of your radii? Using the ratio of circle sides in the scale factor, you understand. Describe the similarity conversions in the list that maps JKLMN to WXYZ. QUESTIONING STRATEGIES How can you use rigid movements to show that two congruent circles maps circles on top of each other. This applies to all circles of all sizes, so you don't need to know the specific lengths. Translate JKLMN right to 7 units so that J maps V. ‹ › − Mirror JKLMN throughout JN . M K 1 Dilate JKLMN with central J and scale factor . 2 J V N Z W X Explain 3 Y Prove All Circles Are Similar You can use the definition of similarity to prove theorem figures. © Houghton Mifflin Harcourt Publishing Company Circle Similarity Theorem All circles are alike. Example 3 Prove Circle Similarity Theorem. Simple: Circle C in mid C and radius r. Circle D in mid D and pathways s. s r D C Prove: Circle C is similar to circle D. To prove similarity, you must show that there is a jam of similarity conversions, which maps round C round D. Module 16 IN2_MNLESE389847_U7M16L2 842 842 Lesson 2 18/04/14 7:57 proving the figures are similar to using conversions 842 → Start by changing the circle C with translation along vector CD Work s questioning strategies D r C What scale factor is needed to expand circle radius 2 to get a circle radius 3? _32 r → CD Circle C translation , the image of point C through point D . Let the image of round C be circle C⃗. The centre of the circle C⃗ coincides with the summary of the lesson . Transform the circle of C⃗ with dilatation in the middle dilation D and scale factor _rs . B How do I use conversions to determine if the two graphics are similar? Define whether one drawing can be mapped to another by a similarity conversion. D Circle C⃗ shall consist of all points from r to point D. _s after expansion, the image of the round C⃗ consists of all points of distance r × r = s from point D. translation followed by enlargement These are the same points that make up the circle D . Therefore, the similarity covers translations of D cards into the round C round. Because and expands is, you can conclude that ring C is similar to ring D . Reflect 11. Can you show that ring C and ring D are similar through another in a jam of similarity conversions? Explain. _ Yes, may reflect round C over the cross bisector CD, mapping point C to point D. © Houghton Mifflin Harcourt Publishing Company Then follow the same steps of dilatation. 12. Discussion Is it possible that ring C and ring D are in sync? If so, does the evidence of the similarity of circles still work? Explain. Yes, if the ratio of each circle has the same positive value. The proof still works because the ratio s and r is 1, so the expansion does not affect the size of the circle. Please specify the 13th paragraph of The New Year's Translations, debates and are rigid movements. What unique feature keeps enlargements considered rigid motion? Unlike other conversions, enlargements do not maintain distance, which means that the length of the sides does not remain the same. Because they compensate for the respective sides shall be proportional according to the scale factor used. 14. The important issue check-In Two squares at the coordinate level are horizontal and vertical sides. Explain how they are similar by using similarity conversions. Possible answer: Convert one square to another square. The first square then expands according to the side-length ratio. Module 16 IN2_MNLESE389847_U7M16L2 843 Lesson 16.2 843 Lesson 2 18/04/14 7:57 PM Evaluate: Homework and Practice Evaluate Exercises 1-4, determine whether these two digits are similar using similar changes. Explain. 1. Looking at the two digits, you can see that EFGH must be EFGH and ABCD 5 enlarged with some factor k to be mapped abcd. Y A E -5 D F H 0 Preliminary image E(0, 1) A(0, 4) G(0, -1) C(0, -4) x B F(1, 0) 5 G -5 • Online Homework • Tips and Assistance • Practice Extra H(-1, 0) C ASSIGNMENT GUIDE B(4, 0) Definitions and Skill Practice D(-4, 0) Explore Confirming Similarity Exercises 11-12 Example 1 To Determine Whether The Figures Are Similar to Exercises 1-4 Example 2 Find a jab of similarity changes Exercises 5-10 Example 3 Proving that all circles are similar to the lesson scale factor 13, you can see that k = 4. Therefore, EFGH can be mapped to an ABCD with a color of 4 with a coordinate notice (x, y) in the middle of the dimension → (4x, 4y). 2. △PQR and △STU, JKLMN and JPQRS. y K value does not exist △PQR and △STU. S x -5 0 5 R Q T U © Houghton Mifflin Harcourt Publishing Company 3. R JKLMN and JPQRS In order to map JKLMN to JPQRS, there must be a scale factor k that reduces JKLMN. In such a situation, the centre of enlargement is not origin, but point J. 3 The scale factor is _ for enlargement. 5 Since angles are congruent, JKLMN is similar to JPQRS. y L We -8 -6 -4 Lesson IN2_MNLESE389847_U7M16L2 844 M 4 Q 2 P -2 J Module 16 6 R 0 x 2 4 6 844 2 Lesson 2 COMMON CORE Mathematical Practices 1-4 1 Recall information MP.1 Problem Solving 1-10 2 Skills / Concepts MP.4 Modeling 11-12 3 Strategic Thinking 18-19 2 Skills/Skills/Concepts MP.2 Reasoning 3 Strategic Thinking MP.2 Reasoning 20 8-2 844 Depth of Knowledge (D.O.K.) 18/04/14 7:57 Proving the figures are similar to those using changes 844 4. AVOID COMMON ERRORS △UVW and △GHI y Some students may confuse that similarity and congruence and similarity. Let them draw two similar triangles that are not congruent and two triangles that are compatible with each and mark each number with an appropriate term. Then you have to repeat them in a few circles, make it clear that all circles are similar, but not all circles are in sync. U W V -5 0 No, the angles are different. Similarity changes, angle-angle measures in triangles are not preserved, so △UVW and △GHI are not similar. x 5 H I G For each pair of similar numbers of 5 to 10 exercises, you can find a similarity in a difference between the changes that map one number to another. Indicate the coordinates of each conversion. 5. Map △ABC △PQR. A -6 -4 B -2 6. y C y 4 B 2 A J 0 R 2 4 5 G F -5 P Q You can map △ABC △PQR reflection followed by translation. JKLMN can be mapped to EFGH reflection, followed by expansion. Reflection: (x, y) → (-x, y) Reflection: (x, y) → (x,y) Dilation: (x, y) → (2x, 2y) Map △CED to △CBA. 8. Map angle ACB △DCE. Translation: (x, y) → (x + 6, y) 7. Y D A C A B C D 5 D1 J -2 K -5 You can enlarge △ACB 5 1 . . . _ x, _ 1 y 2 2 (Translation: (x, y) → (-x, y) ( ) 1 _ enlargement: (x, y) → _ x, y 2 2 Map △JKLMN to JKLMN. y 2 . . . x 2 Module 16 IN2_MNLESE389847_U7M16L2 845 845 Lesson 16.2 He x 16 R C D 4 M ABCD can be mapped to JKLMN reflection followed by the center of origin followed by a translation. A ... (x, y) → (x, y-6) © Houghton Mifflin Harcourt Publishing Company C D 0 E -5 x 4 -7. Map ABCD EFGH. 2 Lesson 2 845 DEPTH OF KNOWLEDGE (D.O.K.) _1 , y − 2) COMMON CORE Mathematical Practices 21 3 Strategic Thinking MP.3 Logic 23 3 Strategic Thinking MP.3 Logic 18/04/14 7:57 9. Map △JKL △PQR card ABCD JKLM. y K -8 -6 -4 L K 6 4 C D -2 B y 4 2 A 0 2 2 4 6 8 -2 J 0 P 2 -4 L -4 2 Q R 4 P 2 4 6 -2 -4 M ABCD can be mapped to JKLM reflection, followed by enlargement by the centre of origin followed by a translation. You can map △JKL △PQR with the PQR, followed by an expansion followed by a 90° clockwise rotation around origin. Reflection: (x, y) → (x, y) 10 1 _ Enlargement: (x, y) → _ _ x, y 1 y 3 3 Rotation: (x, y) → (y,y) enlargement: (x, y) → (3x, 3y) Translation: (x, y) → (x + 1, y − 2) Full proof. 11. Simple: square ABCD side length x. Square EFGH lateral length y. E A Y C H H x D G Step A: Extend ABCD with mid-dilatation A and scale factor _x , producing a square A'B'C'D'. Step B: 12. EF X They're the same side lengths y. The corners are all 90°. So A'B'C'D' is a congruent TO EFGH. → one that produces ABC'D. Step B: Translate A'B'C'D' with translation extension vector A'E this translation, A is mapped to E. It may be true that B' lands F, C is mapped to G and D is mapped to H. If not, rotate the ABCD about E so that B' is mapped to F. Then, C'' lands G and D'' lands H. y module 16 IN2_MNLESE389847_U7M16L2 846 846 © Houghton Mifflin HarCourt Publishing Company Prove: Square ABCD is similar square EFGH. Lesson 2 18/04/14 7:57 proving the figures are similar to using Conversions 846 12. Simple: Equilateral △JKL lateral length j. Proportional △PQR side length p Multiple representations Invite students to rewrite expansion rules that use dialects from everyday. Example: 3 3 (x, y) → (x, y) → _4 x, _4 y , (x, y) → (0.75x, 0.75y) (J P Prove: △JKL is similar to △PQR.) p j R L Q K Step A: Dilate △JKL mid Direct and scale factor _pj , producing △J'K'L'. After enlargement △J'K'L' is three sides p length _j (j) = p. There are the same side lengths as △PQR. By SSS Triangle Congruence, △J'K'L' is in line with △PQR. → that produces △JKL. Through this translation, Step B: Translate △J'K'L' along the vector JP J is mapped P. It may be true that Q and L are mapped to Q and R. If not, rotate △JKL p so that K' is mapped to R. 13. Simple: △ABC AB = c. BC = a. AC = b cx bx △XYZ yz = x, XY = _a, XZ = _a Prove: △ABC is similar to △XYZ. cx a x c Step A: Dilate △ABC with the center dilation B and scale factor k _a , producing △A'B'C'. After expansion, △A'B'C' is the sides length BC' = also = _a (a) = x, bx cx x _ a = b and A'B' = kc = . These are the same side lengths as △XYZ. By SSS Triangle Congruence, △A'B'C'is in line with △XYZ. → that produces △ABC. Through this translation, Step B: Translate △A'B'C' along the vector B'Y B is mapped Y. It may be true that A and C are mapped to X and Z. If not, rotate △ABC y so that A'' is mapped to X. Then L'''s mapped with R. 14. The dimensions of the standard tennis court are 10 meters. × 28 feet. a net that is 3 feet. high in the middle. The court has changed players to age 10 and so that the dimensions are 27 feet × 60 feet., and the same net is used. Use the similarity to determine whether the court that has been amended is similar to the standard court. 4 The width dilation factor is _ and for length 13 dilation factors are _10 The cords are therefore not alike, since the enlargement factor is not constant between width and length. Module 16 IN2_MNLESE389847_U7M16L2 847 Lesson 16.2 847 Lesson 2 18/04/14 7:57 15. To represent the Real-World Problems Scuba flag is used to show the diver below. In North America there are scuba flags with a red and white stripe from the upper left corner of the lower right corner. The parallelation of the triangles formed on the flag of the scuba flag is similar to the triangles. Since the base and the height of the triangles are equal, there has been no expansion. Triangles are 180° reflections of each other. The triangles are similar to each other. 16. The most common image size is 4 inches. ×6 inches. Other normal image sizes (in inches) are 5 × 7, 8 × 10, 9 × 12, 11 × 14, 14 × 18 and 16 × 20. A. Are any of these picture sizes similar? Explain the similarity by using conversions. For a picture to be similar, there must be a scale factor k that maps one image to another. The ratio of the sides of each image is 2:3, 5:7, 4:5, 3:4, 11:14, 7:9 and 4:5 respectively. Therefore, the only similar picture sizes are 8 × 10 and 16 × 20, with a scale factor k = 2. B. What does your conclusion about resizing pictures indicate? When you resize pictures, one of two things: the image is distorted or part of the picture needs to be cropped out. © Houghton Mifflin Harcourt Publishing Company • Picture Credit: t) ©Chuck Wagner/Shutterstock; (b) ©Dave Friday/Design Pics Inc./Alamy 17. Nicole wants to know the height of the snow sculpture, but it's too long to measure. Nicole measured the shadow of the highest point of the snow sculpture to be 10 feet tall. Meanwhile, Nicole's shadow was 40 inches long. If Nicole is 1.5 meters tall, what's the height of the snow sculpture? Since the shadows were measured at the same time of day, the angle dimensions are equal and the triangles are similar. Therefore, a part can be used to determine the heights. 10 feet = 120 inches 5 feet. =60 inches Las h represents the height of the snow sculpture. 60 h = ; h = 180 inches or 15 feet 120 40 18. Which of the following is enlargement? A. (x, y) → (x, 3y) _ _ 19. What is not preserved under enlargement? Select all that apply. B. (x, y) → (3x, -y) A. Angle D. (x, y) → (x, y - 3) C. Collinearity C. (x, y) → (3x, 3y) B. Betweenness E. (x, y) → (x - 3, y - 3) D. Distance module 16 IN2_MNLESE389847_U7M16L2 848 E. Proportionality 848 Lesson 2 18/04/14 7:57 proving the figures are similar to using changes 848 H.O.T. higher Focus order thinking JOURNAL 20. Analysis links Consider the conversions below. Invite students to define similarity to their own words, and then justify the definition with similar characteristics of numbers. I. Translation II. Reflection III. Rotation IV. Enlargement a. Which conversions maintain the distance? I, II, III b. Which conversions retain the ang angle size? I, II, III, IV c. Use your knowledge to stiffen compare and contrasts with congruency and similarity. Rigid transformations retain the angle and distance. Therefore, rigid transformations (translations, reflections, rotations) ensure congressional ity. Changes in similarity (enlargements) only retain angle dimensions. Use the diagram shown for justification exercises 21-23. Determine whether these assumptions are sufficient to demonstrate that the two triangles are similar. Write the correct match of the peaks. If these two triangles must be similar, describe a similarity of the conversions that map one triangle to another. If the triangles may not be similar, explain why. AX DX 21. Lengths AX, BX, CX and DX correspond to equation ___ = ___ . BX CX BX AX __ AX __ AX = DX into __ = CX . Let k = __ . Step A: Arrange __ BX DX CX Step B: Rotate the △DXC 180° around point X so that ∠DXC coincides with ∠AXB. Step C: Dilate ∠DXC by factor k about mid X. This expansion moves to point D A because k(DX) = AX and moves to C B because k(CX) = BX. Because the expansion takes place through point X and the expansions take line segments into line segments, △DXC is mapped △AXB. So △DXC △axb. C X D B © Houghton Mifflin Harcourt Publishing Company 22. The AB and CD lines are parallel.¨is parallel to step A: △DXC so that △DXC coincides with △AXB. Then the image C'D'¨¨cd pre-image. So the new side of C'D' is still the parallel side of AB △. It moves to the top of C′ point B and C'D' ‹ › − − & gt; › Line through B in parallel with the CD . Since AB is parallel to C'D', enlargement moves to C'D' ‹ › › → − − and AB , D' must move A. Thus onto AB . Because D'moves to point XA rotation and expansion map △DXC △AXB. So △DXC △axb. Annex 23 ∠XAB is in accordance ∠XCD. Step A: Draw ∠AXC bisector. I don't know if it's for xA. Because of the reflections in Step B: Reflect △CXD across the corner of the bisector. This maps XC's onto XB¨. Since △XCD ≅ △XAB, image CDs store angles, it also maps XD¨. parallel to AB ‹ › − Step C: expand △XCD point X. It moves to a new point in C A. Because AB is parallel ‹ › ‹ › › › − − CD - the new CD to AB . Therefore, the new point D is mapped to B △ and the △XCD is △ toXAB. Both △XCD is similar to △XAB. Module 16 IN2_MNLESE389847_U7M16L2 849 849 Lesson 16.2 849 Lesson 2 18/04/14 7:57 PM Lesson Performance Task AVOID COMMON ERRORS Students can correctly calculate areas of circles A, B, C and D, but can forget that in order to calculate the required probability, they must find areas of rings. For example, Area Ring C = Area Circle C – Area Circle B. Please reply to the following about dartboard pictured here. 1. Are the circles similar? Explain by using the concept of enlargement in your explanation. 2. You throw a throw and it sticks randomly to the board. What is the likelihood that it will remain in round A? Ring B? In circle C? Laps d? Explain how you found your answers. Integrate mathematical practices into Focus patterns in MP.8 Study pattern probability you 2"A"B C received responses to lesson performance task. Then predict the likelihood of landing darts in each circle if the fifth circle with a radius of 10 inches was 5 ___ 9 1 ___ , 3 , ___ , 7 , ___ attached outside the 8-inch circle. ___ 25 25 25 25 25 D 1. Yes. Each circle can be considered a preview that can be changed by dilation, the centre of which is in the middle of the cash register, any other round. Two. Subring A = πr 2 = π(2) = 4π 2 Region Circle B = πr = π(4) = 16π 2 2 Alaring C = πr = π(6) = 36π 2 2 Region D = πr 2 = π(8) = 6 4π 2 Area Ring B = Alaring B - Area Ring A = 16π - 4π = 12π Area Ring C = Area Circle C - Area Circle B = 36π - 16π = 20π Area Ring D = Area Circle D - Area Circle C = 64π - 36π = 28π © Houghton Mifflin Harcourt Publishing Company Area dartboard = area Ring D = 64π 4π area Circle A 1 P(A) = __ = _ = _ area dartboard 64π 16 area Ring B 12π 3 P(B) = __ = _ _ area dartboard 64π 16 area Ring C 20π 5 P(C) = __ = _ = area of the arrowboard 64π 16 area Ring D 28π 7 P(D) = __ = _ = _ area of the arrowboard 64π 16 Module 16 850 Lesson 2 EXTENSION ACTIVITY IN2_MNLESE389847_U7M16L2 850 Let students design dartboards in the basic shape of different circular board (for example, squares or peer triangles) , but with the same basic characteristics: each should have four sections, and the sections should differ by the same amount from the adjacent sections in the dimensions. Invite students to answer the same questions about their designs that they answered about the circular table. For squares and equilateral triangles, they should get the same results: the numbers are similar and whether each arrow is likely to land (the smallest maximum) 5 7 1 __ , 3 , __ and __ . 16 16 16 16 18/04/14 7:57 Scores Rubric 2 Points: The student's response is accurate and complete in the performance of the task or tasks. 1 point: The student's response contains the characteristics of the appropriate response, but is invalid. 0 points: The student's response does not include the characteristics of the relevant response. The supporting figures are similar to using Conversions 850 LESSON 16.3 Name Corresponding parts similar to numbers Class Date 16.3 Corresponding parts of similar figures Important question: If you know the two numbers are similar to what you can set about the actions of the respective angles and lengths? Common Core Math Standards Resource Locker Student's to: COMMON CORE gross settlement total. A.2 Discover ... explain using similarity changes in the meaning of similarity triangles ..... Also G-CO. A.2, G-CO. A.5 Mathematical practices common core mp.4 modeling DEAL View Deal section online. Discuss the floor plan and invite students to explain the floor plan's relationship with the house. Then, check the hour performance task. N 4 D m∠A = 80,5° m∠K = 80,5° m∠B = 63,4° m∠ = 63,4° m∠C = 116,6° m∠M = 116,6° m∠D = 99,5° m∠N = 99,5° M C x -8 -4 0 4 -4 B A K 8 -8 L Accept reasonable measures; opposite angles should be additional B © Houghton Mifflin Harcourt Publishing Company PREVIEW: LESSON PERFORMANCE TASK y 8 Measure angles. Play similar or not with a partner given a pair of figures in the coordinates of planes. Similar figures shall have appropriate angles which are compatible and have corresponding sides that are proportionate. Think of the graph abcd and KLMN. Are the relevant corners in sync? Yes/no key question for the purpose of the language: if the two numbers are similar, what can you specify for the corresponding angles and lengths? Connection angles and sides of numbers You know that if the numbers are similar, the side lengths are proportional and the angle dimensions are equal. If you have two figures with proportional side length and congruent angles, can you conclude that they are similar? Are the ratios of the corresponding side lengths equal? Yes/No 1 AB = _ _ KL 2 C 1 BC = _ _ LM 2 1 AD = _ _ KN 2 1 CD = _ _ MN 2 Are the figures similar? Describe how you know how to use similarity conversions. y 8 Yes, there is a dilation centered on the origin that N4 D can find when connecting the corresponding peaks. x -8 -4 0 -4 K Module 16 must be EDIT-- Chan NO Key=NL-A; CA-A Correction -8 4 8 B L Lesson 3 851 gh File info made throu M C Date Class Parts sponding 16.3 Corre ilar figures Sim Name HARDCOVER PAGE 851,860 Resource Locker about you can you determine similar to what lengths? the figures are angles and know two ions: if you korres ity measures ng similar ns mean transformatio similarity figures explain using G-CO. A.5 COMMON .2, G-SRT. A.2 ... On the sides of the CORE . Also G-CO. Angles and angle measures triangles ... ting you l Connec corners, can proportiona congruent Discover lengths are lengths and r, half is simila rtional party that if the figures propo You know two If you have r? are equal. is simila that they enter into Quest Essential IN2_MNLESE389847_U7M16L3 851 . and KLMN ABCD graph o Consider uent? Yes/N angles are korres ⚲ Measured s. angle 80,5° m∠A = m∠B = m∠∠K = nable Accept reaso Are ratios ◑ y g Compan 1 AB = _ at 2 m∠M = m∠N = measures; g alekse en a gised written trays Harcour t n Mifflin © Houghto 8 4 0 B A 116,6° K 99,5° ld corners of the show L -8 nt suppit s equal? Yes/N side length 1 CD = _ _ MN 2 1 AD = _ _ CN 2 y similarity knows, using the & x -4 63,4° opposite 1 BC = _ _ LM 2 M 4 D -4 -8 80,5° m∠I = 63.4° 116.6° m∠C = 99.5° m∠D= Watch out for student edition page numbers for this lesson. y 8 N how do you describe s similar? You can be statues. which transform red. is dilati ding vertic Yes, there correspon ecting find conn 8 N -8 M 4 C D 0 -4 K -8 x 8 4 B L L Hour 3 851 Module 16 6L3 851 47_U7M1 THE 851 IN2_MNL 851 lesson 16.3 18.04.14 20:04 PM 18/04/14 9:05 PM D y Think graph ABCD and EFGH. E The numbers are both rectangles, so each -8 B 0-4 D H 4 connector angles and sides of numbers -4 8 C G -8 INTEGRATE TECHNOLOGY Are the corresponding side length ratios equal? Yes/No 4 AB = _ _ EF 7 F F 4 A x angle is 90°. E EXPLORE 8 Are the corresponding corners in sync? Yes/No Explain. 3 BC = _ _ FG 5 4 CD = _ _ GH 7 Students have the opportunity to make an angle measure of activity either in the book or online. 3 AD = _ _ EH 5 QUESTIONING STRATEGIES Are the figures similar? Describe how you know how to use similarity conversions. EF, D'no. If you translate abcd so that maps E, and expand the A'B'C'D' ratio __ Considering the figure that seems to be the expansion of another, how can you check whether the center of enlargement exists? Draw lines through the corresponding peaks of numbers. If they cross the point, it will be the centre of enlargement. If they don't cut the point, the dilation center doesn't exist at the time. AD card H but B' and C' not a map F and G. G. G Think graph PQRS and WXYZ. • Are the corresponding corners in sync? Yes/No 8 Measure angles. Z m∠W = 76° m∠Q = 153,4° m∠X = 104° m∠R = 26,6° m∠Y = 76° m∠S = 153,4° m∠Z = 104° 4 Y R S -8 -4 P W 0 -4 Q 4 x 8 X -8 © Houghton Mifflin Harcourt Publishing Company m∠P = 26.6° Accepted measures; opposite angles should be additional ⏴ are the ratios of the corresponding side lengths equal? Yes/No 1 PQ _ =_ WX 2 ⏮ 1 QR _ _ = XY 2 1 RS = _ _ YZ 2 1 PS = _ _ WZ 2 Are the figures similar? Describe how you know how to use similarity conversions. Not. Dilations map line segments parallel line segments, so there is no expansion, which is¨to ZW¨. Other similarity changes are all rigid movements that retain the length and angle of the MAP SP, so that no similarity conversions are mapped to the WXYZ of the PQRS. Module 16 852 Lesson 3 PROFESSIONAL DEVELOPMENT IN2_MNLESE389847_U7M16L3 852 Math Background 18/04/14 9:05 Students use similarity to solve a variety of real-world problems. The general process is as follows: firstly, indicate that the two digits are similar; then use the fact that the relevant sides are proportional to find an unknown side length. Corresponding parts of similar drawings 852 Reflect 1 If two numbers have the same number of sides and the corresponding angles are compatible, does this mean that the pair of the respective sides is either uniform or proportionate? No, the convergence of the respective corners does not mean that the 1st Century B&B Is not the same. Justify the properties of similar numbers by using conversions to the corresponding pair of sides. For example, consider a square and a rectangle. If two numbers have an enlargement centre, is the respective pair of parties necessarily proportional? Yes. If the two numbers have an expansion center, the figures are similar and any pair 2. INTEGRATE MATHEMATICAL PRACTICES Focus on modeling MP.4 Use drawings to discuss how similar triangles match the sides are proportional. If two numbers have a proportional side, are the figures necessarily dilation? Not. It is possible to establish correspondence between the two digits of the two digits on the disproportionate sides. which are not similar (e.g. if they do not have an enlargement centre). can be used to find the heights of buildings or trees that are difficult to measure directly. The shadows of objects, measured at the same time of day, are proportional to the height of the objects. 1. Justification of the characteristics of similar drawings by means of conversions Two drawings that can be mapped to each other by similar transformations (enlargement disalations and rigid movements) are similar. Similar figures have certain characteristics. Characteristics Similar figures for questioning strategies Corresponding angles similar figures are in sync. How do you know when the two rectangles are similar? They are similar where the relevant sides are proportionate. Y B The corresponding aspects of similar figures are proportionate. If △ABC △ , then ∠A ≅ ∠X ∠B ≅ ∠Y ∠C ≅ ∠Z © Houghton Mifflin Harcourt Publishing Company BC = _ AC AB = _ _ _ XY YZ X A C X Z Show that two digits of all pairs of respective sides with an equal ratio k and all pairs of corresponding angles congruent are similar, you can use similarity changes. Outline one number by using k. The enlarged number is in line with the second figure, meaning congruence. So, there's a jar of rigid movements (which also have a similarity to eliminations) that maps one of the other. Example 1 ⚲ properties of similar numbers. Figure EFGH maps drawing RSTU by similarity transformation. Write a section that contains EF and RU. List all angles that must be in line ∠G or ∠U. EF = _ EH _ RU RS ◑ ∠T is ∠ for the ∠H is ∠ FOR THE ∠U. Figure JKLMN maps figure TUVWX by similarity transformation. Write a section that contains TX and LM. List all angles that must be in line ∠V or ∠K. JN LM _=_ TX VW ∠ L is in ∠V and ∠ U are in ∠K. Module 16 853 Lesson 3 STUDYing IN2_MNLESE389847_U7M16L3 853 Whole Class Action Do students bring magazines and newspapers, and challenge them to find sets of similar figures. Discuss the use of similar figures in art and design, as well as the visual effects that such designs create. 853 Lesson 16.3 18/04/14 21:05 Reflective 4. EXPLANATION 2 If you know that the two numbers are similar, what angle or side measurement you need to know to find dilations by mapping one number to another? To find a scale factor, you need to know the measurements of one of the corresponding sides. Applying features similar to the numbers of your turn 5. The △PQR and △LMN are similar. If QR = 6 and MN = 9, what kind of similarity transformation (coordinate characters) maps △PQR △LMN? POLLING STRATEGIES MN Ratio ___ = 1.5, thus the similarity conversion is (x, y) → (1.5x, 1,5y). QR 6. How can you use similar numbers to fix problems? Use the fact that the corresponding sides are proportional to find an unknown side length. VW Is the statement true? DE = _ Error analysis triangles △DEF and △UVW are similar. _ UV EF DE EF = ___ . Not. The proportions must be compared to the corresponding sides. One option is ___ APPLICATION OF THE PROPERTIES OF SIMILAR DRAWINGS Similar figures The properties of similar figures can be used to find the characteristics of the respective parts. Example 2 AVOID COMMON ERRORS – Given that the numbers are similar, you can find the x and y. ⚲ find x value. Find a value of y. ∠C ≅ ∠R, so m∠C = m∠R AB = _ AD _ PS PQ 3y - 5 4y _ _ = 5 10 3y 5 4y _ 10 = _ 10 5 10 4y = 6y - 10 4x + 27 = 95 4x = 68 x = 17 Students can sometimes write proportions, which do not use the appropriate sides. Although there are several ways to write the correct proportion, encourage students to use a pattern they remember. For example, readers always give side lengths from one triangle, and the denominators always give lateral lengths from the other triangle. 4y ft (3y - 5) ft B D (4x + 27)° P 5 ft Q 10 ft 95° S C R y = 5 (2y - 8) cm Scroll value x.m∠LMN = m∠XYZ 5(x - 5) = 5x - 25 = x = 4x 4x 25 Z L 1 cm Y Scroll to a value of y. JK MN _ =_ VW YZ 5(x - 5)° J 1,5 2x - 8 = _ _ M V 4x° X 4 cm 1,5 cm N 1 4 2x - 8 = 1.5(4) 2x - 8 = 6 © Houghton Mifflin Harcourt Publishing Company K ◑ provide a student with a pair of different images of two digits drawn coordinates. The first student decides whether the figures are similar or not, and reasons for three reasons. The other student must agree or provide an alternative explanation. Students change roles and repeat the process with another number pair. 2x = 14 x = 7 Module 16 854 Lesson 3 DIFFERENTIATE INSTRUCTION IN2_MNLESE389847_U7M16L3 854 Modeling 18/04/14 9:05 Students can work in pairs, The student created a block design and the second create an indicator that is similar but not in sync to the first. For more sets of pattern blocks, search for websites that feature virtual pattern blocks. These sites allow students to select, combine, and rearrange blocks on a computer screen. The corresponding sections 854 of similar drawings reflect the specified 7. QUESTIONING STRATEGIES Discussion What are some of the things you need to be careful about solving problems related to finding values for similar numbers variables? Possible answer: If you find lateral lengths, you need to be sure to create the proportion correctly, and do you have to remember that half the lengths are proportional, rather than being picture always similar to your pre-image? Explain. No, no, for example, disproportionately stretching and shrinking the number can create an image that is not similar. Equal. If you have an algebraic equation set up, you must be careful not to make mistakes in calculations. Use a chart of your △ABE ± △ACD. Can two congatooles have different sides of length? Explain. Yes, the angle measure refers to the opening volume between the two sides; the length of the sides of the angle does not affect its size. 5,6 cm Scroll x. 9. 5 cm 50°C Scroll y. CD AD _ _ = AE 50 = 3x + 14 BE 5.6 + y _ 5 _ = 4 5,6 5,6 + y = _ 5,6 36 = 3x If the two digits are presented in a similar way, what can be inferred about these sides and angles? Similar figures are proportional to the respective sides and the corresponding angles are consistent. 4 cm B m∠C = m∠ABE TOTAL LESSON E (3x + 14)° A 8. D y cm 12 = x 4 y = 1.4 © Houghton Mifflin Harcourt Publishing Company Work 10. Consider two similar triangles △ABC and △A'B'C'. If both m∠A' = m∠C and m∠B' = m∠A, what can you conclude △ABC? Explain your reasoning. Because the two triangles are similar, we know m∠A' = m∠A and m∠B' = m∠B . Together with the information provided, it tells us m∠A = m∠C and m∠B = m∠A. Therefore, ∆ABC is equilateral. 11. Rectangle JKLM maps rectangle RSTU by transformation (x, y) → (4x, 4y). If the perimeter of RSTU is x, what is the perimeter JKLM in terms of x? The ratio of the respective sides is 1:4 and therefore the perimeter ratio is 1:4. x Perimeter JKLM is _ . 4 12. Important question Check-In If the two digits are similar, what can we infer from their respective parts? Similar figures shall have appropriate angles which are compatible and have corresponding sides that are proportionate. Module 16 855 Lesson 3 LANGUAGE SUPPORT IN2_MNLESE389847_U7M16L3 855 Connecting to a word Students can be very familiar with the connotations of word translation because it refers to interlingual translation. In fact, this use of the word comes from its in a strict sense: translation between languages moves meaning from one to another, as the translation indicator moves it from one place to another. Connect to the fact that the mathematical translation does not change the nature of the original figure, because the linguistic translation does not significantly change the original meaning of the message as it enters another language. 855 Lesson 16.3 18.2014.2014 Rate: Homework and internship RATE • Online Homework • Tips and help • Additional practice figures are consistent with the relevant angles? Are the parties concerned proportionate? Are the numbers similar? Describe how you know how to use similarity conversions. 1. 2. GUIDANCE ON DESIGNATION Yes; No, no Rotate the smaller square 90° around its upper right corner, then translate it down and left until the lower left ends coincide. If you extend this scale factor from 3 to 2.5, the short edges are in sync, but the long edges are not. - yes, I don't know. yes, I don't know. Yes. Translate the smaller square down and right so that its top left top coincides with that of a larger square. Then expand the scale factor 2 around this peak. 3. 4. 5. Figure ABCD is similar to figure MNKL. Write the section that contains BC and KL. CD BC=NK KL 6. △XYZ resembles △XVW. Write congruence statements that must be true. 8. _ _ 7. ∠X ≅ ∠X, ∠Y ≅ ∠V and ∠Z ≅ ∠W. 9th CDEF maps JKLM with changes (x, y) → (5x, 5y) → (x - 4, y - 4). What is 1 EF? __ ___ LM 5 Module 16 Execution IN2_MNLESE389847_U7M16L3 856 △DEF value △STU. Write the section that contains THE AND SU. ST SU = DE DF △MNP is △HJK and both triangles are equilateral. If m∠P > 90°, name all angles that are ∠H. If m∠P > 90°, ∠M ≅ ∠N. So, ∠M, ∠J and ∠N are all ≅ ∠H. Practice Explore Connecting Angles and Sides Figures Exercises 1-4 Example 1 Justifying Features Similar Figures Using Transformations Exercises 9-12 Example 2 Applying Characteristics Similar Figures Exercises 5-8 Integrate Mathematical Practices Focus On Modeling MP.4 Compare Symbols of Equality, Similarity, © Houghton Mifflin Harcourt Publishing Company Yes; yes, I don't know. Yes. Rotate the smaller figure 90°, then translate it so that the two corresponding peaks match. Then expand the smaller indicator centers with a factor of 2 around that peak. No, no, no, no, You can rotate and translate the triangle to the right so that the right angles coincide, but no similarity to the transformation makes for fierce angles congruent. Concepts and skills and conjugation. Point out that combining symbols of equality and similarity gives a conjugation symbol. Combine the composition of the symbol with its meaning: congruence combines equality and similarity, because the conglomerate numbers are of the same size and shape. 10. maps △VWX with transformation (x, y) → (x + 3, y - 1) → (2x, 2y). If WX = 12, what QR is equal? QR = 1 (12) = 6 2 __ Lesson 3 856 Depth of Knowledge (D.O.K.) COMMON CORE Mathematical practices 1-4 2 Skills/concepts MP.3 Logic 5-17 2 Skills/concepts MP.5 Use of tools 18-20 2 Skills/concepts MP.2 Recital 21-22 2 Skills/concepts MP.4 Modelling 23-2 4 2 Skills / Concepts MP.4 Modeling 25 3 Strategic Thinking MP.3 Logic 26 3 Strategic Thinking MP.3 Logic 27 3 Strategic Thinking MP.3 Logic 18/04/14 4:05 Corresponding Parts Similar Figures 856 11. △QRS Maps △XYZ with Transformation (x, y) → (6x, 6y). If QS = 7, what is the length of XZ? AVOID COMMON MISTAKES When solving a portion about the scale model, some students may make mistakes because they were unable to read the items carefully. Remind them that the length of the respective sides may not be in the same units, and may need to convert one measure. 12. Algebra Two similar numbers are similar based on the transformation (x, y) → (12x, 3a 2y). What is/ is a value(s)? XZ = 42 If the figures are similar, 12 = 3a 2 and a = 2 or a = -2. 13. The Algebra △PQR is similar △XYZ. If PQ = n + 2, QR = n - 2 and XY = n 2 - 4, what is the YZ value in part n? Since n 2-4 = (n + 2)(n -2), the ratio of the pair to the respective sides is (n -2) :1. Then the value of YZ is (n -2)(n -2) = n 2 -4n + 4.14. Which transformations don't give similar figures? Select everything that applies and explain your options. Options B and E do not produce similar A. (x, y) → (x - 4.y) → (-x, -y) → (8x, 8y) B. (x, y) → (x + 1, y +1) → (3x, 2y) → (-x, -y) C. (x, y) → (5x, 5y) → (x, y) → (x + 3, y - 3) D. (x, y) → (x, 2y) → (x + 6, y - 2) → (2x, y) E. (x, y) → (x, y) 3y) → (2x, y) → (x - 3, y - 2) figures, because each sequence contains changes that are not expansion or rigid movements, and are not balanced elsewhere in the order. Option D contains two conversions that are not enlargements or rigid movements, but create expansion together. 15. The figures in the picture are similar to each other. Find the value of X. The longer bases of trapezoids are 6 units and 3 units, so the scale factor is 2:1. Set the equation x + 1 = 2 (x - 3) and resolve to get x = 7. x-3 6 3 x +1 16. In △ NPQ ± △NLM and PL = 5. M a. Find value x. © Houghton Mifflin Harcourt Publishing Company 3x + 18 = 60 3x = 42 4 cm x = 14 P b. Find the lengths of NP and NL. NP NL = NM NQ y 5-y = 4 3,2 3,2(5 - y) = 4y 3,2 cm (3x + 18)° L 1 6 = 7.2y Module 16 IN2_MNLESE389847_U7M16L3 857 Lesson 16.3 y N 20 _ = y 9 25 20 Np = _ _ and NL = _. 9 857 60° _ Q 9 857 Lesson 3 18/04/14 21:05 17. △CDE maps △STU with conversions (x, y) → (x - 2, y -2) → (3x, 3y) → (x, -y). If CD = a + 1, DE = 2a - 1, ST = + 3 and TU = b + 6, find values a and b. The length of ∆STU is 3 times the length of the corresponding △CDE. It gives equations 3(a + 1) = 2b + 3 and 3(2a - 1) = b + 6. Solve this equation system to get a = 2 and b = 3. 18. If the transformations list contains a transformation (ax, by) with ≠ b, can a pre-image and image represent any concurrent number? Can they represent a similar, non-synchronised number? Justify your answers with examples. - yes, I don't know. if ab = ±1, then the pre-image and image are in sync (and thus similar). The preview and image can represent similar non-congressional numbers if the conversions in the list also contains (bx, ay). The transformation (axe followed by)(bx, ay) is equivalent to enlargement (abx, aby). 19. Do any of them resemble a pair of equilateral triangles? Why or why not? Yes, the triangles are similar. All corners of each triangle are equal to 60° and therefore the respective angles are compatible. The sides of each triangle are consistent and therefore the ratio of the respective sides is constant. 20. Figure CDEF is similar to FIGURE KLMN. What are the claims that are false? Select everything that applies and explain why. CD = _ EF A. _ KL MN CF = _ EF B. _ KN MN CF DE = _ C. _ LM KN LM = _ KL D. _ DE CD KN LM = _ E. _ DE CD Option E is incorrect. The proportion does not match the corresponding sides. © Houghton Mifflin Harcourt Publishing Company • Image Credits: ©Jeff Dalton / Alamy Think about this model of a train locomotive answering the following two questions. 21. If the model is 18 inches long and the actual locomotive is 72 feet long, what is the transformation of the similarity from the model to the actual locomotive? Expression of the response with x → ax, where x is the measurement of the model and the axle is the corresponding measurement on the actual locomotive. The model is 1.5 feet in length and thus the transformation of similarity is x → 48x. 22. If the front wheels of the locomotive are 4 feet in diameter, what is the diameter of the model's front wheels? Express your answer in inches. The model's front wheels have a diameter of 1 inch. Module 16 IN2_MNLESE389847_U7M16L3 858 858 Lesson 3 18/04/14 21:05 Corresponding parts similar to figures 858 Use the following graph to answer the following two problems: MAGAZINE y Let students write a diary entry in which they form their own problems using an unknown length that must be found using similar triangles. Remind students to add solutions to their problems. 8 B C We 4 A -8 D -4 L x 0 4 J -4 -8 8 M 23. Specify a series of two conversions that will match ABCD to JKLM. Possible answer: (x, y) → (2x, 2y) → (x + 14, y - 8) or (x, y) → (x + 7, y - 4) → (2x, 2y) AC + BD 24. Locate the _____ value . JL + VAT AC Two digits are similar, so all the sides/diagonals have the same ratio. ___ JL BD 1 1 1 = __ and ___ = __ and so _____ = __ . 2 2 KM JL + VAT AC + BD H.O.T. Focus on higher-level thinking 25. Counterexamples All rectangles are similar. Is this claim true or false? If that's true, explain why. If false, give a counter-example. The statement is false. For example, a rectangle with dimensions of 5 units by 2 units is not similar to a rectangle with dimensions of 4 units by three units. © Houghton Mifflin Harcourt Publishing House 26. Justify Justification If ABCD is similar to KLMN and MNKL, what type of quadrilateral is KLMN? Justify your reasoning. Looking at the corresponding angles, ∠A ≅ ∠K and ∠A ≅ ∠M, which means ∠K ≅ ∠M by Transitive Property Congruence. It ∠B ≅ ∠L and ∠B ≅ ∠N, which means ∠L ≅ ∠N by Transitive Property Congruence. If both rectangular corners are concentrity, then the rectangular is the parallelogram. Therefore, klmn is a parallelogram. It may also be a diamond, a rectangle or a square, but more information is needed to justify this conclusion. 27. The justification for the criticism is △PQR is similar to △QPR, then △PQR resembles △RPQ. Explain whether this statement is correct or not. The statement is false. If △PQR △ with QPR, ∠P ≅ ∠Q and the triangle are equilateral. This does not demonstrate any ∠P and ∠R or ∠Q and ∠R. For the △PQR to be △RPQ, the triangle must be equilateral. Module 16 IN2_MNLESE389847_U7M16L3 859 859 lesson 16.3 859 lesson 3 18/04/14 9:05 PM Lesson Performance Task You have hired an architect to design your dream house and now the house has been built. Before moving in, you have decided to travel through the house with a yardstick to see how well the builders have followed the architect's floor plan. Describe in as much detail as possible how you can achieve your goal. Then discuss how you can decide whether the space shape and other features of the house are similar to the corresponding shapes on the floor plan. Integrate mathematical practices Focus on critical thinking MP.3 Rectangle 1 is units length and b units in bedroom living RM 17.5 x 18.7' width. Rectangle 2 is obtained by multiplying the sides of rectangle 1 by 10. FOYER • Is a rectangle 2 similar to a rectangle you? Explain. - yes, I don't know. multiplying the poles by the same scale factor changes the size of the drawing but not the shape so that the rectangles are similar. SCALE 1 2 = 10 ft Among students should discuss: •scale plan and how they can use it to check the dimensions of the room; • A method they can use to check that rooms and other floor plan functions are mapped with corresponding shapes house by a number of changes, including enlargement; • How do I compare the area of rectangle 2 with the area of rectangle 1? Explain. Area Rectangle 1 is ab, and area Rectangle 2 is 100ab, so area Rectangle 2 is 100 times larger than area Rectangle 1. • Measurements that should be maintained when switching from floor plan to house (e.g. corners) and measurements that would not be (e.g. lengths). INTEGRATE the mathematical practices of Focus on Reasoning MP.2 Three streets meet to form an equilateral © Houghton Mifflin Harcourt Publishing Company triangle 100 meters on each side. In a photo triangle taken directly overhead, each side measures 4 inches. How is the actual triangle of streets and triangle photo so? How are they different? Explain. Their sizes are different, but their shape is the same; the sides of the photo triangle are shorter than the actual triangle, but because they are both equilateral triangles, their sides are proportional and both have three 60° corners. Module 16 860 Lesson 3 EXTENSION ACTIVITY IN2_MNLESE389847_U7M16L3 860 Supply yardsticks or meter sticks for students and have them make a floor plan in the classroom. Students should focus on the characteristics of the room that are permanent, such as dimensions and shape, excluding those that are not, for example, tables and tables. You can specify the approximate size of the floor plan (for example, 8.5 inches × 11 sheets of paper), thereby forcing students to calculate a suitable scale. Students can switch floor plans with a partner and check the size, shape and calculations of the partner. 18/04/14 21:05 Scores Rubric 2 Points: The student's response is accurate and complete in the performance of tasks or tasks. 1 point: The student's response contains the characteristics of the appropriate response, but is invalid. 0 points: The student's response does not include the characteristics of the relevant response. Corresponding parts similar to blueprints 860 LESSON 16.4 Name AA Similarity Triangles Class 16.4 AA Similarity Triangles Important question: How can you show that the two triangles are similar? Common Core Math Standards To Explore student is expected: COMMON CORE G-SRT. A.3 The study of the similarity of the angle angle in the triangles' resource cabinet is similar if their respective sides are proportionate and their respective angles are consistent. There are several shortcuts to prove triangles. Use the properties of similarity conversions to specify an AA criterion for two similar triangles. So is G-SRT. B.5 Mathematical practices COMMON CORE Date Draw triangle _ and mark it △ABC. Elsewhere on your page, draw a segment that is longer than AB and tick endpoints D and E. MP.5 Using Tools C Language Objective F Explain to the partner how to use the Angle-Angle criterion to show triangles. B A D D B Copy ∠CAB and ∠ABC according to paragraphs D and E. If necessary, expand the rays of the copied corners and mark their intersection F. You have built △DEF. Important question: How can you show that the two triangles are similar? C You built corners D and E to be consistent with angles A and B respectively. PREVIEW: LESSON PERFORMANCE TASK View the Include online section. Discuss this illustration and invite students to speculate on what it represents. Then, check the hour performance task. © Houghton Mifflin Harcourt Publishing Company ENGAGE angles C and F must also be congruent due to the third-corner theorem. D Check the proportionality of the respective parties. Possible answer (ratios should be equal): 4 AB = _ = 0,4 _ DE 10 3 AC = _ _ 0,4 DF 7,5 2 BC = _ = 0,4 EF 5 Since the ratios are equal , the sides of the triangle are proportional . Let's reflect the 1st Discussion Compare the results with your classmates. What assumption can you make for two triangles with two corresponding conjugation angles? If the two triangles have two corresponding corners, the triangles shall be similar. Module 16 must be EDIT--Chan EI key=NL-A; CA-A Correction Lesson 4 861 gh File info made throu Date Class corners rity Tri Name Simila 16.4 AA r? there are simila triangles n two that two AA criterio show to determine how you can rmations Question: ity transfo Essential ties similar to B.5 ilarity COMMON G-SRT. A.3 Use similar ones. So is G-SRT. CORE ngle Sim be triangles that nD angle-IN2_MNLESE389847_U7M16L4.indd 861 HARDCOVER PAGES 861,872 Resource Locker Explori s for Triangle Explore l and their proportions g triangles g sides are uis provin correspondin r if their is several shortc les is simila ent. Two triang g corners are congru eg correspondin draw segme r. page, is simila Elsewhere Watch for hardcover student edition page numbers for this lesson. it △ABC. E. le and label ints D and _ endpo Draw triangular AB and label longer than C F B D A D the △DEF. According to. built D and E, F. You need points for ction point and ∠ABC for these interse therefore copy ∠CAB and label respectively. if necessary, A and B, angles, if angles em. be congruent Third corner theor D and E angles are equal): ucted se s should You constr constr congruent becau answer (ratio also possible and F must be ng sides. angles C ratio 2 Check BC = = 0,4 _ 3 EF 5 AC = 0,4 _ 4 DF7,5 AB = = 0,4 _ al proportion). DE 10 les are equal from the triple, sides y g Compan & n Mifflin Harcour t Publishin What conjec to be simila classmates. les must own? s, s your results congruent angles Compare ng ruent angle Discussion is two vastai sponding cong to two corre triangles les than two triang © Houghto Since reflective 1. Lesson 4 861 Module 16 6L4.indd 47_U7M1 ESE3898 IN2_MNL 861 Lesson 16.4 861 18/04/14 9:00 PM 18/04/14 9:01 Explain 1 Proving Angle-Angle Triangle Similarity EXPLORES recommends the following teorem to determine whether the two triangles are similar. Angle angle (AA) Triangle similarity the works to detect the similarity of the triangles angle when two corners of one triangle are consistent with the two corners of the other triangle, the two triangles are similar. Example 1 Prove Angle-Angle Triangle Similarity to The Yrem. Simple: ∠A ≅ ∠X and ∠B ≅ ∠Y B integrate technology Y Prove: △ABC €△XYZ Students have the opportunity to make similar triangles of activity either in the book or online. C A Z x question strategies XY . Let the △ABC be △A'B'C. Apply the expansion △ABC scale factor k = _ AB B' What angle-angle similarity claim about triangles? According to the angle-angle similarity criterion, the triangles with the two drawers are similar. Y B enlargement A C C C' A' Z X ∠A and ∠B' ≅ ∠B the corresponding angles of similar triangles are in sync. because XY • AB = XY AB also A'B ' = k • AB = . EXPLAIN 1 △A'B'C is similar to △ABC and ∠A' ≅ _ Given that ∠A ≅ ∠X and ∠B ≅ ∠Y Conjugus, ∠A ≅ ∠X and ∠B' ≅ ∠Y transit assets. asa triangle Congruence Theorem . So, △A'B'C' ≅ △XYZ means there's a jam of rigid movements that map △A'B'C'to △YYZ. The expansion followed this series of rigid movements shows that there is a series of similarity transformations that △ABC △XYZ. Therefore, △ABC ± △XYZ. Reflect the 2. Discussion Compare and Contrast AA Similarity Postulate ASA Congruence Postulate. Both postulates require that two pairs of angles be in sync, but congruence you also © Houghton Mifflin Harcourt Publishing Company - Proving angle triangle similarity to integrate mathematical practices focus on math connections MP.1 Compare to prove two triangles similar to proving two triangles congruent. QUESTIONING STRATEGIES need to know that the parties involved are consistent, so the figures are the same. How to use AA similarity to show the two triangles are similar? Indicates that two corners of one triangle are in relation to two corners of the other triangle. This allows you to conclude that the two triangles are similar. AA similarity to Postulate only shows that the two triangles have the same shape. Module 16 862 Lesson 4 PROFESSIONAL DEVELOPMENT IN2_MNLESE389847_U7M16L4.indd 862 Learning Progression 18/04/14 9:01 Students have already turned out triangles of congruent using SAS, SSS, Asa. The same types of reasoning are used here to examine AA similarities, and use this similarity to solve problems. AA similarity Triangles 862 3. EXPLAIN in 2 △JKL m∠J = 40° and m∠K = 55°. △mnp m∠M = 40° and m∠P = 85°. The student concludes that the triangles are not alike. Do you agree or disagree? Why? Disagree; the triangle sum theorem, m ∠ N = 55°, so that the triangles are similar to the AA similarity criterion. Applying Angle-Angle Similarity Explain 2 Applying Angle-Angle Similarity Architects and Contractors use properties similar to figures to find unknown dimensions, such as the correct height of the triangular roof. They may use an angle tool to check that the design angles are consistent with the corners in their plans. INTEGRATE MATHEMATICAL PRACTICES Focus on communication mp.3 Remind students that in the similarity of the triangle, they should identify parties that are proportionate, not congruent. Example 2 If possible, scroll to the length shown. BE First determine whether the △ABC ± △DBE. By Alternate Interior Corners Theorem, ∠A ≅ ∠D and ∠C ≅ ∠E, both △ABC - △DBE by AA Triangle Similarity Theorem. E Find BE by resolving the proportion. © Houghton Mifflin Harcourt Publishing Company • Image Authors: ©John Lund / Drew Kelly / Blend Images / Corbis BD = _ BE _ BA BC A 36 54 = _ BE _ 54 36 54 54 54 54 54 54 = _ BE 54 _ 54 _ 54 36 BE = 81 B D C RT Check ≅ ∠ ∠e, △RSV △ ±TRaviR. ∠R divides both triangles, so ∠R ≅ ∠R by reflexive property Congruence. AA Triangle Similarity Theorem, △RST ± △RTU . So, by R S 10 T 8 12 Find RT solving the part. V TU RT = _ _ RS SV U 12 RT = _ ∠RT = 15 _ 10 8 Module 16 863 Lesson 4 CO-STUDY IN2_MNLESE389847_U7M16L4.indd 863 Small group activity Do students work in small groups and draw diagrams to illustrate all these statements: all squares are similar; not all rectangles are similar; where the two polygons are compatible, they are also similar; not all the correct triangles are the same. 863 Lesson 16.4 18.04.14 21:01 Reflective 4. In example 2A, is there another way you can create a proportion to resolve be? BD BA Yes; = would also give the correct result be. Be BC 5. Discussion When you are asked to resolve y, the student creates a proportion as shown. Explain why this proportion is wrong. How should you adjust the proportion to give the right result? QUESTIONING STRATEGIES _ _ A y y _ 14 _ = 8 10 Variable y does not refer to the side triangle, but only 14 B 10 C How can you use AA similarity postulate to find unknown dimensions? You can use AA similarity to determine that the two triangles of two congruent pairs of corners are similar, and then write the proportions to find the unknown lengths of the sides. D off. +8 14 = _ would be the right way to resolve y. 8 10 8 AVOID COMMON ERRORS E Some students may use a false jad of points for writing a similarity statement. Compare the process with writing a conjugate statement and remind them to list the corresponding peaks in the same order. Your sixth time. The builders were given a design plan with a triangular roof as shown. Explain how he knows that the △AED, △ACB. Then find AB. 2 feet E D 15 feet. C 7. 6 feet. B Corresponding corners theorem, ∠AED ≅ ∠C and ∠ADE ≅ ∠B (or ∠A ≅ ∠A by reflexive property of Congruence), both △AED €△ACB by AA Similarity Triangle Theorem. AB _6 = _ → AB = 10 ft 9 Find PQ if possible. _ _ ∠TSQ is right-angled because PS and TR are cross. So △PRS - △TQS by AA Triangle Similarity Theorem. Let x = PQ. P Q 9 15 Explain 3 S 12 R x+9 _ 12 _ → x = 11 = 15 9 © Houghton Mifflin Harcourt Publishing Company T 15 So, PQ = 11. Application of SSS and SAS Triangle similarity In addition to the similarity of the angular triangle, the two additional shortcuts are similar to the two triangles. Side side (SSS) triangle similarity the works When the three sides of one triangle are proportional to the corresponding sides of the other triangle, the triangles are similar. Side angle (SAS) triangle similarity works If the two sides of one triangle are proportional to the corresponding sides of the other triangle and the angles they are attached are consistent, the triangles are similar. Module 16 864 Lesson 4 Differentiate INSTRUCTION IN2_MNLESE389847_U7M16L4.indd 864 Cognitive Strategies 18/04/14 09:01 There are students to write their AAA Similarity Postulates. Then ask them to explain why this postulate is not necessary. The similarity of AAA to Postulate is not required due to this theorem: taking into account the two triangles, if the two pairs of corresponding angles are compatible, the remaining pair of relevant corners must also be in line. AA Similarity Triangles 864 Example 3 EXPLAIN 3 Determine whether these triangles are similar. Justify your answer. N 4 SSS and SAS in the case of triangle similarity 4 M 8 P Q 6 R You will be given two pairs of corresponding lateral lengths and one pair of corresponding angles, so try using the SAS. INTEGRATE MATHEMATICAL PRACTICES Focus on Math Connections MP.1 Remind students that similarities make sure that the ratios of the respective sides are equal. 8 =_ MN = _ 8 =_ 4=_ MP = _ 2 2 _ _ 8+ 4 3 12 3 6 MR MQ Check that the angles added are consistent: ∠NMP ≅ ∠QMR is shown in the figure. Therefore, △NMP ± △RMQ by SAS Triangle Similarity Theorem. note the corresponding parts in the same way congruence statements made M G 4 8 H 15 12 10 J N L 6 You are given three pairs of corresponding lateral length and zero congruent corresponding angles, so try to use the SSS Triangle Theorem. © Houghton Mifflin Harcourt Publishing Company Checks whether the ratios of the respective sides are equal. 3 12 LM = __ GH 8 2 Thus △ GHJ 3 15 MN = _ __ HJ 10 2 △ LMN 3 6 GJ _ =_=_ LN 2 4 SSS Triangle SimilarityOrem . Since you are given all three pairs of sides, you do not have to check congruent corners. Mirror 8. Are all osceles the true dynasty are alike? Explain why or why not. Let one of the isoki's feet. triangle be x and feet another be y. Then the relationship between the parties would _xy and _xy. Each angle at the triangle is a right angle, so the two rectangles are similar to the SAS. 9. Why is not Angle-Side-Angle (ASA) used to prove two triangles similar? The ASA means that the two pairs of corners are compatible, which is sufficient to show similar triangles, so that it is not necessary to check the attached sides. Module 16 865 Lesson 4 LANGUAGE SUPPORT IN2_MNLESE389847_U7M16L4.indd 865 Connect Vocabulary Associates the idea of proof to justify ideas in mathematics. You use the established rules and conventions to draw some conclusions. In real life, proof means showing something by gathering evidence through established rules and conventions. 865 Lesson 16.4 18.04.2014 21:01 Your Turn Questioning Strategies If possible, determine whether the given triangles are similar. Justify your answer. 10. C 5 10 6 F 8 11. H Two triangles cannot be proven to be similar. Although the 3 two given sides are proportional, there are no G pairs attached to the congruent angles. Avoid common errors Pythagorean Theorem, NO = 6 and GH = 4, M 8 Why are ASA and AAS not similarities between theorem? Both contain two pairs of matching angles, so the ASA and AAS triangles are already similar to the AA similar theorem. Some students may have difficulty identifying similar aspects because of the orientation of the numbers. Show these students how they can copy one triangle to a sheet of paper, then cut it out and rotate it so that the two triangles have the same orientation. HJ GJ GH 1 so ___ = ___ = ___ . △MNO △GHI by MN 2 NO MO SSS Triangle Similarity Theorem. IO J 5 N O G 3 H to develop the 12th EQ 10 J 5 N O G 3 H Is the triangle transitive? If you △ABC ± △DEF and △DEF . △GHJ, is △ABC ± △GHJ? Explain. Yes. If the first two triangles are three pairs of congruent corners and the other two questioning strategies, the two onosceles triangles are in sync with the top corners. Explain why these two triangles are similar. Let the vertex angle measurement x°. The equine triangle theorem must be measured by half (180 - x)°. These two triangles are similar. AA Similarity Postulate applies Is there a AAA resemblance to Postulate for quadrilaterals? Use your geometry software to test your assumptions or create a counter-example. No, no square and rectangular have three pairs of rectangles, but they are not similar. 14. Important question check-in How can you prove triangles are similar? Triangles are similar when their respective angles are in sync and their © Houghton Mifflin Harcourt Publishing Company congruent corners. TAKE STOCK OF THE LESSONS LEARNED THE FOLLOWING SIDES ARE PROPORTIONATE, BUT IT IS SUFFICIENT TO USE AA SIMILARITY (show two pairs of concurrent angles), the similarity of the SSS (shows the sides of the three pairs proportional) or the SAS Similarity (indicate that the two sides of the pair are proportional and the angles they add are each other). Module 16 IN2_MNLESE389847_U7M16L4.indd 866 Lesson 4 18/04/14 9:01 What postulates allow you to conclude that triangles are similar to unused changes to map one to another? What do you need to know about triangles? SAS similarity postulate, SSS similarity postulate; You need to know if the triangles are two pairs of any angles, or that all three pairs of sides are proportional, or that the two pairs of sides are proportional and the added angles are in sync. AA Similarity Triangles 866 Evaluate: Homework and Practice Evaluate • Online Homework • Tips and Help • Extra Practice Show that triangles are similar by measuring the length of their sides and comparing the relationships between the respective sides. 1. 2. F A B D E DETERMINATION GUIDE Definitions and skills D B C A C E F Practice 4.5 DE = _ = 8 A3 Pythagorean theorem, NO = 6 and GH = 4, Explore Exercises 1-2 HJ GJ GH 1 so ___ = ___ = ___ . △MNO ≅GHI by MN 2 NO MO Exploring Angular Angular Similarity SSS Triangle Triangle Similarity Theorem. Example 1 Proving the similarity of the angular triangle 3-6 Example 2 Application of angle similarity exercises 7-10 example 3 SSS and SAS application of triangle similarity exercises 11-14 3 ___ the 1,2 2 C = AC 1,4 _3 or 1. 5 3,9 EF = _ = _ BC 2,6 _3 or 1,5 2 2 3 AB = _ DE 1 _ 1 2 AC = _ DF 4 _ 1 2,5 DC = _ = EF 5 _ 1 2 2 2 Determine whether the two triangles are similar. If they are similar, write a statement of similarity. 3. 4. D B A 65° 16° 48° C E F 65° 67° Exercises IN2_MNLESE389847_U7M16L4.indd 867 Lesson 16.4 B A AA similarity Triangle Postulate, △ABC △DEF. The missing angle in △ABC is 180° - 65° - 48° = 67°. Both ∠A ≅ ∠D and ∠C ≅ ∠F. ∠CE △ACB △ECD. AA similarity, so m∠B = 82. M ∠D = m ∠ACD = 78. None △ angles of the ∠ are consistent with the D, so are not similar. Lesson 4 867 Depth of Knowledge (D.O.K.) COMMON CORE Mathematical Practices 1-2 2 Skills /Concepts MP.5 Using Tools 3-17 2 Skills/Concepts MP.2 Recital 18 2 Skills /Concepts MP.4 Modeling 19 3 Strategic Thinking MP.3 Logic 18/04/14 20:01 PM Determine whether the two triangles are similar. If they are similar, write a statement of similarity. 5. 6. D A C A AVOID common mistakes B Since they need to know only that the two corners of the two triangles are compatible, students may think that they need to show that two corners of the two quadrilaterals to do the same, and so on each n-gon. You can use counter examples to show that it is incorrect. Point out that triangles are a special case because they are rigid structures. C ∠D ≅ ∠C and ∠BAC ≅ ∠DCA by Alternate Interior Angles Theorem. △ADC ≅ △BCA by AA Triangle similarity. E B ∠A ≅ ∠E and ∠ACB ≅ ∠ECD vertical angle theorem. Therefore, △ABC ± △EDC by the AA Triangle Similarity Theorem. Explain how you know if the triangles are similar. 7. AC 8. AD A C 4 C 5 D 10,2 9,0 B A B D 7,5 15 D E F D Triangles are similar to the AA Triangle SimilarityOrem. The marked length cannot be found because the length of the corresponding df is not known. 9. The QR 12,0 C E Triangles are similar to the AA similarity triangle. 5 AD ___ = ___ = 4 AD = 6.72 19 10. Find BD. A P A 2.0 B,1,6 B C R Not possible. △ one single angle between the ABC and the ABC, so the similarity cannot be established. The triangles are similar to the AA similarity. QR 1.2 ___ = ___ → QR = 0,96 1,6 2 Module 16 execution IN2_MNLESE389847_U7M16L4.indd 868 C D Q 1,2 17.0 hour 4 868 depth (D.O.K) © Houghton Mifflin Har Court Publishing Company 8.3 COMMON CORE mathematical Practices 20 3 Strategic Thinking MP.2 Recital 21-22 3 Strategic Thinking MP.3 Logic 18/04/14 9:01 AA Similarity Triangles 868 Show whether each pair of triangles are similar, if possible. Justify your answer and write a similar sentence if the triangles are similar. 11. 12. A B 5 6.3 5 C 4 D 16 Theorem, some students have difficulty matching the respective sides. Invite these students to match the smallest sides, the longest side to the longest side, and to match the sides that are neither the longest nor the shortest. 16 A E 9 D 6.4 E 3 9 F ∠ACB ≅ ∠ECD vertical angle, so the two triangles are similar. 3.5 BC theorem. __ = __ = 0,625; 5,6 CD CE = (4.0/6.4) = 0,625. Therefore, CE △ABC ± △DEC by SAS Similarity. 13. 14. B C 5 _ 16 __ 5 AC 16 DE = _ = __; BC = __ . Ratios are not DE 5 EF DF 3 B H 6 15 D 8 A C 18 20 3,2 6 BD G © Houghton Mifflin Harcourt Publishing Company △ABC ± △BDC by SSS similarity. R Triangles cannot be found to be similar by this information because the conjugating angle is not an added angle. BC 5,5 AB ___ = ___ = 1,15 and ___ = ___ 6 = 1 B 5,5 BC 2 ___ = ___ = 2.5; ___ = ___ = 2.5. Ratios ___ BC and ___ are equal, so the triangles are similar to SSS. DE EF DF 4.7 5.C D is the Student did not compare the ratios of the respective sides. The AB △ the shortest side of the ABC, so that its corresponding side is the △ the DF. AC AB IN2_MNLESE389847_U7M16L4.indd 869 lesson 16.4 869 Lesson 4 18/04/14 9:01 P.D. Algebra Find all the possible values for x where the two triangles are similar. (x + 10)° (2x - 50)° x° x° 70° X possible values x = 70, x = 2x - 50, x + 10 = 70, x + 10 = 2x - 50, x + x + 10 + 70 = 180 and x + x + 10 + 2x - 50 = 180. They result in 50, 55, 60 or 70, of which only 50 results are similar triangles. So x = 50° is the only possible value. 17. Identify two similar triangles in the figure and explain why they are similar. Then find AB. B ∠A ≅ ∠A and ∠ABD ≅ ∠C , both △ABD ≅ △ACB by AA AB AD AB 4 = __ → __ = __ → AB = 8 Triangle Similarity Theorem. __ 16 AB AB AC 4 D C 12 18. The picture shows a person taking a photo of himself with a needlehole. The light entering the opening reflects his image on the wall, forming similar triangles. What is the height of the image to the nearest inch? 15 inches 4 feet 6 5 feet 5 feet 5 '5 65 h _ = _ _ → h = 18 inches 15 4'6 54 H.O.T. Focus on higher-level thinking Y B XZ and ∠A ≅ ∠X XY = _ Simple: _ AB AC Prove: △ABC ± △XYZ A C ge07sec03004a AB Z XXY Enlargement Apply △ABC Scale Factor k = __ and let the image △ABC be △A'B'C'. AB then ∠A' ≅ ∠A. It is ∠A ≅ ∠X, both transit ∠A' ≅ ∠X. XY XY XZ also A'B' = k • AB = __ AB = XY and A'C ' = k = AC = __ = __ = AC = XZ. Therefore, over time, AB AC and Houghton Mifflin Harcourt Publishing Company have been able to © 19. Analyze the connections to prove the SAS Triangle's similarity theorem. △A'B'C ≅ △XYZ by SAS Congruence. So a jam of rigid motion maps △A'B'C to △XYZ. The expansion followed this series of rigid movements shows that there is a series of similarity transformations that △ABC △XYZ. So △ABC ± △XYZ. Module 16 IN2_MNLESE389847_U7M16L4.indd 870 870 Lesson 4 18/04/14 21:01 AA Similarity Triangles 870 20. Analyze the connections to prove the SSS Triangle's similarity theorem. JOURNAL Are students a diary entry that explains what the scale on the map means, how it is used and how it relates to the concept of similarity. XZ = _ XY = _ YZ Simple: _ AB AC BC Y B Prove: △ABC ± △XYZ (Hint: The main steps of the evidence are similar to the evidence of AA Triangle Similarity theorem.) A C Z X XY Apply the △ABC with a scale factor k = __ and let the AB △ABC image △A'B'C'. Then: XY A'B' = k ± AB = __ = AB = XY AB XY XY XZ A'C' = k = __ ± AC = __ = AC = AC = XZ AB AC XY YZ B'C ' = k • BC = __ BC = __ BC = YZ . Therefore, △A'B'C ≅ △XYZ by AB BC SSS Congruence Theorem. This means that there is a series of similarity transformations that maps △ABC △XYZ. So △ABC ± △XYZ. ‹ − − 21. Transmission of mathematical ideas A student is asked to find a point in X BC so that △ABC - gt › △XBA and XB are as small as possible. The student does so by building a cross line at AC at ‹ − point A, and then marking the X cross as a line and BC intersection. Explain why this procedure creates a similar triangle that the student was asked to build. 10 6 © Houghton Mifflin Harcourt Publishing Company X B 8 C Possible answer: In order for XB to be as small as possible, it should respond - △ABC's shortest side, which is AB. Therefore, X corresponds to A. 22. Make Guess builders and architects use scale models to help them design and build new buildings. The architectural student builds an office building model where the height of the model 1 1 is the height ___ height of the actual building, while the width and length of the model is the width and length of each 400,200 actual building. The model contains several triangles. Describe how the triangle of this model might resemble the corresponding triangle of the actual building, then describe how the triangle of this model may not be similar to the corresponding triangle of the actual building. Use the similarity theorem to support each response. The triangle, situated on a level fully parallel to the ground, is the side length which is 1 of the side length of the corresponding triangle of the actual building. So two for every ___ 200 triangles are similar to the SSS Triangle Similarity theorem. The triangle, which has multiple height peaks, has no side lengths which form a constant ratio of the side length of the corresponding triangle in the actual building and are therefore not similar. Module 16 IN2_MNLESE389847_U7M16L4.indd 871 871 Lesson 16.4 871 Lesson 4 18/04/14 9:01 PM Lesson Performance Task CONNECT VOCABULARY Figure shows camera obscura and object captured. Answer the following questions about this number: 1. Explain how this would affect the image of the object when the camera is moved closer. How does it objects that can be photographed? 2. How do you know that △ADC is similar to △GDE? The name of the camera obscura comes from the Latin words of the camera, which means room, and obscura, which means dark. Plural Latin words end with the letter a formed by changing the ae. So, more than one of these instruments would be referred to as camerae obscurae. A B D E G G C 3. Write a part that could be used to find the height of the pine. INTEGRATE mathematical practices with Focus patterns in mp.8 camera obscura is square. This means that 4. DF = 12 in., EG = 8 in., BD = 96 feet. How long is a pine tree? 1. As the object reaches closer to the vertical angle D becomes larger, and because △ADC remains similar to △GDE, the height of the image on the back increases. So the camera must be placed far enough away from the object so that the image is no longer than the height of the rear surface, or otherwise the object cannot be photographed. lesson performance task figure , DF = EG. Let's say you want to take a picture of an object that's n meters from the front of the camera. What is the maximum height of such an object if you want to take a picture of its entire height? Explain your reasoning. The maximum height is n ft. AC EG Sample response: Because DF = EG and , ___ = ___ , BD DF AC = BD = 1. 2. Possible answer: ∠A ≅ ∠G, because the corners are the alternative inner corners - - for the ac and EG parallel lines. ∠ADC ≅ ∠GDE because the vertical angles are compatible. So, △ADC - △GDE by AA similarity theorem. AC EG = __ 3. __ BD DF AC __ = 8 ⇒ AC = 64 feet 4. __ 96 12 © Houghton Mifflin Harcourt Publishing Company Module 16 872 Lesson 4 extension activity IN2_MNLESE389847_U7M16L4.indd 872 Can students research methods make simple models of camera obscura. Some models do not require more than a cereal box or shoebox, ribbon, scissors and pin. A more detailed model can be made by blocking the classroom window and projecting outside the scene on the classroom page. Each method provides reasonable results when students compare the dimensions and angles of an external object with the dimensions of the projected image. 18/04/14 21:01 Scores Rubric 2 points: The student's response is accurate and complete execution of tasks or tasks. 1 point: The student's response contains the characteristics of the appropriate response, but is invalid. 0 points: The student's response does not include the characteristics of the relevant response. AA Similarity Triangles 872 MODULE 16 MODULE STUDY GUIDE REVIEW 16 Similarity and Changes in study guide Review Important question: How can you use similarity and change to solve real world problems? MAIN EXAMPLE OF EVALUATION AND INTERVENTION (lesson 16.1) Specify the centre of dilation and the expansion scale factor. 8 B'y B 6 Specify or customize module comments. 4 000 00 00 A 2 0 C 2 4 C'x 6 Dilation keyword asset centre (centro de dilatación) dilylation (dilatación) scale factor (factor de escala) Side-Side-Side similarity (Similitud Lado -La do -Lado) Side-Angle-Side similarity (Similitud Lado-ÁnguloLado) similar (semejantes) similarity transformation (transformación de semejanza) 8 Draw a line through A and A′. Drawing lines through B and B′. Drawing the line through C and C′. The three lines intersect with point O(2) and (3). Here you can find the distance from O to points A and A′. ___ _ d A = √(2 - 3) + (3 - 4) = √ 2 2 __ d A′ = √ (2 - 4) + (3 - 5) = 2√ 2 2 2 2 Find the distance to point a.' © Houghton Mifflin Harcourt Publishing Company Distance from O to A′ is twice as farther from point O to A. Scale factor expansion is 2-1. IMPORTANT EXAMPLE of similar _figures _are proportional. _ _ _ _ BC AD _ = ____ _. BC = FG and EH = AD, so the corresponding angles of the similar figures _ EH FG are consistent. ∠D ≅ ∠H and ∠E ≅ ∠A. Module 16 IN2_MNLESE389847_U7M16MC 873 873 Module 16 (lesson 16.3) △ABCD maps △EFGH by changing the similarity. Write the proportion that BC and¨EH. Then list all corners that are ∠D or ∠E. includes¨873 Study Guide Review 18/04/14 9:35 PM KEY EXAMPLE (Lesson 16.4) Determine whether △ABC and △DEF are similar. If so, justify it with SSS or SAS. A 3 E B 6 7.5 8 4 C D 10 F Check that the ratios of the respective sides are equal. 3 __ 4 6 = __ 3 __ 8 4 7,5 = __ 3 _ 4 10 Since the ratios of all the relevant sides are equal, the triangles are similar to s. EXERCISES Determine the following dilatation. (Hour 16.1) 8 6 y A B 4 D 2 A′ D′ 0 module 16 IN2_MNLESE389847_U7M16MC 874 (3, 0) 2 O 4 C B' x 6 2.874 8 scale factors © Houghton Mifflin Harcourt Publishing Company 1. C'1-3 Study Guide Review 18/04/14 9:35 P.M. Study Guide Review 874 Determine whether the two numbers are similar using similarity changes. (Lesson 16.2) MODULE PERFORMANCE TASK 3. similar △ABC △DEF 8 B COMMON CORE mathematical practices: MP.1, MP.2, MP.4, MP.6 G-MG. A.1, G-MG. A.3-12-8 D A-4 E STUDENTS SHOULD START THIS PROBLEM BY FOCUSING ON WHAT INFORMATION THEY NEED. Here are some questions they might highlight: B y 0 A x C C C D F 0 -4 2 x G -2 4 F E -8 H △ABC maps △DEF by changing the similarity. (Lesson 16.3) 5. • What should be the maximum height of the model: Students can work back from the desired height or find a percentage of the actual height and use it to determine the scale. 6. _ _ Write the part that contains BC and DF. 7. List all angles that are ∠A or ∠E. • What dimensions are not needed for a scale model: students can decide which dimensions may not be needed, such as height the color of the stone changes. 8.¨___¨AB __ = BC¨DE¨EF¨___¨BC __ = AC¨EF¨DF ∠A ≅ ∠D; ∠E ≅ ∠B Determine whether △ABC and △DEF are similar. If so, justify it with SSS or SAS. (lesson 16.4) is not similar to 6 A 9 13 SAS 9. B C D 2 E 4 E 3 B 9 6 A 76° C D 8 76° 12 F © Houghton Mifflin Harcourt Publishing Company • What materials should be used: Students can study suitable materials or you can propose. is not similar to y 2 4 _ _ Write the part containing AB and EF. • How to determine the dimensions of the model: Students should use the proportions to convert actual dimensions to scaled dimensions. Students may first want to convert actual dimensions to decimal places. △ABCD to △EFGH 4. Module 16 875 Study Guide Review Scaffolding Support IN2_MNLESE389847_U7M16MC 875 • Students should understand that they need to convert the units of these measurements so that all units are the same. Possible units are legs, inches, meters or centimeters. • If students decide to solve the problem by first deciding on a scale factor, they must ensure that the scale factor they use results in a model that fits the classroom. 875 Module 16 18/04/14 21:35 MODULE PERFORMANCE TASK SAMPLE SOLUTION Designing model Washington Monument Use scale1 in. = 10 feet. Your task is to design a scale model of the Washington Monument that would be small enough to accommodate inside your classroom. Here are some important dimensions of the Washington Monument for you to consider determining the scale factor of your model. (Note that the stone color changes the part up to the monument due to the halt construction between 1854 and 1877.) Total height Height at the top of the trapeze side First, convert the units so that all measurements are 1 foot, multiplying the inch by the ratio and 12 inches. The results are: _ Total height: 555.42 feet 555 feet 500 feet 5 in. Width at the bottom: 55.08 ft 500 ft Width is the base of 55 ft. 1 in width at the top of the trapeze side is 34 feet 5 inches. The height at which the stone color turns to 16 Width at the top of the trapezoidal side: 34.42 ft To find model dimensions, solve the proportion: 151 ft What scale factor do you use in your model? What are the basic dimensions of your model? model height 1 in. = _____ __ 10 ft Start by making some comments on the model design strategy in the room below. Then use your paper to complete the task. Submit your plan using charts, words, and/or numbers. Using the actual height Of proportion, check the total height of the model to see if it fits in the classroom. 1 in. = ___ x __ → x ≈ 55.5 inches ≈ © ©. scale factor is (____ 12 years) This is 55.54 reasonable. The other dimensions are as follows: Longline side height up: 50 in. Width north: 5.5 inches. Width at the top of the trapeze side: 3.4 inches. Height at which the stone color changes: 15.1 in. Study Guide Review DISCUSSION OPTIONS IN2_MNLESE389847_U7M16MC 876 18/04/14 9:35 p.m. • If students decide to work back at the desired altitude of exactly six feet, how does it change the calculations and scale factor used? • What are some advantages of choosing a scale that makes the model no more than a foot long? Sample answer: The amount of material needed to build a model is much smaller, which in turn reduces both the cost and weight and make the model more portable. Evaluation rubric 2 points: The student correctly solves the problem and explains his reasoning. 1 point: The student shows a good understanding of the problem, but is not fully solved or explained. 0 points: The student does not show an understanding of the problem. Study Guide Review 876 ready to go ahead? You ready to move on? 16.1–16.4 Similarity and transformation PRICEMastership Use the assessment on this page to determine whether students have acquired the definitions and standards covered in this module. • Online Homework • Tips and Help • Extra Practice Answer related to each problem picture. (Lesson 16.1) 1. ASSESSMENT AND INTERVENTION Yes Are these two shapes similar? 16 3 y B′ 2. Scroll to scale factor k. 3. Find an expansion center. (4) 12 4th △A′B′C′D′ area. Compare k ratio __ area △ABCD 8 A′4 A C Ratio equals 9, which is the square scale factor. D 0 Determine which of the following conversions are enlargements. (Lesson 16.1) 5. Access ready to go ahead? and receive instant scoring, feedback and tailored intervention or enrichment. x, y → (4x, 4y) 6. x, y) → (x - 2, y - 2) 8. Response to intervention resources 9. • Success of English Learners • Challenge Worksheets Assessment Resources © Houghton Mifflin Harcourt Publishing Company Find Missing Length. (Lesson 16.3) • Reading Strategies D' x 16) △XYZ maps △MNO with transformation (x, y) → (7x, 7y). If XY = 3, what is the length of MN? Find suitable statements about triangles. (Lesson 16.4) 10. △ABC is △RTS. Write the part that contains AC and RT. Also write congruence statements that must be true. AC _ __ = AB , ∠A ≅ ∠R, ∠B ≅ ∠T, ∠C ≅ ∠S RS RT IMPORTANT ISSUE 11. How can you determine whether a shape resembles another shape? The answers may vary. Example: Two shapes are similar only if one shape can be mapped to another through similarity conversions. These changes are rigid movements, which means reflections, translations and rotations, as well as enlargements. Module 16 COMMON CORE 877 Module 16 12 21 • Leveled module quizzes 877 (1 x, _ 1y (x, y) → _ 3 Dilation additional resources differentiated instruction resources 8 (x, y) → (-x, 3y) Non enlargement • Reteach Worksheets 4 Does not extend to expansion 7. C' B Study Guide Review 877 Common Core Standards Lesson Themes 16.2 1 16.1 18/04/14 09:35 Content Standards Mathematical Practices G-SRT. A.2, G-SRT. A.1, G-CO. A.2 MP.7 2-4 G-SRT. A.1 MP.7 16.2 5-8 G-SRT. A.1, G-CO. A.2 MP.2 16.2 9 G-SRT. A.1, G-CO. A.2 MP.4 16.3 10 G-SRT. A.2 MP.4 MODULE MODULE 16 MIXED OVERVIEW MIXED ASSESSMENT Assessment Preparedness assessment Preparedness 1. Consider each transformation. Does conversion maintain distance? Select Yes or No A to C. A. Dilations Yes No B. Reflections C. Rotations Yes Yes ESTIMATE AND INTERVENTION No ( ) 1 _ x, 1 y . Select True or False 2. △MNO cards △RST with transformation (x, y) → _ 3 3 for each application. A. If RT = 3, MO = 9. Real wrong B. If RT = 12, MO = 4. Real wrong C. If RT = 9, MO = 27. Real false 3. Determine whether the following triangles _ _are similar. If so, explain how. Please note that AC ∥ Bd. A 70° Possible answer: ∠ACB and ∠CBD are compatible because they are alternative interior corners. ∠ABC and ∠CDB are compatible because they have been granted the same measure. So the two triangles are similar to the Angle-Angle similarity. Set ready-made or customized exercise tests to prepare students for high-stakes tests. B ADDITIONAL RESOURCES Assessment tools 70°C • Leveled module quizzes: Modified, B D 4. If △ABC ∼ △XYZ and △YZX, what type of triangle is △ABC? Justify your reasoning. COMMON CORE AVOID COMMON MISTAKES Section 2 Some students have problems after the expansion rule is rejected. Remind students that when you move from a picture to the original drawing, multiply it by the interaction of the original expansion. © Houghton Mifflin Harcourt Publishing Company Responses may vary. Sample: Examine the relevant angles. Since △ABC is △ to XYZ, this ∠A ≅ ∠X. Since △ABC is △ to YZX, this ∠A ≅ ∠Y. Thus, by transitive property congruence, it can be said that ∠X ≅ ∠Y. It ∠B ≅ ∠Y and ∠B ≅ ∠Z, which means ∠Y ≅ ∠Z by Transitive Property Congruence. By putting these facts together, it can be shown that ∠X ≅ ∠Y ≅ ∠Z. Therefore, △XYZ, △YZX and similarities are all triangles. Module 16 16 Study Guide Review 878 Common Core Standards IN2_MNLESE389847_U7M16MC 878 18/04/14 9:35 P.M. A.2 MP.7 16.3 2 G-SRT. A.1 MP.7 16.4 3 G-SRT. A.3 MP.7 16.3, 15.2 4* G-SRT. A.2 MP.3 * Point combines mixed review concepts with previous modules or previous course. Study Review 878 MODULE 17 using similar triangles using similar IMPORTANT QUESTION: Answer: For example, similar triangles can be used to find the height of a high object by measuring your shadow. 17 MODULE Important question: How can you use similar triangles to solve real-world problems? LESSON 17.1 Triangle Proportionality Theorem Lesson 17.2 Segment Sharing Given Ratio This version is designed for Algebra 1 and geometry for only professional development video author Juli Dixon models successful teaching practices in the actual high school classroom. LESSON 17.4 Similarity right triangles Professional Development © Houghton Mifflin Harcourt Publishing Company • Image Authors: ©Lightguard/iStockPhoto.com my.hrw.com REAL WORLD VIDEO Check how properties of similar triangles can be used to determine real world areas in geographical areas such as the Bermuda Triangle. MODULE PERFORMANCE TASK PREVIEW How big is the Bermuda Triangle? In this module, you will be asked to use a map to determine the Bermuda triangle area. How do indirect measurement and similar triangle properties help you find the answer? Let's start by solving the mystery of this Triangle of Bermuda! Module 17 DIGITAL TEACHER EDITION IN2_MNLESE389847_U7M17MO 879 Access to a full set of educational materials where and where you need them: • Access content online or offline • Customize lessons to share with your class • Communicate with your students in real time • View student scores and data instantly to guide your instruction, where it is required for most 879 module 17 879 MATHEMATICS PERSONAL COACH EVALUATION AND INTERVENTION Set automatically sorted homework, quizzes, tests and intervention activities. Prepare your students with updated, common basic practice tests. 18/04/14 10:00 PM ARE YOU READY? Are you ready? Complete these exercises to view the skills you need in this module. Scale and scale drawings Example 1 ESTIMATE THE PREPARATION Specify the AB = 4, BC = 3 and B'C' = 6 length given in segment A'B'. A' A △ABC image is created as a result of a scale drawing, so that conversion is an expansion. C Use the assessment on this page to determine whether students need strategic or intensive intervention for module prerequisites. • Online Homework • Tips and Help • Extra Practice B 2BC = B'C', so 2AB = A'B'. ASSESSMENT AND INTERVENTION B' C' A'B' = 2(4) = 8 Specify the length of the edge. 1.6 BC, considering AC = 5, A'C' = 15, and B'C' = 18 the similar number of Example 2 figures PQRS and KLMN are similar. The angle is determined in the FIGURE KLMN, which is in ∠Q, and the measurement shall be determined by m∠Q = 45°. ∠Q ≅ ∠L The corresponding angles of similar numbers are appropriate. m∠Q = m∠L = 45° Angle concealability definition. 3 2 1 Personal Math Trainer creates automatically a tailored intervention and strengthening for your students, tailored to the needs of each student! Give it to every corner size. 2. 3. 67° m∠A, given △ABC ≅ △DEF and m∠D = 67° m∠E, given △PQR ≅ △DEF, m∠R = 13° and m∠D = 67° " 100° Example 3 The right triangle △ABC side length is AB = 3 and BC = 4. Find the length of hypotensive AC. △ABC is the right triangle, so Pythagorean Theorem can be used. __ ac = √ ab 2 + bc 2 __ AC = √ 3 2 + 4 2 = √ 9 + 16 AC = 5 Write Pythagorean Theorem. Replace and simplify. Simplify. Find the side length of each right triangle. 4. 5. DE, given AB = 15, AC = 17, and AC is hyponeuse IN2_MNLESE389847_U7M17MO 880 ADDITIONAL RESOURCES See the table below for the full list of intervention resources available in this module. Response to intervention resources also include: • Pre-flight tests for the second stage for each module • Second-stage skill tests for each skill 13 DE, taking into account DF = 5, EF = 12, and DE is the hypotensive module 17 © Houghton Mifflin Harcourt Publishing Company The Pythagorean Theorem TIER TIER, TIER 2, TIER 3 SKILLS 8 880 Response intervention Tier 1 lesson intervention journals Tier 2 strategic intervention skills intervention worksheets reteach 17.1 reteach 17.2 reteach 17.3 reteach 17.4 40 scale factor and scale drawings 41 Similar Drawings 43 Pythagorean Theorem 44 ... Tier 3 Intensive Intervention Worksheets Available online Building Block Skills 36, 38, 46, 48, 50, 63, 80, 82, 86, 90, 95, 100 Challenge Worksheets Expand Math Lesson Activities TE Module 17 880 LESSON 17.1 Name Triangular TheoD Important Question : If a parallel line with one side of the triangle intersects with the other two sides, how does it divide these sides? The source locker for the construction of similar triangles explore student is expected: In the following action you will see one way to build a triangle similar to a given triangle. G-SRT. B.4 Prove theorem triangles. Also G-CO. C.10, G-CO. D.12, G-SRT. B.5 Mathematical practices COMMON CORE Date 17.1 Triangle Proportionality Theorem Common Core Standards Resource Locker The student is expected to: the Triangle Proportionality Theorem ... how the proportionality of triangle and its opponent. B C _Select point A0. Label this E. E ENGAGE Joon cuts the two sides of the triangle into segments, and the relationship between the two segments on both sides is equal. PREVIEW: LESSON PERFORMANCE TASK View the Include online section. Discuss the photo, and ask students to look for geometric shapes in the sail. Then, check the hour performance task. C © Houghton Mifflin Harcourt Company Essential Question: If the line parallel to one side of the triangle intersects the other two sides, how does it divide these sides? B C Build angle tops E, which is in line with ∠B. Mark the point where the angle _ side of the corner you built intersects with AC when F. A E B D E F C _ › Why are EF and BC parallel? ‹ › – – Coplanar lines EF and BC are cut by horizontal AC to ∠AEF ≅ ∠B. By the Converse _ ‹ › – – Theorem of the respective angles, EF ∥ BC. ___ Graphics\02192014\From Rames\Econ 2016\W Use ruler to measure AE, EB\\192.168.9.251\07Macdata\07Vol1Data\From _ AF, and FB. Then compare the AE AF ratios ___ and ___. EB FB measurements are different, but if the structures are accurate, the ratio should be roughly equal. Module 17 must be EDIT--Chan El key=NL-A; CA-A Correction Lesson 1 881 gh File info made throu Date class by portionali Name ge Pro 17.1 Trian rem Theo Resource Locker two sides that cuts triangle side in one line parallel Explore constructing similar triangles in this lesson. nts 'Check studs ns. Construction 'C Tl E 1Tl, A ..., Coplanar Angles Theo ics\02192014 Sponsoring From Graph e the1Data_ corre compa\07Vol _ Thenacdata FB.1\07M _ \\192.1 68.9.25 ld be, AF and re, AE, EB relations show measure ate, Use ruler _AF__ ns have ae and ___ FB constructo ratios eb, but if different, ents is a lesson 1 Measurem t. equa approximat Why is © Houghton n Mifflin Harcourt 8 - 881 Module 17 ESE3898 7L1 881 47_U7M1 IN2_MNL 881 Lesson 17.1 18/04/14 10:03 PM 18/04/14 10:04 PM Reflect 10:04 PM Reflect 1. EXPLORE Discussion How can you show that △AEF ∼ △ABC? Explain. You can indicate that there are three pairs of synchronized corners. ∠A ≅ ∠A (Reflexive Property) ∠AEF ≅ ∠B (construction). Also because EF ∥ BC, ∠AFE ≅ ∠C. Then build similar triangles that can be used Criterion, because you can show that there are two pairs in sync with corners. For example, 2. INTEGRATE TECHNOLOGY AE AF What do you know about the ratios ___ and ___? Explain. EB FB Since △AEF ∼ △ABC, the triangles have the opportunity to do explore activities either in the book or online. AE AF proportional, __ and ___? Explain. AB AB 3. Make a guess Use your answer step E to make an assumption about the line segments produced when the line parallel to one side of the triangle intersects the other two sides. The parallel line divides the other two sides so that the length of the segments is the POLLING STRATEGIES How to use the AA similarity criterion to show the two triangles are similar? Indicates that two corners of one triangle are in relation to two corners of the other triangle. This allows you to conclude that the two triangles are similar. Proportional. Please explain 1 Proof of the triangle's theorem of proportionality. As you can see in the explore, when a parallel line parallel to one side of the triangle, intersects the other side of the triangle, the length of the segments is proportional. Triangle Proportionality Theorem Hypothesis A E B EXPLAIN 1 Conclusion AE = _ AF EB FC F C EF ∥ BC Example 1 Prove Triangle Proportionality Theorem ‹ › Simple: EF ∥ BC AE AF = ___ Prove: ___ EB FB Step 1 Show that △AEF ∼ △ABC ‹ › – As EF ∥ BC, you can conclude that ∠1 ≅ ∠2 and ∠3 ≅ ∠4 corresponding angles of the theorem. E 1 The theory of proportionality of © Houghton Mifflin Harcourt Publishing Company, if the line is parallel to the triangle side intersects both other parts, it divides those parties proportionally. 3 4 F C INTEGRATE mathematical practices in focus on Reasoning MP.2 Review with students, which geometric statements can be used to justify each stage of the certificate. 2 B Questioning Strategies So, △AEF ∼ △ABC by AA Similarity Criterion . Module 17 882 x+y Is it always true that the fraction ___ z can be y x + __? Yes, provided that z ≠ 0, written as __ z z z Lesson 1 PROFESSIONAL DEVELOPMENT IN2_MNLESE389847_U7M17L1 882 Mathematics background Triangle The triangle proportionality theorem and its conversion can be a single double condition: a line that intersects two sides of the triangle is parallel to the third side of the triangle, if and only if it divides the connected sides proportionally. 18/04/14 22:04 How do you take the interaction on both sides of the equation like x = y? The mutual part of the expression is 1 divided by an appropriate expression. One option is to multiply both sides ___ (xy ≠ 0). Triangle Proportionality Theorem 882 Step 2 Use the fact that the corresponding sides of similar triangles are proportional to the ae at, proving that __ = ___ . EB FC EXPLAIN 2 AC AE ___ = ___ Application of the proportionality of the AE Triangle AF+ FC ___ AE + EB __ = AE Segment Adding Postulate AF EB = 1+_ AB integrate mathematical practices Focusing on communication with MP.3 Some students may incorrectly meet the parties are proportional. AF 1+ EB = __ AE FC _ a+b a b ___ Use real estate that ___ c = c + c. AF FC ___ Subtract 1 from both sides. AF AE = __ EB FC Mirror the triangle Proportionality theorem, which means that the parallel line parallel to one side of the triangle divides the other two sides instead of proportional segments. Show these students that synchronized segments result only in a special case where a line intersects both sides of their centers. 4. Explain how you conclude that △AEF ≅ △ABC without ∠3 and ∠4. A △ A Kongruence reflexive property and ∠1 ≅ ∠2, as these are the appropriate angles; △AEF ≅ △AAA AA similarity criterion. Example 2 Triangle Proportionality Theorem Find the length of each segment. ___ CY __ AX ___ AY by Triangle Proportionality It should be taken into account that XY ∥ BC so ___ = YC BY Theorem. B 4 X 9 Substitute 9 AX, 4 XB and 10 AY. © Houghton Mifflin Harcourt Publishing Company Then settled cy. C 9= __ 10 _ 4 CY Take mutual on both sides. 4= CY _ 9 10 Next, multiply both sides by 10. () (CY 10 4 = __ 10 _ 9 10 NP ___ PL L Theorem of triangle proportionality. N Q 2 Multiply both parties by 3 A () _ NQ It is given that PQ ∥ LM, so ___ = QM 5= _ NP _ 2 3 10 40 = CY or 4 _ 4 = CY _ 9 9 → Find PN. Substitute 5 Y 3 5 QM and 3 PL . : 3 Module 17 () 5 = 3 _ 2 NP → _ 3) 883 M Q 2 P N 15 2 _ or 7 1_ 2 = NP Lesson 1 collaborative learning IN2_MNLESE389847_U7M17L1 883 Small group action Give small groups of students each page with three parallel lines cut by cross-section. Invite students to measure cross-cutting segments and check the relationship formed. Then let them draw another cross over the parallel lines and measure the resulting segments. Invite students to form a hypothesis based on their results. 883 Lesson 17.1 18.04.14 22:04 Your turn Find the length of each segment. _ 5. Dg E 32 24 F 6. QUESTIONING STRATEGIES _ RN 40 C 8 D M G How can you use the triangle proportionality theorem to find unknown dimensions? If the line cutting the two sides of the triangle is parallel to the third, you know that it cuts both sides proportionally. Knowing that it allows you to create and resolve part of the missing half length. 5 P Q 10 N R European Economic and Tav Directorate-General for Justice 32 24 40 24 10 _ 8 RN 5 5 __ = Mq _ ; _ = _ ; _ = _ ; _ = _ ; 40(_) = dg MR = ; _ = _ ; RN = (_)10 DG 24 CF DG 32 40 32 RN 8 8 4 4 8 Explain 3 5 10 QP RN 1 50 25 RN = _ or 6_ Proving the opposite triangular proportionality theorem AVOID COMMON ERRORS Some students may write proportions that do not compare the corresponding the indicator staa- Let them use different colors to show proportional parts. It is also theorem of the proportionality of the triangle. On the other hand, the theorem hypothesis of the proportionality of the triangle Where the line divides the two sides of the triangle proportionally, it is parallel to the third side. Example 3 Converse of the Triangle Proportionality Theorem AE AF = EB FC E F C B ‹ › _ EF ∥ BC EXPLANATION 3 C Proof of the theory of proportionality contrary to the triangle proportionality theorem AE reverse = _ AF Simple: _ EB FC _ ‹ › – Prove: EF ∥ BC A F E AE = AF, and having reciprocal It has been granted that _ EB FC EB = FC AF . Now add 1 on both sides shows that AE ___ C B AF on the right side. AE on the left and _ on both sides, adding _ AE and AF AF AE FC EB + AF + AF + AF It gives AE _ AF = _ ___ = ___ = Adding AC and using Segment Addition Postulate provides AE AF . _____ Since ∠A ≅ ∠A, △AEF, △ABC SAS on the basis of the similarity criterion. ‹ › _ – 2 ‖. Integrate Mathematical Practices To Focus On Math Connections MP.1 Remind Students that opposite © Houghton Mifflin Harcourt Publishing Company Show to △AEF ± △ABC. Theorems turn off the condition and conclusion. Review conversations about previous lessons, such as the theorem of triangles, chat, and discuss why they are useful. ∠AEF ≅ ∠ ABC and has corresponding angles and ∠AEF ≅ ABC . ‹ › – & So, EF ∥ counterparty to the theorem of the respective corners. Module 17 884 QUESTIONING STRATEGIES What are the main steps in the evidence? Firstly, they show that the two triangles are similar to the SAS. Then use the conjugating corresponding angles to indicate that the lines are parallel. Lesson 1 DIFFERENTIATION GUIDE IN2_MNLESE389847_U7M17L1 884 Using transparencies to explain the Triangle Proportionality theorem. Place the parallel line on transparency separate from the triangle chart. Slide the line up and down against the triangle to indicate that the sides are distributed proportionally regardless of the position of the line. 18/04/14 22:04 How does this evidence resemble the Triangle proportionality theorem? Both evidence uses the postulate properties of the addition of reapers and segments to work with proportions. Both evidence also uses the fact that you can take mutual both sides of the role. The Golden Relationship Proportionality theorem 884 R Mirror 4 _ Criticism Reasoning Student argues that UV must _ be parallel to ST. Do you agree? Why or why not? 7. RU RV Yes; for RU = USA and RV = VS, ___ = ___ = 1. USA VT Applying the converse triangle proportionality theorem S T - Both UV ∥ ST by converse triangle proportionality theorem. Application of the opposite direction of the triangle proportionality theorem Example 4 MATHEMATICAL PRACTICES Focus critical thinking on MP.3 Can students discuss a special case You can use the Converse of Triangle Proportionality theorem to check that the line is parallel to the side of the triangle. Example 4 Make sure that the line segments are parallel. _ _ MN and KL JN JM 30 = 2 42 = 2 __ = _ 21 15 MK NL JM JN _ = _ Since _ _ MN ∥ MK NL Q A parallel line intersecting two sides at the centre. 42 21 M K L 15 J 30 N Theorem of proportionality of the triangle. DOUBT STRATEGIES __ DE and AB (considering at AC = 36 cm and BC = 27 cm) Ⓐ A D 20 cm AD = AC - DC = 36 - 20 = 16 B BE = BC - EC = 27 - 15 = 12 15 cm E C 20 5 CD = _ DA 4 16 15 5 CE _ = EB 12 CE _ DE ∥ AB, which is the opposite of the theorem of the proportionality of the triangle. CD = _ Since _ DA B © Houghton Mifflin Harcourt Publishing Company How does the Converse Triangle Proportionality theorem differ from using the theorem itself? TPT is used when the two sides of the triangle is parallel to the third side and you can use it to determine that the two sides are proportional. You use, on the other hand, when you are given that the line cuts the two sides proportionally and you want to prove that it is parallel to the third side. V U Reflects 8. Submit mathematical ideas to △ABC, for example, what is AB ___ ? Explain how you know. DE __ 5 CD AB ___ = _ ; Possible answer: for DE ∥ AB, the corresponding angles A and CDE are 4 DA DE CD CE congruent, as well as the corresponding angles B and CDE. So, △ABC △EC and ___ = __ , DE EA EB your turn 9. _ Make sure that TU and RS are parallel. R 72 TR U 4 54 108 4 __ 90 67.5 72 US ___ VU VT = , so RS ∥ TU. TR US V 90 _ 5 VU 67.5 135 5 VT __ = _ = __ 54 S Module 17 885 Lesson 1 Language Support IN2_MNLESE389847_U7M17L1 885 Connect Vocabulary Some students may find it difficult to identify the field. Remind them that dowlars mean over-and-trans fixate, such as transport, transfer and transatlantic transport. Transverse - location or lying over. Therefore, there is a crossline of a line that is located over other rows. 885 Lesson 17.1 18.204 22:04 To Work _ 10th in △ABC, XY ∥ Bc. Use the notification of similarity and proportionality to identify as many different proports as possible. AC AX YC B AY XB AX = ; = ; = ; Possible answers: AY XB AY AB AC AX YC AX ________ DEVELOP X Y C QUESTIONING STRATEGIES What are triangles with synchronized sides and triangles with proportional sides? The congressional party has equal dimensions, but the proportional parties have equal relations with each other. 11. Discussions Which theories, characteristics or strategies are common in demonstrating the theorem of proportionality of the triangle and demonstrating the opposite of the triangle's proportionality theorem? Possible answers: Segment Postulate; the characteristics of the political groups; Taking the reciprocal from both sides of the section, and using the similarity criteria (AA for the Triangle proportionality theorem and SAS its reverse) - 12. An important issue in Check-In Let's say the line parallel side of BC - - AX △ABC intersects the side of AB and AC respectively in paragraphs X and Y, and ___ = 1. What XB do you know X and Y? Explain. ___ AY X and Y are the centre points of parties AB and AC. If AX = 1, then ___ = 1. XB YC _ Then AX = XB, so X is the center _ AB. Similarly, Y is the center _ AC. SUMMARIZE LESSON If the line cut two sides of the triangle proportionally? What does it mean that the sides are cut proportionally? If it is parallel to the third page, the ratio of the respective components are equal. Evaluate: Homework and practice Get ready • Online Homework • Hints and Help • Extra Practice Check student designs. _ ‹ › XN XM ZY ∥ Mn. Write a paragraph certificate indicating that ___ = ___ . NY MZ Y X N _ ‹ › From ZY∥ MN , ∠XNM ≅ ∠XYZ and ∠XN M≅ ∠XZY theorem of the respective angles. Z M So △XYZ ∼ △XNM by AA similarity criterion. Since the respective sides of the like XZ XY triangles are proportional, ___ = ___ . Use Segment XN XM Adding Postulate to write XZ as XM+MZ and write XY as XN+NY. So. ______ XM XM + MZ © Houghton Mifflin Harcourt Publishing Company 2. Copy the ABC triangle you drew to explore the activity. Create a line _ ‹ › – FG in parallel with AB using the same method as you used in the investigation. XN + NY XM MZ XN NY = ___ . It can be rewritten ___+___ = ___ + ___. Since each number shared itself XN XM XN XN XM XN MZ NY MZ ___ is 1, you can write the expression 1 + XM = 1 + XN . Subtracting both sides, ___ = ___ . XN NZ NY Module 17 NZ_MNLESE389847_U7M17L1 886 Tutorial 886 Lesson 1 1/18/04/14 10:04 AM TRIANGLE Proportionality Theorem 886 Find the length of each segment. _ 3. CLICK H G 6 4. X m M n K Definitions and skills 18 Construction of similar triangles 1 Example 1 Evidence of the triangle theorem 2 Example 2 Implementation of the theoretical triangle proportionality 3-5 Example 3 Triangular proportionality theorem exercise 13 Example 4 Implementation Of Triangular Proportionality Theorem Exercises 6-9 6 8 _ AB and CD 6. 7. D 14 B © Houghton Mifflin Harcourt Publishing Company 9. 18 30 _ MN and QR 8.9 10 N 3 R 2,7 M Q PM = 9 - 2,7 = 6,3 PN = 10 - 3 = 7 6,3 7 PN = 7 PM = 2,7 3 N MQ _ | Sex, MN ∥ Qr. __ _ CA 3.5 2.5 E 1,5 FW ___ = ___ = 0,6; 2,5 WD 2,1 FX __ = ___ = 0,6 3,5 XE 3.5 From 16 20 16 5 AM CN 12 _ 14 CL 4 AL _ = = _ ; _ = = _ 15 2 15 L5 LC 4 MB 16 20 8 NB __ CL CN __ = and LN∥ Ab. So, LA NB __ AM AL So, __ ≠ , and LM are not parallel BC. FX __ FW ___ = , WX ∥ DE. XE WD M 16 A 8 B 15 20 MB LC N L 16 12 C 10. The map has a 1. Town Hall is 2. 3.3 km 3.8 km 3.8 km 1. Exercise IN2_MNLESE389847_U7M17L1 887 ( ) 2,1 x ___ = ___ 2,4 (2,1 x ___ = 1.4 = 1.8 2.8 2.8 1.8 ( ) 1.8 = 2.4 = 4.8 miles Distance is 4.8 miles. Module 17 Lesson 17.1 14 _ WX and CF, CD WX _ ∥ CF. Using the theorem of the proportionality of the triangle, find the length of each segment. Truly Hall 887 49 V 74 1 D E _ 14 _ = 14 _ 3 = 14 2 DB 4 3 EC _ 12 = _ 3 = 3 4 CA __ EC ED Sou __ So, AB ∥ CD.DB U V P 2 43 8 35 __ XY XZ VN _ VM _ VM _ XY = 12; _ = _ ; _ = _ ; 8 YU ZV NT MU 14 35 XZ 12 _ 12 _ = _ ; XZ = (_)30 VM = (_)8 = 20 3 AC 4 12 E 30 U KL 4 1 _4 = _ ; KL = (_)8 = 5_ Practice Z Y 30 L 6 _ VM 5. 8 J 4 ASSIGNMENT GUIDE _ XZ Lesson 1 887 Depth of Knowledge (D.O.K.) COMMON CORE Basic Mathematical practices 1-9 2 Skills /Concepts MP.4 Modelling 10-12 2 Skills/Concepts MP.4 ge07se_c07l04007a Modelling 13-15 2 Skills / Definitions MP.2 Recital 16 3 Strategic Thinking MP.2 Recital 17 3 Strategic Thinking MP.3 Logic 18 3 Strategic Thinking MP.1 Problem Solving 18/04/14 10:04 PM 11. The map is parallel to 5th Avenue, 6th Avenue and 7th Avenue. How long is Main Street between 5th and 6th Avenue? 5th Ave. 6th Ave. Main St. g. gS 0.3 5th Avenue and 6th Ave. 0,4 0,5 km per 0,3 0,3 m ___ = ___ 0,4 0,5 In exercises giving length with an algebraic expression, it is a common error to give the variable value in response. Remind students to use the variable value to evaluate expressions and find segment length. ( ) 0,3 m ___ = ___ 0,4 = 0,24 0,5 The length is 0,24 km. 0.5 AVOID COMMON MISTAKES Let you be the length of kilometres of Main Street between 7th Ave. KM integrate mathematical practices Focus on modeling MP.4 Problems with two triangles, to encourage G 12. A multi-stage recording device is a horizontal line that has a parallel base. a. Find LM. 2.6 GL GH ___ 10.4 ___ 2,6 ___ = ___ ; 11.3 = ___ ; LM = 11.3 (___) = 2,825 ft LM HJ LM 2.6 11.3 11.3 ft 10.4 10.4 b. GM Scroll. L GM ≈ LM = 11,3 + 2,825 = 14,125 M N H 2,6 ft E F J 2,2 ft K Find MN to the nearest tenth of a foot. c. 10.4 feet students draw (or color) two triangles separately to identify the corresponding parts. GJ ___ 2,2 GM 13 ___ 2,2 ___ = ___ ; 14.125 = ___ ; MN ___ ; MN = 14,125 (___) = 2.4ft MN JK MN 2.2 ft 13 ft 13.13 HP HJ HjM ___ = 1.18 and ___ = 1.18. LM and __ d. Make a guess write ratios ___ MN JK decimal places in the nearest hundredth and compare them. Make a guess between the parallel ‹ › ‹ › ‹ › – lines LD , ME and NF and cross-directions GN and GK. MN JK Parallel lines divide cross-sections proportionally. E-phrases ‹ › - 1. AB ∥ CD , CD ∥ AF ‹ › – 2. Draw EB intersecting CD X. F For reasons 1. Given 2. Two dots determine the line. AC BX 3. ___ = ___ 3. EF AC BD 5. ___ ___ DF CE 5. Equality Subtract AB + CD ∥ AF 4. ___ = ___ 4. CE AF 4. Triangle Proportionality Theorem BD BX 4. __ = __ DF CE 5. Equality Subtract AB ∥ CD, CD ∥ AF Module 17 888 © Houghton Mifflin Harcourt Publishing Company 2. This is accompanied by the theorem of the proportionality of the triangle, that when three or more parallel lines intersect two crosses, they divide the crossings proportionally. Fill out the proof. ‹ › ‹ › ‹ › – given: Parallel lines AB ∥ CD, CD ∥ EF E F A B AC AC BX BX BD BD ____ Prove: ___ XE XE DF CE XE DF CE D C 888 Hour 18/04/14 10:04 PM Lesson 17.1 Triangle Proportionality Theorem 888 14. Assume that LM = 18.75. Use the triangle proportionality theorem to find the PM. JOURNAL L 15 ___ 15:00 ___ = ___ ; PM = 10 24 PM - 2 LP 3(PM 3 16) PM 2 4 ... JM ___ ; PM = 36 - _ PM ___ = (22 = 14.08 2 22 22 25 25 25) _ PM = 14 or 14.4 Do students write a diary entry explaining in their own words the theorem of proportionality of the triangle and its opponent. Invite students to add numbers with inscriptions. Sun 10 2 15. - Which measures allow you to conclude that UV ∥ ST? Select all that apply. a. SR = 12, TR = 9 B. SR = 16, TR = 20 C. SR = 35, TR = 28 D. SR = 50, TR = 48 E. SR = 25, TR = 20 20 5 _ 16 _ = _ ; 20 ≠ _ 4 20 5 16 16 20 20 16 20 16 4 ___ = ___ = 16 _ 12 _ 12 12 9 20 5 ___ 16 4 ___ = ___ ; 16 ≠ ___ ; 20 ≠ _ 4 20 5 16 16 20 20 16 20 16 4 ___ = ___ ; explanations: 30 12 5 ___ ; ≠ 37 28 7 35 28 20 16 20 16 4 ___ = ___ ; ___ 9 ___ ; ___ ; 20 16 S T V R U 16 4 ___ = ___ ; 5 ≠ ___ = ___ 12 4 Reply: Only C and E allow you to conclude that UV ∥ ST. H.O.T. Focus on higher-level thinking 16. Algebra What value x is GF ∥ HJ? G 30 F 45 - H EJ + 1 EH ___ GF ∥ HJ as ___ = ___ ; 5x ; 360 45 40 JF HG 4x + 4 J 5x + 1 4x + 4 36x + 36 = 40x; x = 7 to - . E Then x = 7, GF ∥ HJ 17. To pass on the mathematical ideas John used △ABC to write the evidence - of Centroid Theorem. He started drawing medians of AK and CL, - in sections Z.s. Then he drew midsegments LM and NP, both - parallel to the median AK. © Houghton Mifflin Harcourt Publishing House +4 5x + 1 = 360(______) ; (4x_____ 45) 40) - Simple: △ABC with medians OF AK and CL and midsegments LM - and NP A L B Prove: Z is located 100,000 from each △ABC peak to in the opposite place. M L a. Complete all the statements to justify the first part of John's certificate. By definition of the central segment, MK = _12BK. Definition sMK KC = median 2, BK __ 12KC. So, replacing MK __ 12KC _ MC & ZC ___ KC by Triangle Proportionality consider △LMC. LM∥AK, both ___ LZ Z K N P C Z K P C MK ZC 2LZ = ___ = ___ 2 and is located ... _23 Theorem and ZC = 2LZ. Because LC = 3LZ, ___ LC 3LZ distance to peak C △ the centre of the opposite side of the ABC. b. Explain how John can complete his evidence. Z without distance, the same process may be repeated twice to show that Z is located _ 3 from highlights A and B △ABC to the centers on the opposite side. Module 17 IN2_MNLESE389847_U7M17L1 889 889 Lesson 17.1 18/04/14 10:03 18. Fixed problem solving given △ABC FC = 5 th that you want to find BF. First, find the value that must be the proportionality of the triangle. Then describe more than one way to find BF and find BF. AVOID common mistakes When students try to find an area △AEF, they can get lost in the triangle of the right triangle 1 bh find in the area. at right angles △AEF, then use A = __ 2 It gives the wrong surface area of 31.6875 feet 2, slightly larger than the actual area that can be found by adding areas of two different right triangles, △AGE and △EGF. This triangle applies to the theorem of proportionality, y y - 9 B (5.5, y) 8 7 F (7, 6) 6 5 4 3 2 C (10, 2) 1 A(1, 2) E (4, 2) x 0 1 2 4 5 6 7 8 9 — shall be such that AB and DE are parallel. - y-2 y-2 - 2 4 EF inclination = 6 ___ =_ and AB inclination = ___ 5 - 1 - 7-7 y-2 5.5 - 1 4 ( ) 4,5 4 4 Let ___ =_ and solve y: y - 2 = 4.5 _ 6 y = 8. 4.5 3 3 BF, you can use Pythagorean theorem, distance formula or triangle proportionality theorem: (3 3 BF AE __ AE = 3, EC = 6 , FC = 5 ___ = BF ___ ; _ BF = 5 _ 2,5 6 6 FC EC 5 INTEGRATE Mathematical Practices Focus on recital MP.2 rectangles similar to the lesson Performance Task Shown here is a triangular striped sail, showing some of its dimensions. Find one of the following. A 1. H C 8 5 A J C 1 5 Feet C 2 AGH 5 ft E BE 3. D 4 C 4 E 7th number sails will make $10,000 if sail material costs $30 per square metre SQUARE YARD HG 4. 1.5(HG) = 42 1,8 BC ___ = ___ 1 ,5 1,2 2(BC) = 27 BC = 2.25 2.5 AJ ___ = ___ 3. 2.25 2 2 6.2 (GF) = 6,25 1 • base area = _ 2 = 30.9 2 5. F 6,5 ft 1(= 10.3) (6) 2 36 + (GF) = 42.25 HG ≈ 2.8 2.2 6.2 + (GF) = (6.5) E GF 2.5 Perimeter = 2,5 + 2,25 + 1,5 + 3,5 + 6,5 + 2.5 + 2.8 + 1.2 + 1.8 + 2 = 26.55 7. 30.9 ft ___ = 3.43 yd 2 9 feet $30 3.43 yd 2 ___ = $103 per sail 2 yd $2 10,000 total ___ = 97 sails $103 per sail © Houghton Mifflin Harcourt Publishing Company 3.5 G 3.5 ft 6. △AEF 1.5 1.2 ___ Region ___ Triangle The theorem can go like this: if a line segment parallel to one side of the triangle intersects both sides perpendicularly proportionally. Is the theorem true? Give an example to support your answer. - yes, I don't know. possible response is compiled: rectangle ABCD, segment EF ∥ AD, and intersecting AD ' and BC in parallel with AB FD = 5. Since AB = FE, ABEF is a rectangle, so BE = 3. Since FE = DC, FECD is a rectangle, both EC = 5.3 and segment divides BE ___ = AF ___ So, ___ FD EC 5 perpendicularly proportionally. 1,8 ft 1.2 ft R H D 5 A(AEF) = 1.25 (AJ) 3 4 AJ Module 17 890 Lesson 1 EXTENSION ACTIVITY IN2_MNLESE389847_U7M17L1 890 Recommended Students sail lesson Performance Task. Then give these instructions: _ _ 1. Use your knowledge of similar triangles to find lengths of BJ, CI and DH. Rounded up to the nearest century. 1.54 feet; 2.92 feet; 3.85 feet 2. Find perimeters △AHD. Estimated △ABJ, △ACI, △ADH and △ITME. 6.04 feet; 11.47 feet; 15.1 ft; 23.55 feet 3. Are the perimeters of the triangles in the same relationship as the side lengths? Explain. The perimeters are in the same relationship as the side lengths. Students' △AC AC = 1.9 = _____ . should give sample results such as ___ AB perimeter △ABJ 18/04/14 10:03 Scores Rubric 2 Point: The student's response is accurate and complete execution of the task or tasks. 1 point: The student's response contains the characteristics of the appropriate response, but is invalid. 0 points: The student's response does not include the characteristics of the relevant response. Triangle Proportionality Theorem 890 LESSON 17.2 Name Segment Division given ratio in class 17.2 Segment sharing with a given ratio on a given issue: how can I find a point in a directed line segment that divides a segment into a given ratio? A common Core Math Standards Resource Locker Student is expected to: COMMON CORE G-GPE. B.6 It takes only one number to determine the exact position on the number line. Therefore, a number line is sometimes called a one-dimensional coordinate system. Mile markers on a straight stretch of highway make it part of the highway with a one-dimensional coordinate system. Mathematical practices common core MP.5 Tools E.M.A Straight Highway, exit from Arthur Avenue is mile marker 14. Collingwood Road exit is 44. The state Highway Administration plans to set out a exit on Briar Street at 23 Arthur Avenue to Collingwood Road. Do the following to determine where the new exit should be placed: Work in groups to find the ratios of divided segments. IF the segment is on the number line, subtract the coordinates to find the distance between the endpoints. Then multiply the segment to find the coordinate of the point to be shared in the segment ratio. If there are no coordinates, use a compass and a straight line to divide a given segment into equal parts, and identify the point that divides the segment by a given ratio. Mark Arthur Avenue (point A) and Collingwood Road (point C) on the number line. © Houghton Mifflin Harcourt Publishing Company • Image Credits: ©Condor 36/ Shutterstock Important question: How do you find a point-oriented line segment that divides a given segment into a given relationship? Partition a segment in a one-dimensional coordinate system Find out what the point of the directed line segment is between the two given dots that divide the segment by a given ratio. Also G-CO. D.12 10 B D B 20 C 30 40 50 What is the distance from Arthur Avenue to Collingwood Road? Explain. 30 miles; absolute value of the coordinate difference: │44 – 14│= 30. C How far does Briar Street come from Arthur Avenue? Explain. 2 20 miles; · 30 miles = 20 miles 3 D What's the marker number for the briar street exit? Why? - 34. 14 + 20 = 34 E Plot and mark the exit of Briar Street (point B) on the number line. Check the student number lines. PREVIEW: LESSON PERFORMANCE TASK Module 17 MUST EDIT- Chan NO KEY=NL-A; CA-A Correction Lesson 2 891 gh File information made throu Date Class Segment ividing 17.2 Subd en Ratio Giv Name ions eg that partit ed line segme directly to the point you find given relationship? partitions ion: How to score that eg two given segme eg between d line segme on directe 0.12 Also G-CO. D Find a point .6 relationship. G-GPE. B given segment Resource Locker Quest Essential common CORE hardcover pages 891,902 a ment in System ing seg coordinates partition ensional line. For that. One-Dim number Discover Watch for hardcover student edition page numbers for this lesson. n Nate system accurate locatio mensional koordi ay highw r specify is called a one-diay turn, that part of one numbe is sometimes it takes just highw number line straight stretch reason, a . Coordinate system markers exit. plans mensional e is a mile istration into a single-di Arthur Avenu ay admin r Avenue exit 44. State highw ce from Arthu should r t highway, Straigh Road is a mile of stamps that are _23 is distan's new exit, where Collingwood Briar Street has point steps to determine Follow them to put the exit on OD Road. collingwo C). mark the tape of Mark Arthu Avenue. A (point A) gwood Road and Collin (point C B D 50 40 30 Authors: 20 Explain. 14│= 30. gwood Road? s: │44 e that Collin coordinates Arthur Avenu ence distance different What is the value of absolute ; find Explain. far = 20 miles 2 ; · 30 miles 20 miles 3 Why? A street exit? Briar r number mile stamps What is + 20 = 34 er 34; 14 er line. mile sign of dull (point B) Street exit Briar etiquette Plot and er lines. • Image g Compan IN2_MNLESE389847_U7M17L2 891 View part Participates In. Discuss the photo and note that the Golden Relationship is found throughout the wild–even, some say, the ratio of the length of a person's forearm to the length of a person by a person. Then, check the hour performance task. Harcour t n Mifflin © Houghto erstock 36/Shutt ©Condor _ Lesson 2 891 Module 17 7L2 891 47_U7M1 ESE3898 IN2_MNL 891 Lesson 17.2 18/04/14 10:08 PM 18/04/14 10:08 p.m. - The Highway Administration also plans to put the exit of Dakota Lane at the point that divides the highway on Arthur Avenue to Collingwood Road at 2-3. What's the Dakota Lane mile badge number? Why? (Tip: Let the distance from Arthur Avenue to Dakota Lane be 2x and let the distance from Dakota Lane to Collingwood Road to be 3x.) DISCOVER the separation segment of the OneDimension coordinate system mile marker 26; because the distance from Arthur Avenue to Collingwood Road is 30 miles, 2x + 3x = 30, 5x = 30, and x = 6. Thus, the distance from Arthur Avenue to Dakota Lane is 2x = 12 miles. integrate technology painter and mark the Dakota Lane exit (point D) on the number line. Shows how to use geometry software to partition a segment. Check the student number lines. Let's reflect the 1st How can you say that the place where you drew point B makes sense? 2 path (or about 67% of the way) from point A to point B seems to be about _ 3 polling strategies under point C.2. What does it mean to say that point P _ divides the AB ratio from 3 to 1? Point P divides the segment into two segments and one length is three times the length. Would your answer to Step F be different if the exit from the Dakota Avenue shared highway on Arthur Avenue to Collingwood Road ratio 3-2? Explain. - yes, I don't know. Then the way out would be closer to Collingwood Avenue. This would be 32.1 Explanation 1 Segment separation in a two-dimensional coordinate system EXPLAIN 1 The directed line segment is the segment between two points A and B, the direction of which is defined, from A to B or B to A. For partitioning a targeted line segment, it is divided into two segments that have a given relationship. Example 1 A (-8, -7), B(8) (5); 3 to 1 Step 1 Write a ratio that represents the distance from point P to B. 3 =_ 3 from distance A to B. Point P is _ 4 3 + 1 Step 2 Find the run and the increase of the directed line segment. 8 y B(8, 5) 4 x -8 -4 0 -4 A(-8, -7) -8 run = 8 - (-8) = 16 4 Segment two-dimensional coordinate system © Houghton Mifflin Harcourt The company Find point P coordinates that divide the directed line segment from A to B. INTEGRATE MATHEMATICAL PRACTICES into Focus on Patterns MP.8 Can students draw a segment and find a center to divide it into two segments with a ratio of 1 to 1. Then let them find the center of one segment to divide the segment by a ratio of 3 to 1. Ask students what other ratios they can divide segments by center. Rise Run (8, -7) Rise = 5 - (-7) = 12 Module 17 892 Lesson 2 PROFESSIONAL DEVELOPMENT IN2_MNLESE389847_U7M17L2 892 Learning Progression 18/04/14 10:08 Students have previously learned the inclination and learned to find the slope line. They also used a slope to write an equation line, either in tilt-intercept form (y = mx + b) or in point-tilt form (y – y 1 = m (x – x 1)). Here, students use the term tilt (relationship surge run) to help them find a point that divides a given line segment given the relationship. Segment division in a given ratio 892 from 3 point A to B, so find _ 3 both rise and run. Step 3 Point P is _ 4 4 3 run = _ 3 (16) = 12 3 elevation = _ 3 (12) = 9 _ 4 4 4 y Step 4 to find the coordinates of point P, Add 8 i step 3 values to point A. 4 P x x-coordinates point P = -8 + 12 = 4 -8 -4 0 4 y-coordinates point P = -7 + 9 = 2 Rise -4 QUESTIONING STRATEGIES How can you predict whether a point to split the line in a given relationship is closer to one or the other? If the ratio is less than half, the point is closer to the first endpoint and if it is greater than half, the point is closer to the second endpoint. FPO The coordinates of point P are (4, 2). Can I compare measurements with different units in proportion? Explain. - yes, I don't know. if the readers are both in one unit and the denominators in another unit, or if one fraction is in one unit and the other fraction is A(-8, -7) -8 Run (8, -7) A(-4, 4), B(2, 1); 1 to 2 B Step 1 Write a ratio that represents the distance from point P in segment A to B. 1 Point P is __ = _ distance from time B to B. 1 + 2 3 A P 4 Step 2 Graph-directed line segment. Find the increase of the directed line segment and run y 2 B. -4 run = 2 - (4) = 6 ascent = 1 - 4 = 3 -2 0 2 x 2 -2 -4 1 Step 3 Point P is _____ from point A to point B. 3 ( ) © Houghton Mifflin Harcourt Publishing 1 1 _ run = _(6) = 2 3 3 1 1 _ run = _-3 = -1 3 3 Step 4 koordin To find theates , point P = -4 + 2 = -2 (coordinates of point P are -2 , 3) y-coordinates to P = 4 + -1 = 3 , add 3 coordinates to the coordinates . Figure point P on the graph above. Mirror3. In Part A, show how you can use the distance formula to check this point divides the targeted line segment in the correct relation. 2 2 2 AP = (4 - ((-8)) + (2 - (-7)) = √225 = 15; PB = √(8-4) + (5-2) = √25 = 5. ——----√ - THE AP ratio to BP is 15-5 or 3-1, which is the correct ratio. 4. Discussion What can you infer from a point that divides the segment ratio from 1 to 1? How do you find the coordinates for a point like this? The dot is the center of the segment. Use the center formula. Module 17 893 Lesson 2 Collaborative Learning IN2_MNLESE389847_U7M17L2 893 Peer-to-Peer Activity Do students work in pairs to draw and mark a line, line segment, and a targeted line segment. Let them discuss their similarities and differences. 893 Lesson 17.2 18.218.2014 10:08 Your turn EXPLAIN 2 Find the coordinates of point P that divides the directed line segment from A to B in this ratio. 5. A(-6, 5), B(2, -3); 5-3 5 5 Point P is = distance from 5+3 to 8 between A and B. _ 6. Create segment section _ _ run = 2 - (-6) = 8; increase = -3 - 5 = -8 5 5 runs = 5; elevation = -5 8 8 x-coordinates at point p = -6 + 5 = −1; run = -6 - 4 = −10; elevation = −13 - 2 = −15 QUESTION STRATEGIES 5 5 x-coordinates at P = 4 - 6 = −2; The coordinates of point P are (−1,0). The coordinates of point P are (−2, -7). Do you partition the line (as opposed to the line segment)? Explain. No, no, no, because the line continues infinitely in both directions, it cannot be divided into segments with a specific ratio. _ _ _3 run = -6; _3 elevation = -9 y-coordinates at point p = 2 + (9) = -7 y-coordinates at P = 5 + (5) = 0 Explain 2 Example 2 A(4, 2), B(-6, -13); 3-2 3 3 = from a distance, point P 5 is 3+2 between A and B. Create segment section Considering the directed line segment between A and B, build point P that divides the segment from a given relationship from A to B. to B →. The exact measure of angle Step 1 Use a straight line to draw ac is not important, but the design has the easiest angles from about 30° to 60°. C A C E D A Step 3 Use a straight line to connect points B and F ∠. Create an angle congruent ∠AFB with E at its peak. Step 4 construction partitions AB into three equal parts. Point P of the label at point P, which divides the segment with a ratio of 2 to 1 between A and B. Module 17 894 B C F E D P B © Houghton Mifflin Harcourt Publishing Company → . Label step 2 Place the compass point on A and pull the arc through ac at intersection D. Using the same compass device, draw an arc in the middle of D and mark the intersection E. Using the same compass device, draw the arc in the middle of the E and label at the intersection F. Lesson 2 Guide to differentiate IN2_MNLESE389847_U7M17L2 894 Multiple performances 18/04/14 10:0 PM whether it matters whether a segment with endpoints C and D is called DC _ or CD. The order does not matter if C and D are endpoints. If the segment is directed to a line segment, the order is important. Segment division in a given ratio of 894 B AVOID COMMON ERRORS 1-3 C G Remind students to pay attention to the order of the letters of the problem so that they can find the right point in a given direction. F E D A B P 1 Step 1 Use a straight line to control the AC. Intersection d. Intersection d. Draw an arc centered on D and mark the intersection with E. Using the same compass setting, draw the arc centered on f, draw an arc centered on F, and mark the intersection G. Step 3 Use a straight line to connect points B and G. Constructing joomine, ∠AGB d-, E and F to the top. Step 4 construction partitions AB into four equal parts. Point P of the label at point P, which divides the segment with a ratio of 1 to 3 between A and B. Reflect 7. © Houghton Mifflin Harcourt Publishing House 8. _ _ Why is there a parallel to the FB in Part A of the EP? ∠AEP was built in such a way as to ∠AFB. These are in sync corresponding _ _ corners, so EP∥FB. How do I use the triangle proportionality theorem to explain why this construction method works? Construction ensures that the rai segments you built are in sync. The segments drawn up in step 3 are also all parallel. By Triangle Proportionality _ Teorem, you can conclude that segments along the AB are compatible with each other. Your turn – considering the line segment from A to B, integrated a section P that divides the segment into a given relationship A to B. 9. 1 to 2 10. 3 to 2 C F A E D E F G H C D A P B Module 17 895 Lesson 2 LANGUAGE SUPPORT IN2_MNLESE389847_U7M17L2 895 Connect Vocabulary Help students develop an understanding of the meaning of dimensions by considering one-dimensional numbers such as a line whose only dimension is length, and two-dimensional figures, for example, square. The two dimensions of the square are length and width. 895 lesson 17.2 18.04.14 22:08 to work to work 11. How does a one-dimensional coordinate system resemble a two-dimensional coordinate system? How's that different? For both types of coordinate systems, numbers are used to determine the locations of points. QUESTIONING STRATEGIES A single number is used in a one-dimensional coordinate system to determine the location Do proportions compare measurements with different units? Explain. Yes, but each ratio must compare measurements with the same unit; For example, you can write a proportion that has a ratio of legs to legs equal to the ratio between meters and meters. point on the line. In two-dimensional two digits shall be used to determine the position of the point on the aeroplane. 4 _ 12. Is finding a point that _ is from point A to B to B 5 the same as finding a time point that divides AB ratio 4 to 5? Explain. 4 No; a point _ from a distance from A to point B is 80 % from point 5 _ 4(A)(B); the point that divides AB with a ratio of 4 to 5 is _ or approximately 44% of the 9 distance points A to point B. TAKE STOCK OF LESSON 13. Important question Check-In What are some different ways to divide segment ratio 2-1? If the segment is on the number line, subtract the coordinates to find the distance between the direction row segment in a given ratio? Directed line segments are always read in the order in which points are presented. Segment section divides segment from point A to second point B: (b). At coordinate level, the rise and run can be used with a segment ratio a: b. Segment sections can also be built using a compass and a straight line. 2 result indicators. Then find a point that is _ distance from one end point to another. 3 If the segment is at the coordinate level, use the run and ascent to find a point that is in the range from a certain indicator to another. If there are no coordinates, use three compasses and straight line edges to divide a given segment into three equal parts. Then define the point that divides the segment by ratio 2 and 1. The choreographer uses a series of numbers to place the dancers. Dancers A and B have coordinates 5 and 23, respectively. Find 1-4 coordinates for each of the following dancers in exercises based on these locations. 1. Dancer C stands at a point that is _56 distance from dancer A dancer B. AB = │23 - 5│= 18 2. 3 6 Dancer C is 15 units dancer A, so the coordinate dancer C is 5 + 15 = 20. IN2_MNLESE389847_U7M17L2 896 Dancer D stands at a point that is _13 distance dancer A dancer B. AB = │23 - 5│= 18 _1 ± 18 = 6 _5 ± 18 = 15 Module 17 • Online Homework • Tips and Help • Extra Practice © Houghton Mifflin Harcourt Publishing Company • Photo authors: ©mcpix/iStockPhoto.com Rate: Homework and practice dancer D has 6 units from dancer A, so dancer D coordinates 5 + 6 = 11. 896 lesson 2 18/04/14 10:08 Dividing segment given ratio 896 3. EVALUATE Dancer E stands at the point that divides the line segment dancer from A to dancer B to dancer B with a ratio of 2 to 1. AB = │23 - 5│= 18 Let distance dancer A dancer E be 2x and let distance Dancer E dancer B be x. Then 2x + x = 18, 3x = 18 and x = 6. So the distance from dancer A dancer E is 2x = 2(6) = 12. The coordinates of Dancer E are 5 + 12 = 17. 4. Dancer F stands at the point that divides On the segment from dancer to dancer B with a ratio of 1 to 5. AB = │23 -5│= 18 Let distance dancer F be x and let distance Dancer F dancer B be 5x. Then x + 5x = 18, 6x = 18 and x = 3. So distance from dancer to dancer F is x =3. DETERMINATION GUIDE Concepts and skill practice Explore the separation segment of OneDimensional coordinate system Exercises 1-4 Example 1 separation segment two-dimensional coordinate system exercises 5-8 Example 2 Build partition Segment Exercises 9-12 Coordinate dancer F is 5 + 3 = 8. Find the coordinates of point P that divide the target line segment from a to B in the target line segment. 5. A(-3, -2), B(12, 3); 3 to 2 3 3 P is ____ =_ distance from A to B 5 3+2 A(−1, 5), B(7, -3); 7 to 16.7 7 P is ____ =_ from distance A to B. 7+1 8 run = 12 - (−3) = 15; elevation = 3 - (−2) = 5 The coordinates of point P are (6, 1). The coordinates of point P are (6, -2). 5 © The Constructions and Labels of Houghton Mifflin Harcourt Publishing Company when splitting segments. _7 run = _7(8) = 7; _7 increase = _7(-8) = -7 5 5 5 8 x-coordinates at P = -3 + 9 = 6; y-point coordinate P = −2 + 3 = 1 7. INTEGRATE MATHEMATICAL PRACTICES Focus on modeling MP.4 Discuss the importance of careful run = 7 - (−1) = 8; increase = -3 - 5 = -8 _3 runs = _3(15) = 9; _3 elevations = _3(5) = 3 A(−1,4), (B-9,0); 1 to 3 1 1 P is ____ =_ distance A to B. 4 1+3 8 run = -9 - (−1) = -8; increase = 0 - 4 = -4 _1 run = _1(-8) = -2; _1 increase = _1(-4) = -1 4 4 4 4 x-coordinates at P = -1 + (−2) = −3; The y-point coordinates E = 4 + (−1) = 3 The coordinates of point P are (−3, 3). 3 3 =_ distance A to B. P is ____ 7 3+4 run = -7 - 7 = -14; elevation = 4 - (−3) = 7 _3 runs = _3(-14) = -6; _3 increase = _3 (7) = 3 7 7 7 x-coordinates to p = 7 + (-6) = 1; y-point coordinates P = −3 + 3 = 0 The coordinates of point P are (1, 0). Taking into account the targeted line segment from A to B, point P was built, which divides the segment from time to time to Point B.9. 3 to 110. 2 to 3 C H C G G F F E D A D P B module 17 Exercise IN2_MNLESE389847_U7M17L2 897 lesson 17.2 8 A(7, −3), B(-7, 4); 3 to 4 8. Item P 897 8 x-coordinate = −1 + 7 = 6; y-point p coordinates = 5 + (-7) = −2 B P Lesson 2 897 Depth of knowledge (D.O.K.) COMMON CORE Mathematical practices 1-4 2 Skills/concepts MP.4 Modelling 5-8 2 Skills/concepts MP.2

Recital 9-12 2 Skills/concepts MP.5 Use of tools 13-16 2 Skills/concepts MP.4 Modelling 17-20 2 Skills / Concepts MP.4 Modeling 21 2 Skills /Concepts MP.2 Recital 22 3 Strategic Thinking MP.3 Logic 23 3 Strategic Thinking MP.3 Logic 18/04/14 10:08 CONSIDERING the targeted line segment A-B, creates point P, which divides the segment with a given ratio of A B. 11.1 to 4 12.4 to 1 D A F E G H C AVOID COMMON ERRORS Some students may confuse different ratios. Remind them that the relationship of similarity refers only to the relationship of the length of the sides. This is equal to the square root of the perimeter ratio and area ratio. A D E P F G H B B Find the coordinates of point P, which divides each directed line segment into a given ratio. J We -10 -20 13. J to M; 1 to 9 JM = |10 - (-15)| = 25 L M 0 N 10 20 14. K to L; 1 to 1 KL = |5 - (-6)| = 11 Let JP = x and let PM = 9x. Shoot KP = x and shoot PL = x. The coordinates of point P are −15 + 2.5 = 12.5 The coordinates of point P are -6 + 5.5 = -0.5 Then x + 9x = 25, 10x = 25 and x = 2.5.15. N to K; 3 to 5 Then x + x = 11, 2x = 11 and x = 5.5. 16. K to J; 7 to 11 KJ = |-15 - (-6)| = 9 NK = |-6 - 18| = 24 Shoot KP = 7x and let PJ = 11x. The coordinates of point P are 18-9 = 9. The coordinates of point P are -6 - 3,5 = -9,5. Then 7x + 11x = 9, 18x = 9, x = 0.5. So KP = 7(0.5) = 3.5. Then 3x + 5x = 24, 8x = 24 and x = 3. So NP = 3(3) = 9.17. To convey mathematical ideas, Leon built point P, which divides the targeted segment from B to A with a ratio of 1 to 2. Here are points P and Q related? Explain. 2 Points P and Q are the same point. Explanation of the sample: point P is 3 1 points from distance B to A. This means that the distance from A to B. point Q is _3 points located in the same place along the line segment. Module 17 Execution IN2_MNLESE389847_U7M17L2 898 Lesson 2 898 Depth of Knowledge (D.O.K.) © Houghton Mifflin Harcourt Publishing Company Let NP = 3x and let PK = 5x. COMMON CORE Mathematical practices 24 3 Strategic thinking MP.2 Recital 25 3 Strategic thinking MP.2 Recital 18/04/14 10:08 subning segment given ratio 898 18. City planners use a number line to place landmarks along a new street. Each unit in the number line represents 100 feet. Fountain F is located in the coordinates -3 and plaza P is located at the coordinates of the 21st century. City planners place two benches down the street at points that divide segments from F to P in ratios 1-2 and 3-1. What is the distance between the benches? FP = |21 - (-3)| = 24; Then x + 2x = 24, 3x = 24 and x = 8. Allow the distance from F to the first ping to be x and allow the first ping to P to 2x. The coordinate of the first ping is -3 + 8 = 5. Allow the distance from F to the second ping to be 3x and the second bench up to P to be x. Then 3x + x = 24, 4x = 24, x = 6. So the distance from F to the second stand is 3x = 3(6) = 18 units. The coordinates of the second ping are -3 + 18 = 15. The distance from the benches is 15−5 = 10 units or 1000 ft. 19. Marathon course includes a direct segment of the city main library. The planning committee wants to put water stations on this part of the course so that the stations are divided into three equal parts of the segment. Find the coordinates of the points where the water stations should be placed. 4 2 y) Main Library M x © Houghton Mifflin Harcourt Publishing house • Picture authors: ©sportgraphic/Fotolia -4 0 -2 2 4 -2 C Town Hall -4 from C to M, run = 3 - (-3) = 6; rise = 3 - (-2) = 5 1 Let the water stations be at points P and Q. Point P is _ from distance C to M. 3 _1 from the distance = _1(6) = 2; _1 increase = _1(5) = 1_2 3 3 3 3 3 3 2 1 x-coordinate p = -3 + 2 = −1; y-coordinates P = -2 + 1_ = -_1 3 3 ( ) 1 The coordinates of point P are -1, - _ 3 2 point Q is _ from distance C to M. 3 _2 from the race = _2(6) = 4; _2 elevation = _2(5) = 3_1 3 3 3 3 3 x-coordinate Q = -3 + 4 = 1; y-coordinates Q = -2 + 3_1 = 1_1 3 3 ( ) The coordinates of point Q are 1,1_. 3 1 1 Water stations must be placed -1, - _ and 1, 1_. 3 3 Module 17 IN2_MNLESE389847_U7M17L2 899 899 Hour 17.2 ( ) 899 ( ) Lesson 2 18/04/14 20:08 20. Multi-stage Carlos drives straight from Ashford to Lincoln, Ashford is 433. The rest stop is on interstate _23, which is from Ashford to Lincoln. Assuming Carlos drives 60 miles an hour all the time, how long does it take to drive from Ashford to the rest of the stop? The distance from Ashford to Lincoln |553 – 433| = 120 miles. AVOID COMMON MISTAKES Students may forget to use in the right direction to partition a series of segments. Remind students to check the direction to choose the right partition point. _2 • 120 = 80, so Carlos must travel 50 miles from Ashford for the rest of the stop. 3 1 d = rt where d is the distance, r is the speed, and t is the time, so that 80 = 60t and t = 1_. 3 1 This takes 1_ hour (1 hour and 20 minutes) to get from Ashford to the rest of the stop. 3 21. The illustration shows the targeted segment from J to K. 4 y J Points divide segment J to K in each of the following ratios. Which points have integer coordinates? Select all that applies from 2 x A. 1 to 1 -4 0 -2 2 4 B. 2 to 3 C. 1 to 3 D. 1 to 3 E. 1 to 1 K -4 Distance J to K, run = 0 - (−3) = 3; rise = -3 − (-6) = -1 1 1 1( ) 1 A. Point is ___ = _ distance from J to K. run = _3 = _1; 1 1+1 2 2 2 2 1(1 1 1 −6) = -3. x-point x-coordinate is -3 + 1_ = -1_; The y-coordinate 2 2 2 points are 3 + (-3) = 0. This point does not have integer coordinates. 2 2 2 (2) 2 B. Point is _ = _ from a distance of J to K. _ run = _3 = 2; 2+ 1 3 3 3 3 2 2 2 2 (2) 1 1 C. Point is ___ = _ distance c-d = _3 = 1_ ;2 5 5 5 2+3 2(2 1 4th = rise _(-6) = -2 _. x-point x-coordinate is -3 + 1_ = -1_; 5 5 5 5 (y 2 3 points are 3 + -2 _ = _. This point does not have integer coordinates. 5 5 1 1 1 1 (( )) 1_ = _1 __ D. Point is 1 + 3 = 4 from distance J to K. _ run = _3 = 3/4 ; 2 1 4 4 3 1(1 1 _ the x-coordinate point is -3 + _ = -2 _; 4 4 4 4 4 4 2 This point does not have integer coordinates. The point is 3 + -1_ = 1 4 4 ( ) 1 1 1 1 1 () E. _ run = _3 = 1, _1+2 3 3 3 3 1(elevation = _ -6) = -2. point x-coordinate is -3 + 1 = -2; 3 © Houghton Mifflin Harcourt Publishing Company 2(elevation = _ -6) y-coordinate = -4th point x-coordinate is -3 + 2 = -1; The y-coordinate of 3 points is 3 + (-4) = -1. This point has integer coordinates. point is 3 + (−2) = 1. This point has integer coordinates. Module 17 IN2_MNLESE389847_U7M17L2 900 900 Lesson 2 18/04/14 10:08 Dividing segment given ratio 900 H.O.T. Focus on higher order thinking journal 22. Critique Reasoning Jeffrey was given a targeted line segment and was told to use a compass and a straight line to build a point that divides the segment into a ratio of 4-2. He said he had to do a quick and then build 6 synchronized segments along the ray. Do you agree? If so, explain Tamara's shortcut. If not, explain why not. - yes, I don't know. ratio 4 to 2 equals a ratio of 2 to 1. To create a segment ratio of 2-1, you only need to build three synchronizing segments along the beam. Invite students to summarize the process of finding a point in the targeted line segment that divides the segment in a given ratio. 23. Explain that error item A has a coordinate -9 and point B has a coordinate 9. The student was asked to find the coordinate P between A and B 23. The student said the coordinates of point P are -3. A. Without any calculations, how can you say that the student made a mistake? Point P shall be closer to point B than point A, so that the coordinates of point P should be positive. B. What kind of mistake do you think the student made? distance from B to A. Sample answer: the student found the reference point , which is _ 3 24. Point P of the relationship between the analysis and the ratio of the segment is 2 to 5. Coordinates of point P. 3 can be found: Point P is ____ 5 3+2 3 Race from A to P 2 - (4) = 6. Let him run from A to B. Then 6 = _ x and x = 10.5 3 The increase from A to P is 1 - (2) = 3. Let's get up from A to B. Then 3 = _ and y = 5.5 © Houghton Mifflin Harcourt Publishing Company x-point B coordinates = -4 + 10 = 6; Coordinates of y point B = -2 + 5 = 3 point B(6), (3) ___ 25. Critical thinking RS passes through R(-3, 1) and S(4, 3). Find point P RS so that the ratio between RP and SP is 2 to 1. Is there more than one way? Explain. _ 5 If P is on RS, point P is ___ distance from I to S. 2+4 to 3, run 4 = (-3) = 7; increase = 3 - 1 = 2 _5 runs = _5(7) = 3_8, _5 increases = _5(2) = 1_1 9 9 9 9 9 8 _8; in point P = 1 + 1_1 = 2_1 x-coordinates for point p = -3 + 3_ = 9 9 9 8 ... In this case, the coordinates of point P are (_, 2 1). 99, it's beyond point S. Let P have the coordinates (x, y). There is also point P, not RS ¯x - (3) RP ___5 ____) Then rise to bioni = 4 both x - 4 = 4, 4 (x + 3) = 5x - 4 , 4x + 12 = 5x - 20, 12 = x - 20, rise SP and x = 32. 'y-1 RP ___5 ____5 Also rise = 4 , so y - 3 = 4 , 4(y - 1) = 5(y - 3), 4y - 4 = 5y - 15, -4 = y - 15, rise SP and y = 11. In this case, the coordinates of point P (32, 11). Module 17 IN2_MNLESE389847_U7M17L2 901 901 Lesson 17.2 901 Lesson 2 18/04/14 10:08 Result Task In this lesson you divide line segments into three ratios. The chart shows a row divided into two parts so that the shorter part divided by the longer part: a+b a=__ b Each such relationship is called a golden ratio. To find a point in the line segment that divides the segment in this way, study this figure. L S M R Q integrate mathematical practices Focus by focusing on modelling MP.4 Opening lesson performance task N provides that the split line segment a+ b longer, a is longer, b the split shorter part equals the total length divided by the longer part. a + b a a = __ So, __ a . Use the same relationship to complete this equation: _LM LN=?. __ ___ ML invite students to reflect on math connections mp.1 Lesson performance Task shows line LN Figure, LMQS is a square. ___ equals the golden ratio (the length of the entire segment divided by the longer part). _ 1. Describe how, starting with the LM line segment, you can find the location of N. LN LN 2. Let the LM length is 1, find ___ = ___ = LN, Golden Ratio. Describe your 1 LM method. _ 1. Use LM as one side, build the _ LMQS square. Constructor R, SQ center. Module 17 902 © Houghton Mifflin Harcourt Publishing Company 2.RM equals the hypotens of the right triangle with sides 1 and 0,5. By Pythagorean Theorem, RM =1,118. So, RP = 1.118. SR = 0.5, so SP = 1.118 + 0.5 = 1,618. LN = SP, so LN, Golden is approximately 1,618. divided into two parts a and b such a+b a+1 a = ______ a = _____ ______ et __ a . f b = 1, 1 = a . Solve equation b a. Compare your results with the results of the lesson results task and guess - 1 + √5 per value a. a= __ = 1.618; a seems 2 equal to Golden Ratio. Lesson 2 EXTENSION ACTIVITY IN2_MNLESE389847_U7M17L2 902 The first two digits of the Fibonacci sequence of natural numbers are 1 and 1. To find each new term in. So, the first few terms are 1, 1, 2, 3, 5 and 8. Invite students to use calculators to write a series of ratios of the first ten digits to the previous number, rounding to the nearest two thousandths. Here are four of the first 3 = 1.5; __ 5 = 1.333. When students finish their 2 = 2; __1 = 1; __ ratios: __ 1 1 2 3 calculations, let them compare their results with the ratio in the lesson performance task and then make assumptions about their calculated ratios. Ratios approach and approach the Golden Ratio, the farther the ratios continue. 18/04/14 22:08 Scores Rubric 2 Points: The student's response is accurate and complete performance of the task or tasks. 1 point: The student's response contains the characteristics of the appropriate response, but is invalid. 0 points: The student's response does not include the characteristics of the appropriate response. Segment Sharing In a Given Ratio 902 LESSON 17.3 Name Using Proportional Relationships Class Date 17.3 Using Proportional Relationships Important Question: How can you use similar triangles to solve problems? Common Core Math Standards Student is expected: COMMON CORE Explore G-SRT. B.5 Study of indirect measurement In this study, you will consider how to find heights, lengths or distances that are too large to measure them directly, i.e. measuring instruments such as rulers. Indirect measurement shall include the use of similar triangular properties to measure such heights or distances. Use the same and similarity criteria for triangles to solve problems and prove relationships in geometric indicators. Mathematical practices COMMON CORE Resource Locker MP.5 Using tools during the day sunlight creates shadows as shown in the figure below. The intermittent segment represents the ray of sunlight. What triangle is formed by the flagpole, its shadow and the ray of sunlight? Language objective Explain the difference between direct and indirect measurement of the partner. A better triangle of ENGAGE You can use similar triangles to measure things indirectly, using the fact that similar triangles have a side length that is proportional. PREVIEW: LESSON PERFORMANCE TASK View the Include online section. Discuss the photo and ask if students can cite evidence that suggests the Earth is round. Then preview the lesson Task. B © Houghton Mifflin Harcourt Publishing Company • Image Authors: ©naphtalina / iStockPhoto.com Important question: How can similar triangles be used to solve problems? Let's say the sun shines and you stand near the flagpole, but from the shadow of its. You're casting a shadow, too. You can assume that the sun's rays are parallel. What do you know about the two triangles formed? Explain your reasoning. The triangles are alike. The sun's rays are parallel, and the line depicting the ground is a cross. Both the acute angles formed by the bases and hypotensive correct triangles are in sync, as are the correct corners. So the triangles are similar to the AA similarity criterion. C What heights or lengths do you already know on the chart? You probably know your height. D What heights or lengths can be measured directly? Your highness (if necessary) and the length of your shadow and shadow flagpole. Module 17 must be EDIT--Chan El key=NL-A; CA-Correction Lesson 3 903 gh File Info Made throu Date Class al Osa17.3 Usingionships Relat Name IN2_MNLESE389847_U7M17L3.indd 903 problems? solve and prove r triangles problems you use simila es solve How can triangl Question: ity criteria important and similar congruence ment COMMON G-SRT. B.5 Use geometric numbers. CORE in Measure relationships into great indirect ces, which is involves exploring s, or distan rement heights, length of indirect measu explore how rulers. consider measure ance, e. then the heights or measurement in this Explordirectly, it is, les measure such a red triang to be measu below. similar properties in the figure using ws, as shown in Resource Locker HARDCOVER PAGES 903,912 Watch for hardcover student edition page numbers for this lesson. shado What kind of th Text created by the sun ray sunlig Turn on the shadow its triang rmed by flagpole? is shadow and that for creates he? Sunlight, day sunlig ray of sunlig Time represents s of segment le, this should shade triangle is ♀ You to your shadow. But will you flag the rod, El. With ng near the sun is parall is standi rays g, and you e that the sun is shining. may be a step in the oletame n your cause well. You d? Explai shadow as les forme casts two triang tea correct triang y • Image g Coman Authors: ☺ Publishin Harcour t n Mifflin .com © Houghto lina/iStockPhoto ©naphta sending a line repre led and is paral tenuses and hypo sun AA r. Rays by bases r by s formed les on simila les is simila acute angle triang l. So the corners. So the accountante is the right la sent in ounts, are as les is cong right triang criterion. The resemblance is already known? s s or length m, which height. In Diagra, you know that. You proba flag? shadow red direct and shadow s can measu Lesson 3 s or length of you're length What height aame) and and I (if neces Your heigh - 903 Module 17 ESE389847_7L3.indd 47_U7M1 IN2_MNL 903 Lesson 17.3 903 18/04/14 10:12 18/04/14 10:13 EXPLORE How could you indirectly use similar triangles to measure the height of a flagpole? Let someone measure the length of your shadow and the shadow of the flagpole. Write to detect indirect measurement and solve a part where the unknown is the height of the flagpole. Use the fact that you can use similar triangles to solve problems Use geometry software to calculate missing measurements. The dimension shown can be found using the measurements and similar triangular properties shown in the figure. A To find the height of a palm tree, you measure the shadow of a tree, and at the same time you measure the shadow poured by the meter with the stick you hold in X at right angles to the earth's surface. Find tree height h. ___ Since ZX ll CA, ∠Z ≅ ∠C. All the right angles are in sync, so ∠Y ≅ ∠B. So △XYZ ≅ △ABC. Set up a aspect ratio. Substitute. Multiply by multiplying each of the 7.2. Simplify. QUESTION STRATEGIES 1m Z 1.6 m Y C What assumptions are made, this type of indirect measurement, about the sun's rays and the time at which the shadows are measured? The sun's rays are parallel and shadow measurements are made at the same time. 6 7,2 m BC AB = __ XY YZ h = __ 1 _ 7.2 1,6 1 h = 7,2 _ 1,6 h = 4,5 ( ) The tree is 4,5 meters high. ◑ explain 1 h 72 inches. The triangles are similar to the AA similarity criterion. Set up a aspect ratio. 48 inches. () Multiply both sides by use. 128 h 72 _ 128 Simplify. X = 192 Finding unknown height © Houghton Mifflin Harcourt Publishing Company Sid is 72 inches long. To measure the flagpole, Sid stands near the flag. Sid's friend Miranda measures Sid's shadow and the shadow of the flagpole. Find flagpole height h. INTEGRATE MATHEMATICAL practices Focus on Math Connections MP.1 Do students compare the process using similar triangles in the process using synchronized triangles. The flagpole is 192 inches long. Module 17 904 Lesson 3 PROFESSIONAL DEVELOPMENT IN2_MNLESE389847_U7M17L3.indd 904 Learning Progression 18/04/14 10:13 Students have already used congruence to solve real-world problems. For example, to find the distance across the pond, the students showed that the two triangles were in sync and then used the CPCTC to find an unknown lateral length that corresponded to the distance over the pond. This lesson presents similar problems (problems that need to be fixed at an unknown length), but now students are using and the proportionality of the parties in order to find an unknown length. Use of proportional relationships 904 Reflect QUESTIONING STRATEGIES 2. Which row in the drawing is a cross that cuts two parallel lines? earth line for example, how can you check whether your answer makes sense? The shadow length of the meter stick is a little more than 1.5 times longer than the length of the meter stick. So the length of the tree shadow, should be a little more than 1.5 times the height of the tree. Since 1.5 (4.5) = 6.75, the answer to 4.5 is reasonable to EXPLAIN 2 Your turn Finding an unknown distance liam is 6 feet long. To find the height of a tree, he measures its shadow and the shadow of the tree. The dimensions of the two shadows are displayed. Find tree height h. INTEGRATE MATHEMATICAL PRACTICES in Focus communication with MP.3 Whether students name corresponding parts of the triangles are similar to the AA similarity criterion. tree height tree shadow = Liam height Liam's shadow 28 28 h h= 6 = 6 8 8 168 x = 21 h = 8 The tree is 28 ft long. 3. ___ before they begin to resolve any length. Explain 2 h Liam 6 ft 28 ft 8 ft 8 ft. Find an unknown distance in real world situations, you may not be able to measure an object directly because there is a physical barrier that separates you from the object. You can use similar triangles in such situations as well. QUESTIONING STRATEGIES Example 2 ◑ © Houghton Mifflin Harcourt Publishing Company How could you predict whether an unknown length is longer or shorter than the corresponding given length? If a triangle of unknown length looks smaller than a triangle of a given length, you can predict that the unknown length will be smaller. If the triangle of unknown length is larger than the triangle of the given length, you can predict that the unknown length will be greater. Tree Explain how to use the information in the illustration to find the distance down. The hiker wants to find the distance over the canyon. He's looking for points as described. 1. He identifies the landmark X. He places the marker (Y) directly over the canyon from X. 2. In Y, he turns 90° from X and walks 400 feet in a straight line. He puts a marker (Z) in this place. 3. It continues to walk another 200 feet, and places a marker (W) in this place. 4. He turns 90° away from the canyon and walks until marker Z aligns with the X._ He places the marker (V) in this place and measures the WV. ∠VWZ ≅ ∠XYZ and ∠VZW ≅ ∠XZY (Vertical Corners are in sync). So, △VWZ ≅ △XYZ by the AA similarity criterion. d =_400 or _ d = _XY = _YZ , So _2 _ 327 327 3 600 VW WZ 2 = 218. Then d = 327 _ 3 X W d Z 600 ft 400 ft Y 327 ft V () The distance over the canyon is 218 feet. Module 17 905 3. 905 Small group activity Do students work in a small group to assess the length of something in the school building that would be difficult to measure. Examples include the height of a staircase or flagpole. Then write an explanation of the structure, including the measurement of the charts. 905 Lesson 17.3 18.2014 22:13 B To find distance over the abyss, the student identifies the points as shown in the illustration. Find d. AVOID COMMON MISTAKES 42 m ∠JKL ≅ △NML by AA similarity criterion. K 24 m Remind students to pay attention to units in their answers and ensure that the length found for real-world problems makes sense. M L JK = _ KL _ NM ML 35 m d = _24 _ 35 24 N 24 d = 35 _ 7 42 PREPARE 140 d=_ 7 QUESTIONING STRATEGIES = 20 What is the direct and indirect measurement? If you can use a unit of measure, such as a ruler, to measure a dimension, the measurement is direct. If you use a measurement tool to measure the corresponding dimension and associate it with the desired dimension by using a proportion, the measurement is indirect. The distance across the ravine is 20 meters. Let's reflect the 4th Why is ∠EGV ≅ ∠IGH? ∠EGF ≅ ∠IGH are vertical angles and vertical angles are compatible. Your turn on the 5th. To find the distance d over the stream, Levi is located on the points, as shown in the illustration. Use this information to find d. Summary lesson △JKL ≅ △NML by AA similarity criterion. d BC _ AB _ 12 12 _ = = _; d = 12(_) = 24 DE EC 12 6 d = 24 meters 6m 12 m C E Work D Discussion Let's say you want to help a friend prepare to solve indirect measurement problems. What topics would you recommend your friend review? Possible answers: triangle similarity criteria, theorems around congruent angles (such as Vertical Angles Theorem), writing and settling proportions, and characteristics of similar triangles, including the connections between the respective angles and between © Houghton Mifflin Harcourt Publishing Company B 12 m 6. How can you use indirect measurement and similar triangles to solve problems? You can indicate that the two triangles are similar by using the AA or SAS similarity criterion. You can then use the fact that the corresponding sides are proportional to find an unknown side length. 6 the corresponding sides. 7. An important issue Check-In You is a given indicator, including triangles that represent the real world situation. What is the first step you should take to find unknown measurement? First, you need to make sure that the triangles are similar. Module 17 906 Lesson 3 DIFFERENTIATE GUIDANCE IN2_MNLESE389847_U7M17L3.indd 906 Modeling 18/04/14 10:13 If possible, encourage students to go outside to build and solve indirect measurement problems with shadows. Find a distance that can be difficult to measure directly and use similar triangles to find distance. Direct students to take a diagram of the problem, make measurements, write and solve measurements with the steps they need to find the distance. Using Proportional Links 906 Evaluate: Homework and InternshipS • Online Homework • Tips and Help • Additional Practice Finding Distances using Similar Triangles is an indirect measurement . 1. Use similar triangles such as ABC and △△XYZ to find the missing height h. 2. B 3. h h DETERMINATION GUIDE A Concepts and Skills Practice Exploring exploring indirect measurement exercise 1 Example 1 Finding unknown altitude exercises 2-5 Example 2 Finding unknown distance exercises 6-9 C 60 ft h 6 ft 60 AC _ AB _ h = h = (_) = 24 ft XZ 6 XY 15 Y X 15 ft Z 15 4. A C 156 ft XZ 156 16.5 5. X 14 feet Z 4 feet 16.5 14 C 208 ft X 15.2 feet Y 3.8 feet Z XY XZ 208 h = 52 feet 14 15.2 15.2 Use example problems △EFG and △IHG to find missing distance. INTEGRATE mathematical practices focus on modeling MP.4 Discuss how you could use astick or 6.7. Mon 8 May 2015 © Houghton Mifflin Harcourt Publishing Company 6 60 m F d H 48 m G F 78 m G 80 m I d 48 64.8 FG EF _ = ; = _; 60 IH HG 80 48 d = 80(_) = 64 60 d = 34 metres 9. d E F 45 m H 180 m H 140,4 m I FG h EF _ = _; 45 IH HG 180 78 EF = 180(_) = 312 45 d = 312 metres 388.8 m 211,2 m P F 27 m G d EF _ 27 FG _ = _; = _ = _; 18 IH HG 140,4 27 EF = 140,4(_) = 210,6 18 d = 210,6 metres f 64,8 m G d = 64,8 FG EF _ = _; = _; d = 211,2(_) = 35,2 = d = 35,2 metres ; IH HG 211.2 388.8 388.8 Module 17 Lesson 3 907 IN2_MNLESE389847_U7M17L3.indd 907 907 lesson 17.3 18/04/14 10:13 18/04/14 10:13 □) = 31 XY XZ 15,5 h 31 15 6 ft 5 5 h ft 5,5 ft 5,5 ft 5,5 AC _ AB _ h = _ = (_) 156 = 52 ft XY B Y ruler to perform indirect measurements of objects in the classroom that are difficult to measure directly, such as a whiteboard or door. B 907 Depth of Knowledge (D.O.K.) COMMON CORE Mathematical Practices 1-5 1 Recall Info MP.4 Modeling 6-9 2 Skills / Definitions MP.2 Recital 10-16 2 Skills/concepts MP.4 Modelling 17-19 2 Skills/concepts MP.2 Recital 20 3 Strategic Thinking MP.3 Logic 21 3 Strategic Thinking MP.3 Logic 18/04/14 10:13 PM 18. To find the height of the h dinosaur at the museum, Amir placed a mirror on the ground 40 feet from his base. Then he stepped back 4 feet so he could see the top of the dinosaur in the mirror. Amir's eyes were about 5 feet 6 inches above the ground. How high is the dinosaur? These two triangles are similar to the AA similarity criterion. All dimensions must be in the same unit, so convert 5 feet 6 inches by 5.5 feet. 40 ft h = ; h = 5,5 = 55; The dinosaur is 15 feet tall. 5.5 4 4 5 6 in. 4 feet (_) __ 11. Jenny's 5 feet tall. To find the height of the light pole h, he measured his shadow and the shadow of the pole. What is pole height? The 40 feet go?secO7805005aa AB Two triangles are similar to AA 5 ft d to AA similarity criterion. All dimensions must be in the same units, so write 5 feet 2 as 5___ 5 ft. Write 7 feet 9 inches 6 12 3 9 1 ___ 7 = 7 feet. To avoid mixing decimals and fractions, write 15.5 ft to 15__ feet. 12 _4 _1 3 7_ 2 56 31 31 31 __ 31 _ 62 h 2 ___ = (_ ___ ÷ ___ 20 _)= __ (31)(_4) = __ (4) = __ 1 15_ 2 4 2 6 4 2 3 3 3 2 ft on 2 ft 9 inches long. The light pole is 20 __ 7 feet 9 in. 3 3 D 12. The student wanted to find the height of the h statue of pineapple in Nambour, Australia. He measured the shadow of pineapple and his shadow. The student's height is 5 feet 4 inches. What is pineapple height? △ the = △AAAA AA similarity criterion. AC = 5 ft 4 in = 64 year. BC = 2 ft = 24 in. Pineapple height h is 280 inches or 23 feet 4 inches. B 2 1 2 ft C E 8 ft 9 feet 13. To find the height of the h flagpole, Casey measured his shadow and flagpole shadow. Considering that Casey's height is 5 feet 4 inches, what's the height of the flagpole? ge07se_c07105002aa Triangles are similar to the AA similarity criterion. AB Casey height: 5 feet 4 in. Casey shadow: 3 feet = 36 in flagpole: 14 feet 3 in. Casey shadow 64 36 x 5 feet 4 inches 3 feet flagpole height __ flagpole shadow = h 171 171 __ = ; = _; h = 64(__) = 304 in __ 64 36 36 © Houghton Mifflin Harcourt Publishing Company EF = 8 ft 9 in = 105 in 64 _ 105 105 AC _ BC _ 24 _ h _ = ; = ; h = 64(_) = 280 14 ft 3 in 36 Flagole height h is 304 inches or 25 feet 4 inches. Module 17 IN2_MNLESE389847_U7M17L3.indd 908 908 lesson 3 18/04/14 10:13 Using Proportional Relations 908 City Plans an Outdoor Concert independence day celebration. To store the speakers and lights, a team of technicians creates a two-platform scaffold by stage. The first platform is 8 feet 2 inches above the ground. The second platform is 7 feet 6 inches above the first platform. The shadow of the first platform extends from 6 feet to 3 inches above the ground. The second platform casts a shadow. AVOID COMMON MISTAKES Some students may not think that they may find a similarity in the relationship between the two digits with unknown lengths of sides. Remind them that the perimeters have the same similarity ratio to the corresponding sides, and the areas have the same ratio as the squares of the respective sides. E 7 Feet 6. 8 feet 2 in. Six feet three inches. B D 14. Explain why the △ABC resembles △ADE. (Hint: The rays of light are parallel.) Transmission of mathematical ideas Possible answer: Since the light rays are parallel ‹ › and AD is △ABC △ADE are the appropriate angles so that they are compatible. ∠A is common to both triangles. So △ABC ~ △ADE by AA Similarity Postulate. 15. Find the length of the shadow of another platform with feet and inches to the nearest inches. First, convert all lengths in inches: AC = 8 feet 2 in = 96 in CE = 7 feet 6 inches = 90 in © Houghton Mifflin Harcourt Publishing Company AB = 6 feet 3 inches = 75 inches. AE = AC + CE = 98 + 90 = 188 188 AD 188 AD AE = _ ; AD = 144 75 98 AB AC _ shadow of the second platform is represented in BD. _ _ _ _ BD = AD - AB = 144 - 75 = 69 So the shadow of the second platform is 69 inches or 5 feet 9 inches. 16. The technician is 5 feet 8 inches long. The technician's standing on the other platform. Find the length of the shadow that is cast from the scaffolding and technician nearest inch. Technician height: 5 feet 8 in. = 68 years. Scaffolding height with technician: 188 + 68 = 256 in 256 256 s = ; s = 75 = 196 75 98 98 Shadow cast scaffolding and technician is 196 inches or 16 feet 4 inches. _ _ Module 17 IN2_MNLESE389847_U7M17L3.indd 909 909 Hour 17.3 (_) ( ) 909 Lesson 3 18/04/14 10:13 17. To find the distance XY over the lake, you will find the points as shown in the illustration. Explain how to use this information to find XY. 300 feet △XYZ ~ △VUZ by SAS similarity criterion, XY XY XY 800 = . Then = , so XY = 1000 feet so 500 VU VZ 400 _ _ 300 ft U V 400 ft Z _ _ 500 feet 800 ft Y X 18. To find the height of the rock, you stand at the bottom and the cliff, walk 60 metres from the bottom and place a mirror on the ground. Then face the cliff and step back 5 feet to see the top of the cliff on the mirror. Assuming your eyes are 6 feet above the ground, explain how to use this information to find the height of the cliff. (The corners marked with the corners are a con kongaad due to the nature of the reflection of the mirror light.) J P M 6 ft Q 5 feet 60 feet K Mirror △JKM ~ △PQM by AA similarity criterion, both JK=72 and the height of the cliff is 72 feet. JK JK 60 MK _ = _. Then _ , PQ 5 6 MQ 19. To find the height of the tree, Adrian measures the tree shadow and then his shadow. What part could Adrian use to find the height of the tree? Select all that apply. D © Houghton Mifflin Harcourt Publishing BC AC = _ A. _ DF EF DF = _ EF B. _ AC BC B AB = C. _ DF EF DF = _ EF D. _ BC AC AC BC = _ E. _ EF DF Tree Adrian A 5.6 ft 8 4.2 ft C E 42.3 ft F DF - and AC are hypotenuses of the triangle, so are EF and BC. · AB and DF are not the respective sides. · DF and BC have no corresponding sides, as well as EF and AC. · BC and EF are the respective sides, as are AC and DF. A. AC and DF are the respective sides, as are BC and EF. B.C. D. E. Module 17 IN2_MNLESE389847_U7M17L3.indd 910 910 Lesson 3 10.13 Using Proportional Relationships 910 JOURNAL H.O.T. Focus on higher order thinking Do students account for their problem where unknown length must be found using similar triangles. Remind students to add solutions to their problems. 20. The critique of the recital Jesse and Kyle is hiking. Jesse's wearing a walking stick. They spot a long tree and use a walking stick as a vertical marker to create similar triangles and measure the tree indirectly. Later in the day, when they come to the rock formations. They measure the shadow of rock formation and want to use similar triangles again to measure its height indirectly. Kyle wants to use the length of the shadow they used earlier for the stick. Jesse said they should measure it again. Who do you think is right? Jesse's right. The length of the shadow depends not only on the height of the object, but also on the position of the sun in the sky. To create similar triangles, the shadows of two objects must be measured at the same time. 21. Error analysis Andy wants to find the distance d across the river. It located the points, as shown in the illustration, then use similar triangles to find that d = 220.5 feet. How do you know, without realizing he's going to be wrong? Tell me what you think he did wrong and correct his mistake. C A 300 ft E 200 ft D 147 ft B d Is the shortest side of the right△ABE, so the corresponding side of DC △DCE must be shorter than DE, i.e. DE ∠BE 200. triangles are similar, Andy must have used wrong ( ) d 200 200. The correct proportion is _ , so d = 147 ____ = 98. The distance over 147,300,300 © Houghton Mifflin Harcourt Publishing Company River is 90 feet. Module 17 IN2_MNLESE389847_U7M17L3.indd 911 911 Lesson 3 18/04/14 10:13 Lesson Performance Task AVOID COMMON MISTAKES About 240 B.C., Greek astronomer Eratosthenes lived in Alexandria, Egypt. He believed that the Earth was spherical and created an experiment to measure its circumference. At noon in the town of Syene, the sun was right above his head. Jammed vertically, the earth does not cast a shadow. At the same time in Alexandria, 790 miles from Syene, a vertical stick cast a shadow that veered 7.2° from the vertical. Error Percentage compares measurement error to correct measurement. Some students may confuse this with the relative error calculation, students should compare the 401-mile error with the correct circumference, not the wrong girth. 401 401, not ___ ___. The correct ratio is ___ 24 901 24500 shadow alexandria 7,2° 490 mi 7,2° Syene does not shadow 1. Look at the chart. Explain why Eratosthenes reasoned that the angle in the middle of the Earth, which hit the 490-mile arc, measured 7.2 degrees. INTEGRATE MATHEMATICAL PRACTICES Focus on Math Connection MP.1 Old Greeks use a number of different stages of the circumference of the Earth using eratosthenes figures. Explain how you got your answer. 3. Calculate the radius of eratosthenes figures. length units. One was a stadium (plural, stadia) which, with a modern rating, was 185.4 metres. How far was Alexandria from Syene, stadias? (1 ≈ 1,609 meters) about 4,252 stadia 4. Earth's recognized circumference today is 44,901 miles. Calculate the percentage of error in the calculation of eratosthenes. 1. line from the middle of the earth to Syene, and the line from the tip of Alexandria to the bottom of the shadow of Alexandria is parallel. By Alternate Interior Angles Teorem, the angle at the center of the Earth is consistent with the 7.2° angle of Alexandria. 2. In full rotation 360° in the middle of the earth, each 7,2° angle lasts 360 = 50 such 490-mile arcs throughout the circumference. A 490-mile arc. On ____ 7.2 Eratosthenes figures, circumference of the Earth measures 50 × 490 = 24, 500 mi(s) C = 2πr 24, 500 = 2(3.14)r 24, 500 _ ≈r © Houghton Mifflin Harcourt Publishing Company the earth has a radius of approximately 3901.4 km 401 4. 24 901 - 24 500 = 401 miles; __ 1.6% 24, 901 Module 17 912 3 IN2_MNLESE389847_U7M17L3.indd. 3. Select one of the planets in the solar system Mars 2(3.14) 3901,3 = r Of the mind, then make a 24,500 = 2(3.14)r 24, 500 = 2(3.14) 3901 r km 6.28 Lesson 3 18/04/14 10:13 Ratio 1° 3. 24.5 Use your knowledge of the radius of planets or radius of the planet. Calculate the planet's circumference (π 3.14 for the problem). ( ) Find the location around 490 miles. Imagine that the sun was directly over a stick in one of the point's planets. Currently the sun was directly over one of the sticks, at what angle vertical would the shadow of the second stick cast? Example: Example: Mars: diameter 4200 mi; circumference: 13 888 mi; 490 = ____ x; x =12.7° ___ 13888 360° 18/04/14 10:13 PM Scoring Rubric 2 points: The student's response is accurate and complete execution of the task or tasks. 1 point: The student's response contains the characteristics of the appropriate response, but is invalid. 0 points: The student's response does not contain the characteristics of the relevant response. Using Proportional Relationships 912 LESSON 17.4 Name Similarity Right Triangles Class Date 17.4 Similarity Right Triangles Important Question: How Do Height Hypotensive Right Triangles Help You Use Similar Right Triangles to Solve Problems? A source at Locker Common Core Math Standards To examine a student is expected to: COMMON CORE Class G-SRT. B.4 Identify similar triangles A Make two copies of the right triangle on paper and cut them out. B Select one of the triangles. Fold the paper together to find the height of the hypothesis. C Cut the second triangle along the fold. Number the angles as shown. Making triangles Appears. Prove theorem triangles. So is G-SRT. B.5 Mathematical practices COMMON CORE MP.8 Patterns Language objective Explain to the partner how to use the angle/angle criterion to show similarity in triangles. Important question: How does the right triangle's hypotensive height help you use similar correct triangles to solve problems? The height of the hypotensive makes two triangles that are similar to the original triangle. The triangles can then be used to find an unknown length by measurement by means of indirect measurement. PREVIEW: LESSON PERFORMANCE TASK © Houghton Mifflin Harcourt Publishing Company DEAL 2 1 3 View Deal Section Online. Discuss the photo by asking students if they can describe what's going on and where. Then, check the hour performance task. Module 17 must be EDIT—Khan El Key=NL-A; CA-Correction Lesson 4 913 gh File info made throu date Class ilarity 17.4 Sim corners Right Tri le can help you right triang ensure to height using the Hyp otenuse use ° the similar s can help you use similar G-SRT. Triangles. ms about Resource Locker Student Essential Discover IN2_MNLESE389847_U7M17L4.indd 913 ♀ s Theory of G-SRT. B.4 Prove COMMON CORE identification copies Make two y Compare ◑ choose one triangle on triangl Cut another piece of tes. Fold the triangle along to cut them on paper and paper to find HARDCOVER PAGES 913,924 out. See the hardcover student edition page numbers for this lesson. Introduced. E hypotly prone e. Mark the bottomed triangles publishin the right angle Similar Tria . as shown in Mifflin Harcour t 2 1 © Houghto 3 lesson 4 913 Module 17 7L4 913 47_U7M1 ESE3898 IN2_MNL 913 Lesson 17.4 18/04/14 10:18 PM 18/04/14 10:19 D Place triangle 2 on top of triangle 1. What do you notice about the corners? EXPLORE The corresponding corners are in sync. E Detection of similar triangles What applies to triangles 1 and 2? How do you know that? They are similar. They are in line with the AA similarity criterion. F INTEGRATE TECHNOLOGY Use geometry software to examine the figure of right angle and height of hypotenus. Repeat steps 1 and 2 for triangles 1 and 3. Does the same ratio apply to triangles 1 and 3? - yes, I don't know. the corresponding angles are congestate and the triangles are similar to the AA similarity criterion. THE POLLING STRATEGIES REFLECT THE RESULTS OF THE 1990S AND 1990S. How do hypotensive triangles 2 and 3 relate to triangle 1? The hypotensive of the smaller triangles is the feet of the original triangle. 2. What is the link between triangles 2 and 3? Explain. They're similar because the resemblance of the triangle is transitive. If you draw a length of height on the hypothesis, what figures are produced? Two triangles that resemble the original triangle and each other. 4. Let's say you draw ∠ABC so that ∠B is right angle and the length of the hypotensus intersects with the hypotenus AC in P. Match each triangle similar triangle. Explain your reasoning. B. △ABC ∠PAB B. △PBC C △CAB C. △BAP A ≅ △BPC What does it mean when you say that the two triangles are similar? Triangles are of the same shape; it has the appropriate angles in line and the respective sides are proportionate. © Houghton Mifflin Harcourt Publishing House 3. Which angle is formed from height to hypotheses? The height of hypotenus forms a right angle with hypotensiveness. A. Angles B and P are the corresponding right angles. Angle A corresponds to itself. Angle C corresponds to itself. ∠BPC ≅ ∠APB and ∠BCP corresponds ∠ABP. C. Angles B and P are the corresponding right angles. Angle A corresponds to itself. Module 17 914 Lesson 4 PROFESSIONAL DEVELOPMENT IN2_MNLESE389847_U7M17L4 914 Math background 18/04/14 10:19 In mathematics describes the word meaning the relationship between numbers. The arithmetic mean of two numbers is the sum of the two numbers divided by 2. It is different from the geometric mean, which, for positive numbers a and b, is the positive square root of their product. A and b geometric mean is √ab. In later math courses, students can also learn harmonic instruments. Similarity in right triangles 914 Explain 1 Explain _ bx_ ax GE 1. Number x is a and b geometric mean. The geometric mean of two positive numbers is the positive square root of their product. So the geometric mean of a and b is a positive number x such that x = √ab or x 2 = ab. - Finding geometric mean of a pair of numbers Example 1 ♀ Find the geometric mean of the numbers x. 4 and 25 4 x = _ x 25 x 4 = 25x · 25x · 25x · x 25 2 100x = _25x _ 25x 100 = x 2 Write aspect ratio. INTEGRATE MATHEMATICAL PRACTICES Focus on recital MP.2 Note that only a positive number can be multiplied by both parties' product denominators. Example 1 ♀ √ 100 Take the square root on both sides. segment length in the triangle. 10 = x Simplify. ◑ 9 and 20 QUESTIONING STRATEGIES 9 x _ _ x 20 Write the aspect ratio. How are the geometric mean between the two digits, which includes a and b? The geometric mean is the denominator of one fraction and the reader of the other. _ = √x 2 9 x _20x = 20x · 20 Multiply both sides by the product of the nominees. 180 x _ 20x = 20 Multiply. 180 = x 2 Take the square root from both sides. © Houghton Mifflin Harcourt Publishing Company Simplify. Mirror 5 - How can you show that if the positive numbers a and b are such that ___ a= b = x, then x = √ab ? Multiply both sides by the proportion of xb, then take the square root of both sides: () () (_x 2 2 xb _ = xb b ; ab = x ; √ab = x. Your turn Find the geometric mean of each pair of numbers. If necessary, give the answer in the simplest radical form. 6. 6 and 24 16 24 √-6 · 24 = √-144 = 12 Module 17 7. 5 and 12 = √60 = √4 · 15 = 2√15 Lesson 4 COLLABORATIVE LEARNING IN2_MNLESE389847_U7M17L4 915 Small group activity Ask the students to draw several types of similar triangles and tell them to assign the values of the two sides. Ask students to use proportions to find the missing pages. 915 Lesson 17.4 18/04/14 22:19 Explain 2 By Proving the Geometric Tools of Theorems EXPLAIN 2 In Exploring The Action, you discovered the theorem about the correct triangles and similarity. The height of the hypotensus of the right triangle consists of two triangles similar to each other and the original triangle. By proving the geometric means of theorem, the height of the hypotensus suggests that theorem led to two additional theorems about the right triangles. Both theorems involve geometric means. Geometric means Theorem Example The height of the hypotensus of the right triangle is the length of the hypotensus segments. h = xy or h = √xy The length of the hypotensus of the right triangle is the geometric mean of the hypotensus segment √ √ of the hypothesis and its footing. _ Simple: Right triangle ABC height BD A CD = _ BD Prove: _ BD AD B A Example 2 Statements c h geometric means of the hypotenuse segments a length of a right triangle consists of two triangles similar to the original AB triangle and each other. Then show that __ = __ because the corresponding sides are similar to AB AC How could you express the geometric mean length of the hypotensive and length is equal to the height and the other segment length. The ratio between one segment and height is equal to the height for the other segment. © Houghton Mifflin Harcourt Publishing Company AD What is the relationship between height hypotenuses and segments of hypotenuses? This is the geometrical average of segment lengths. C 2. ∠CBD ≅ ∠BD angle and hypotenuse, as well as the height of hypotenuse. D 1. The first Simple CD _ BD _ = y b _ 1. ∠ABC is a right angle. Angle C and 3. INTEGRATE MATHEMATICAL PRACTICES focus on communication with MP.3 Make sure students can identify the correct DC BC triangles are proportional. Then prove that _ = _ in the same way. BC AC Module 17 916 Lesson 4 Differentiate Instruction IN2_MNLESE389847_U7M17L4 916 Multiple Modeling 18/04/14 10:19 Ann Whether Students Cut from 15-20-25-unit triangle out of the paper on the schedule paper. Ask them to draw height to the hypotenus, forming a 12-16-20-unit right triangle and a 9-12-15-unit right triangle. Then ask students using side lengths in three triangles to confirm the similarity of the legal triangles 916 Using Similar Triangles Explain 3 Explain 3 You can use geometric means theorems to find unknown segment lengths in the correct triangle. Example 3 Use of geometric mean theorems Find the value shown. 2 10 QUESTIONING STRATEGIES ♀ x If you know the length of segments that height divides the hypotensive into, how can you find the length of the height? You can write the proportion length of the segments and length of the hypotensive and resolve without knowing. x 2 = _x 10 x 2 = 10x _ 10x = 2 10x = 20 √ x= 10x 2 20 Write a proportion. Write the geometric average. Write the proportion x - 10 = x = 20 Square root both sides. Simplify. - √ 2√5 = √20 = x2 5 Take the square root of both sides. Simplify. ◑ AVOID COMMON ERRORS y 6 12 Some students may find it puzzling that a leg in one triangle can become a hypotenus-related triangle. Use colored pencils to distinguish between different triangles. Keep them with darker line. y 2 = 12y _ 12y _ y 12 Multiply both parties by the product of the denominators. 12y 2 120y _= y 12 Simplify. 120 = y 2 Simplify © Houghton Mifflin Harcourt Publishing Company z x - √ 120 = √ y Take the square root of both sides. - ≈y 2√30 Simplify. 2 Reflect the 9th Discussion How can you check your answers? Possible answer: Use Pythagorean Theorem. Your turn on the 10th. Find x. y 5 x z _5 = _ x; 7x(_5) = 7x(_x); 35 = x ; √- 35 = x 16 cm x 7 x 7 2 7 Module 17 917 Lesson 4 LANGUAGE SUPPORT IN2_MNLESE389847_U7M17L4 917 Combine vocabulary Distinguish between arithmetic mean, where you add numbers and divide by number of values. The geometric mean is the basket of n-numbers. This means that we multiply the numbers, then take the nth root where n is the number of values multiplied. 917 Lesson 17.4 18.18.2014 10:19 Explain 4: There is a lot, a lot of evidence of the Pythagorean theorem. You're proving it now by using similar true ideas. Proving the Pythagorean theorem using a resemblance to the Pythagorean theorem in the correct triangle, the square amount of length of the legs is equal to the square of length of the hypotenuse. Fill in the evidence pythagorean theorem. - Simple: Right Height CD A Example 4 c b Prove: 2 + b = c 2 2 Draw hypotensive. The intersection x. A shall be marked. ∠B ≅ ∠B kongruence reflexive property . B A C Part 1 ∠BXC ≅ ∠BCA because all right angles are in sync to integrate mathematical practices of Focus math Connections MP.1 Pythagorean Theorem was known in many ancient civilizations, including Egypt, India and China. The oldest known aksiomamatic evidence of theorem, however, dates euclid's Elements (about 300 bce). Since then, hundreds of evidence has been provided, including evidence that depends on the surface area, evidence that uses calculus, and evidence that is given in this lesson, based on similar triangles. x c b B a C So △BXC ~ △BCA AA similarity criterion . ∠AXC ≅ ∠ACB, because all the right angles are in sync. ∠A ≅ ∠A by Reflexive property Congruence. So, △AXC ~ △ACB by AA similarity criterion . Part 2 Be the length of the hypotensive segments (d) and (e) as shown in the illustration. Use the fact that the corresponding sides of similar triangles are proportional to writing two proportions. e a = _ Share 1: △BXC ≅ △BCA so _ a c d x c e a B d b = ____ Share 2: △AXC ≅ △ACB, so _ c b Module 17 IN2_MNLESE389847_U7M17L4 918 918 © Houghton Mifflin Harcourt Publishing Company Lesson 4 18/04/14 10:19 Similarity right triangles 918 Part 3 QUESTIONING STRATEGIES Now execute some algebra to complete the certificate as follows. Which triangles does Pythagorean theorem apply to? Why is it useful? Only right triangles; this allows you to pair the length of the right triangles. Multiply the proportion1 of both sides by ac. Write the equation you received. a2 = ce b2 = cd 2 2 Adding two resulting equations give it: a + b = ce + cd 2 2 Factor on the right side of the equation: a + b = c (e + d) Finally, use the fact that e + d = c. 2 2 adding Postulate write equation + b = c. 2 2 2 To reflect the 11. Error analysis The student used the example 2. Criticism of student evidence. ∠BXC - ∠BCA, ∠BCA ~ ∠CXA, so ∠BXC ~ ∠CXA transit similarity. Since the respective sides of the like triangles are proportional, _e = _ and f 2 = ed. Since △BXC and △CXA f d are letter triangles, a2 = e2 + f 2 and b2 = f 2 + d 2. Add equations. a2 + b2 = e2 + 2f 2 + d 2 Substitute. = e2 + 2ed + d 2 Factor. = (e + d) Segment Adding Postulate = c2 2 The certificate is incorrect because the student assume that the result is verified. The student assumes that Pythagorean Theorem is true to write that © Houghton Mifflin Harcourt Publishing Company 2 = e 2 + f 2 and b 2 = f 2 + d 2. Module 17 919 Hour 17.4 Lesson 4 18/04/14 10:19 AT TO WORK OUT 12. How would you explain to a friend how to find the geometric mean of two digits? Possible answer: Write the part and multiply both sides by the product of the nominees, then simplify and take the square root of both sides; Multiply the numbers of two and take the square root of their product. So what is the relationship between the original triangle and the two smaller triangles formed by the height of the hypotensive? All three triangles are similar to each other. ∆ 13. △XYZ is a _isosceles triangle and the right corner is ∠Y. Assume that the height of the hypotenuse XZ intersects in XZ section P. Describe the relationships between the triangles △XYZ, △YPZ, and △XPY. All three triangles are similar. Additionally△YPZ ≅ △XPY. How can you identify hypotenus in the correct triangle? It's against the right angle, and it's the longest. 14. Can two different pairs of numbers have the same geometric mean? If so, an example. If not, explain why not. - yes, I don't know. possible answer: geometric mean 4 and 16 are 8 and geometric mean 2 and 32 are also 8.15. Important question Check-In How is the correct triangle hypotensive segments it generates? Height length is the geometric mean of the length of the sections of the lesson How are the geometric means associated with the Pythagorean theorem? Geometric tools can be used to prove pythagorean theorem. hypotheses. © Houghton Mifflin Harcourt Publishing Company Module 17 IN2_MNLESE389847_U7M17L4 920 920 Lesson 4 18/18/14 10:19 Similarity Right Triangles 920 Evaluate: Homework and Practice To Assess • Online Homework • Tips and Help • Practice Extra Write a

Similarity Statement Compared to Triangles in Each Chart. 1. 2. P B 3. C X W R DETERMINATION GUIDE E Q S △PQR , △SPR , △SQP D Y △BDE , △DEC ± △BEC Z △XYZ , △XWY , △YWZ Concepts and Skills Explore The Practice to Identify Similar Correct Triangles Exercises 1-3 Find Geometric Mean X For Every Pair of Numbers. Write your answer in the simplest radical form. Example 1 Finding geometric mean for pairs of numbers Exercises 4-9. 4. Example 2 Proving geometric theorems 13-18 Example 3 Use of geometric mean exercise 10-12 Example 4 Evidence of Pythagorean theorem by similarity Exercise 22 5 and 20 x ;x _5 = _ x 6, 2 x 7. - _ _ _ 1.5 and 84 1.5 x __ 84 2 9. - 2 = 36 ; x = 6 3,5 and 20 3,5 __ = x ;x 20 2 27 2 and __ 40 3 2 3 √5 x 3 9 2 x = 27 ; x = 20 ; x = 10 40 __ 126; x = 3 √14 - = 70; x = √70 _% , Find x, y and z. 10. 11. 25 65 x y x y z --- 30 y = √652 - 252 = √3600 = 60 2 2 2mine 40 30 _x _ √ x x x = 40 ; x = 1200; X = 20 3 y 70 _ √9 = 30 ; y = 2100; y = 10 21 70 _z _ = 7 ; z = 2800; z = 20√ 25 25 _60 _y _ y = x ; 60 = x ; 25x = 3600 ; x = 144 25 + x _z z z 169 _ _ = x; _z _ = 144 ; z = 24,366; -- Z = √24 336 = 156 IN2_MNLESE389847_U7M17L4 921 lesson 17,4 12 X Module 17 921 3 and 12 x _3 = _ 100; x = 10 8 and 13 x © Houghton Mifflin Harcourt Publishing Company with height drawn hypotenus. Have them identify three similar triangles. Invite them to write three different proportions that include a geometric average. 20 8 x 2 √ x = 13 ; x = 2 26 8. INTEGRATE MATHEMATICAL PRACTICES Focus on Modeling MP.4 Can students draw and mark the right triangle 8 x 20 26 x √ 8 x 13 2 x 7 2 √ 26 ... Module 17 ... Practices Reasoning 10-12 2 Skills/concepts MP.2 Reasoning 10-12 2 Skills/concepts MP.1 Problem solving 19-22 2 Skills/concepts MP.5 Using Tools 23 3 Strategic Thinking MP.1 Problem Solving 24 3 Strategic Thinking MP.3 Logic 18/04/14 10:19 -- 12. Some students may have difficulty visualizing the corresponding corners of overlapping triangles. Refer to them to re-draw the chart to separate the three triangles. 12,8 9,6 _ = _; 12.8z = 92.16; z = 7,2 y 9,6 AVOID COMMON ERRORS 2 2 2 y2 = (12,8) + (9,6) = 256; y = √256 = 16 12,8 z 9,6 z 12,8 + 7,2 _ x _ =;x x 7,2 = 144; x = 12 2 Use the chart to fill each equation. (c) (a) d e b e 13. _ec = _d c _ a 14. _ a = c + d c + d b 15. _ _ b d 17.c(c + d) = a d = _e 16. - c e 2 2 18. = ab © Houghton Mifflin Harcourt Publishing Company Find height under these conditions. B D A 19. BC = 5 AC = 4 _ AB = 3; AD = 4 ; 5 3 AD = 2.4 Module 17 IN2_MNLESE389847_U7M17L4 922 C 20. BC = 17 AC = 15 __ 15 ; AB = 8; AD = 17 8 120 = 7,06 AD = 17 922 21. BC = 13 AC = 12 __ 12 AB = 5; AD = ; 5 13 60 = 4.62 AD = 13 Lesson 4 18/04/14 10:19 Similarity right triangles 922 22. Transmission of mathematical ideas The length l and width of the rectangle area are the geometric mean of the side length. Indicates that the side length of the rectangle is the geometric mean of the length and width of the rectangle. - Square area = s2 = Rectangular area = lw; s2 = lw; s = √l w JOURNAL Invite students to show examples and reasons that the geometric mean of two positive numbers is always less than their arithmetic mean. H.O.T. Focus on higher-level thinking 23. The height of the algebra's 8-inch right triangle divides the hypotenuse into two segments. One segment is 4 times as long as the other. How long are the segments of hypotenuse? x in. = shorter segment; 4x in . The length of the hypotenus is 8 x geometric mean, with a length of 8 x, so = . Then 4x2 = 64, x2 = 16 and x = 4.4x8 Segments hypotensive measure 4 inches and 16 inches. _ 24. An error analysis of Cecile and Amelia both found the value of EF △DEF. Both students work. Which student's solution is right? What's the other student mistake? 12 EF = _ Cecile: _ 8 EF E So EF 2 = 12(8) = 96. _ 8 = ... © Houghton Mifflin Harcourt Publishing Company EF Then = √96 = 4√6 _ F _ Then EF = √32 = 4√2 Cecile's solution is right. Amelia wrote the wrong proportion. IN2_MNLESE389847_U7M17L4 923 Lesson 17.4 D 5V EF 2 = 8(4) = 32. Module 17 923 G 4 EF 923 Amelia: __ 4 EF 923 Lesson 4 18/04/14 10:19 PM Lesson Performance Task Integrate Mathematical Practices Focus on Patterns Example at the beginning of the lesson. The $100 investment grew at a one-year rate of 50%, to $150, then fell one year at a rate of 50%, to $75% The arithmetic average of +50% and -50%, which is 0%, was not a good predictor of change because it predicted the investment would still be worth $100 after two years, not $75. • Scroll to a geometric mean of 1, 1, 2, 4 and 4.2 1. Find a geometric mean of 1 + 50% and 1 - 50%. (Each 1 represents the fact that, at the beginning of each year, the investment is worth 100% of itself.) Round to the nearest thousand. • Write numbers a, b, c, d, 5 -- 5 - a) · (b) · c · (d) · (e) or 3abs and e. 3 2. It's a geometric mean, not an arithmetic mean, that tells you what the interest rate should have been for the entire investment period to achieve the end result. To check this requirement, you can use your answer for exercise 1. Find a value of $100 for an investment after it increased or decreased at the rate found in Exercise 1 for two years. Show me your work. AVOID common errors in the fourth issue of lesson performance task, students may calculate the geometric mean and then assume that it represents the interest rate they are looking for. Remind students that the geometric mean of 1 represents a 0% change, and they need to subtract the average they found out of 1 to find the average percentage change per year. 3. Copy the right triangle shown here. Write the terms Year 1 Rate, Year 2 Rate and Average Rate to show geometrically how the three interest rates are linked. Average rate. Year 1 Rate Year 2 Rate 4. The geometric mean of N numbers is the nth root of the number product. Find out what the interest rate should have been over 4 years to achieve the result of a $100 investment that increased by 20% in Year 1 and 30% in Year 2, then lost 20% in year 3 and 30% in Year 4. Show me your work. Round the answer to the nearest tenth. _--- = √√ 0.75 --- √(1 + 0.5)(1 - 0.5) = √(1.5) (5) © Houghton Mifflin Harcourt Publishing Company 1. = 0.866 2. After 1 year: 100 × 0.866 = 86.6 After 2 years: 86.6 × 0.866 = 74.9956 or $75 3. Look at the chart. 4 4 .-- 4. √(1.2) (1.3) (0.8)(0.7) = √0.8736 = 0.967 Since 1 stands for 0% change, 0.967 represents (1 - 0.967) = 0.033 or 3.3% a year average loss. Module 17 IN2_MNLESE389847_U7M17L4 924 Lesson 4 extension activity IN2_MNLESE389847_U7M17L4 924 Pythagorean Theorem is just one of many contributions the Greek mathematician and philosopher Pythagoras made in mathematics. He was the first to identify three ways to weigh numbers: arithmetic mean (or average), geometric mean, and harmonic mean. Then they should show how to find all three Pythagorean means numbers 4 and 9. arithmetic: 6.5; geometric: 6; 72 ÷ 5.54 harmonious: __ 13 18/04/14 10:19 Scores Rubric 2 points: The student's answer is accurate and complete performance of the task or tasks. 1 point: The student's response contains the characteristics of the appropriate response, but is invalid. 0 points: The student's response does not include the characteristics of the relevant response. Similarity to the Right Triangles 924 MODULE 17 MODULE STUDY GUIDE REVIEW 17 Using Similar Triangles Study Guide Review Important Question: How Can You Use Similar Triangles to Solve Real-World Problems? ASSESSMENT AND INTERVENTION KEY EXAMPLE Key Vocabulary indirecta(medición indirecta) geometric mean (media geométrica) (lesson 17.1) Find missing length x. ZU = _ ZV _ UX VY 16 x = ___ 10 9 16 (10) x = ____ 9 ( ) Determination or adaptation of module overviews. x = 17.8 IMPORTANT EXAMPLE MODULE PERFORMANCE TASK Mathematical practices: MP.1, MP.2, MP.4, MP.6 G-SRT. B.5, G-MG. A.1 • The best way to work with GPS coordinates: Students may want to express coordinates as (latitude, longitude) so that they can use the distance formula to find the length of the sides of the triangle (see Module 1 Performance Task). It gives the following coordinates: Miami: (-80.226529, 25.789106); San Juan: (-66.1057427, 18.4663188); Hamilton: (-64.781380, 32.294887) • How to find the length of the sides of the area: Students can test how to find this area, or you can propose. 925 Module 17 © Houghton Mifflin Harcourt Publishing Company 10 U x Z 16 Multiply by 10 on each side. Y Simplify. 9 V (lesson 17.2) Use a straight line to combine points B and H. SUPPORTING STUDENTS' REASONING • GPS coordinates the peaks of the Bermuda Triangle: Students can study them. Miami GPS coordinates are (25.789106, -80.226529), For San Juan (18.4663188, -66.1057427) and hamilton (32.294887, -64.781380). X Substitute. Taking into account the targeted line segment A to B, build point P, which divides the segment with a ratio of 2 to 3 between A and B → . Use a straight line to draw AC → . Position the compass at point A and pull the arc through the AC marking at junction D. Continue at junctions D to H. COMMON CORE Focus on what information they need. Here are some questions they might highlight: Write the aspect ratio. C H H G F E D A B P B Build the arabs ∠AHB where points D to G. _ AB are divided into five equal parts. Label point P at point P, which divides the segment by a ratio from 2 to 3 a to B.KEY EXAMPLE (lesson 17.3) A 5.8-foot-tall man stands next to a basketball ring that casts a 11.2-foot shadow. The man's shadow is 2.5 feet long. How long is a basketball ring? May x be the height of the basketball ring. x = _5.8 _ Write aspect ratio. 11,2 6.5 5,8 (11,2) x = ____ Multiply both sides by 11,2. 6.5 x ≈ 10 ( ) Module 17 925 Study guide Dating guide Scaffolding with support IN2_MNLESE389847_U7M17MC 925 • For this rural area, assume that each latitude or longitude unit is 100 km. • One way to find an area is to encircle the triangle shown in the rectangle, then to enclose the rectangle area and subtract the areas of the triangle surrounding the right triangles. • Students can also use a, b a+b+c and c side length a, b a+b+c and c triangles, provided by A = √s · a) · b) (s - c) where s = _ 2 -- 18/04/14 10:23 EXERCISES Find missing lengths. (Lesson 17.1) 4.5 A 1. _ BG = SAMPLE SOLUTION B 3,5 E 8 F G 5D 15 7 Find an unknown length. (Lesson 17.3) A 5.9-foot-tall-man stands near a 12-foot statue. The man places a mirror on the ground at a certain distance from the bottom of the statue and then stands another 7 feet from the mirror to see the top of the statue. How far is the mirror from the bottom of the statue? 14.2 feet 4. C 3. The first time in a G F E D A Find lengths. (Lesson 17.4) 9 X 6. x = 16 Z 2. ─ The 45-foot flagpole casts a 2.7-foot shadow. How tall is this woman? 5.5 meters 5. B P Locate the GPS coordinates of the three locations and write them in the form (y, x) = (latitude, longitude). Then use the distance formula → d = 3(x 2 - x 1)2 + (y 2 - y 1) 2 to find the length of the triangle, a, b and c lengths. To find the distances, multiply the results by 100 km. Finally, use a triangle to determine the region, a+b+c A = √ s - a · b) (c) where s = ____ 2. C _1 _ CE = 10 2 2. Based on the target line segment A to B, build point P that divides the segment with a ratio of 3 to 1 between A and B. (lesson 17.2) Methodology: 7 coordinates: Miami: (-80.226529, 25.789106); San Juan: (-66.1057427, 18.4663188); Hamilton: (-64.781380, 32.294887) 20 y z y = 13.4 8. The distance by formula and multiplying the results by 100 km gives the following distances: z = 24.1 Miami San Juan: 1,590 km San Juan Hamilton: 1,499 km Hamilton Miami: 1,676 km MODULE PERFORMANCE TASK LIMITS The triangle is not well defined, but the region is often represented in a triangle with peaks in Miami, Florida; San Juan, Puerto Rico; and Hamilton, Bermuda. The distance between Miami and San Juan is about 1,034 miles. What is the approximate area of this area? One tool that can find useful in solving this problem is a similar triangle shown here in the angle dimensions marked. Use your paper to complete the task. Be sure to save all your data and assumptions. Then use charts, charts, words, or numbers to explain how you came to your conclusion. H 61° M 50° 69° S © Houghton Mifflin Harcourt Publishing Company How big is the Bermuda Triangle? Calculate s, s - a, s - b, and s - c. s = 1590 + 1389 + 1676 ___ = 2327,5 2 s - a = 2327,5 - 1590 = 737.5 s - b = 2327,5 - 1389 = 938.5 s - c = 23 27.5 - 1676 = 651.5 Find the area: -- √2327.5(737.5)(938.5)(651.5) A= module 17 926 ≈ 1, 024, 472 km 2 Study Guide Review DISCUSSION OPPORTUNITIES IN2_MNLESE389847_U7M17MC 926 √s (s a) (s) b)(s - c) 18/04/14 10:23 • If students decide to add height to the triangle figure they are intended to provide, and then use a familiar triangle area formula, how will it affect their accuracy in their calculations? • How does the accuracy of the calculations affect the fact that the Bermuda Triangle is not really an aircraft triangle, but a sphere triangle? Is the actual area greater or lower than the calculated area? Evaluation rubric 2 points: The student correctly solves the problem and explains his reasoning. 1 point: The student shows a good understanding of the problem, but is not fully solved or explained. 0 points: The student does not show an understanding of the problem. Study Guide Review 926 Ready to move on? You ready to move on? 17.1-17.4 Use similar triangles MASTERY Use the assessment on this page to determine whether students need the strategic or intensive intervention. Find the angle dimensions each triangle. 17.2 Use your paper to complete the task. • Online Homework • Tips and Help • Extra Practice Find Missing Lengths. (Lesson 17.1) 1. 5 ASSESSMENT AND INTERVENTION A 3 2.7 D E 15 y x 20 12 G x = 4,2 z = 16 ; y = 9 In view of the directed line segment from A to B, create point P that divides the segment from A to B in relation to a given relationship (lesson 17.2) C 3. 3-4 G Access ready to move forward and receive instant scoring, feedback and tailored intervention or enrichment. D A • Reading Strategies • Success of English Learners • Challenge Worksheets Assessment Resource B P 6.25 feet © Houghton Mifflin Harcourt Publishing Company Dif Instruction Resources F Height street light 20 meters. It casts a 12-foot shadow. At the same time, a man standing next to a streetlight casts a 3-foot shadow. How tall is this man? ADDITIONAL RESOURCES • Reteach Journals I Find Missing (Lesson 17.3) 4. Response to intervention resources E H IMPORTANT QUESTION How can you use similar triangles to find missing parts of the triangle? You can use triangular proportions or corresponding parts to find missing parts of another similar triangle. 5 = Equals module quizzes module 17 COMMON CORE IN2_MNLESE389847_U7M17MC 927 927 module 17 Tutorial Overview 927 Common Basic Standards 18/04/14 10:23 Content Mathematical Standards Lesson Points 17.1, 17.3 1 G-SRT. B.4, G-SRT. B.5, G-GO. C.10 MP.7 17.1, 17.3, 17.4 2 G-SRT. B.4, G-SRT. B.5, G-GO. C.10 MP.7 17.2, 17.3 G-CO. D.12, G-GPE. B.6 MP.5 17.1, 17.3, 17.4 4 G-SRT. B.4, G-SRT. B.5, G-CO. C.10 MP.4 MODULE 17 MIXED REVIEW MIXED REVIEW Assessment Preparedness selected response 1. △XYZ shall be obtained by points x(-1, -1), Y(3, 5) and Z(5). Consider all the below. Is each point the peak of the image under the transformation (x, y) → (x + 3, y - 2) → Select Yes or No A to C. (ASSESSMENT AND INTERVENTION) 1 _ x, y → (y, -x)? 2 A. X'" (-3, -1) B. Y'" (3, -3) C. Z' (-1, -4) Yes Yes No No 2. Which of the following statements is the correct triangle on the right? Select True or False for each sentence. A. X is 15. The real false B. Y value is 12. The real false C. Y value is 16. True False Set ready or customized practice tests to prepare students for high-stakes tests. 28 30 Y 7 ADDITIONAL INSTRUMENTS X 3. △ABC is given in accordance with points A(-1, 2), B(2, 5) and C(4, -1). What are (2 2) assessment tools 1 _ point _ 5 ? Explain what that means. • Leveled module quizzes: modified, B possible answer: orthocenter; If the height is drawn from the three tops of the triangle, point 1 _ 5 . intersecting is _ 2 2 ( ) COMMON CORE to avoid common errors Section 1 Some students stop working on this problem so early, do not comply with all three changes. Encourage students to review the problem to make sure they are fully ready before making their choice. © Houghton Mifflin Harcourt Publishing House 4. Taking into account the targeted segment from A to B, build I C H point P, which divides the segment with a ratio of 1 G F to 5 between A and B. Explain your process and how this ED is associated with similar triangles. → A B . Use the compass to draw 6 arcs Draw AC P → and label (distance) through ac for each intersection D to I. Connect points B and I. The design angles are divided into 6 ∠AIB at the remaining intersections. AB equal parts. Mark point P, which divides the segment with a ratio of 1 to 5 between A and B. Each created triangle is similar to △AIB. Module 17 17 Study Guide Review 928 Common Basic Standards IN2_MNLESE389847_U7M17MC 928 18/04/14 10:23 Content Mathematical Standards Lesson points IM1 17.1 1* G-SRT. B.5, MP.3 17.4 2 G-SRT. C.8 MP.6 13 G-CO. D.12, G-CO. C.10, G-GPE. B.5 MP.6 17.2 4 G-CO. D.12, G-GPE. B.6 MP.5 17.3 G-CO. D.12 MP.5 * Point combines mixed review concepts with previous modules or previous course. Study Guide Review 928 MODULE 18 18 MODULE Trigonometry correct triangles Important question: How can you use trigonometric ratios to solve correct triangles to solve real-world problems? Answer: Trigonometric relationships allow you to use a given length of the side and the size of the angle to find the length of a given angle from a right triangle or vice versa. This can be useful if the triangular shape appears in the real world, such as a metal frame leather or sculpture. LESSON 18.1 Tangent Relationship LESSON 18.2 Sine and Cosine Relations LESSON 18.3 Special Right Triangles LESSON 18.4 Solving Problems with Trigonometry This version is Algebra 1 and for professional development geometry only VIDEO LESSON 18.5 Pythagorean Identity Author Juli Dixon models successful teaching practices in the actual high school classroom. Professional Development my.hrw.com © Houghton Mifflin Harcourt Publishing Company • Image Authors: Gartzenter/Shutterstock Professional Development Video REAL WORLD VIDEO Find out how proper triangle trigonometry is used in real world warehouses to reduce the space needed for items to be delivered or stored. MODULE PERFORMANCE TASK PREVIEW How much shorter are timed pipe stacks? In this module, you will find out how much space can be saved by stacking the pipes with a timed pattern, not directly on top of each other. How does trigonometry help you find the answer to this problem? Get ready to discover incredible results! Module 18 DIGITAL TEACHER EDITION IN2_MNLESE389847_U7M18MO 929 Access to a full set of educational materials where and when you need them: • Access content online or offline • Customize lessons to share with your class • Communicate with your students in real time • View student scores and data instantly to guide your instruction, where it is required for most 929 module 18 929 PERSONAL PERSONAL TRAINER EVALUATION AND INTERVENTION Set automatically scored homework, quizzes, tests and interventions. Prepare your students with updated, common basic practice tests. 18/04/14 11:08 PM YOU READY? Are you ready? Complete these exercises to view the skills you need in this module. EVALUATE READINESS ANGLE Relationships Example 1 Locate the angle attached to the given angle. 75° + 75° = 90° Use the assessment on this page to determine whether students need the strategic or intensive intervention for module prerequisites. • Online Homework • Tips and Help • Extra Practice Write as an Equation. x = 90° − 75° Resolve x. x = 15° Find an additional angle. ASSESSMENT AND INTERVENTION 1. 20° 2. 35° 55° 3.67° 23° Search angle. 4. 80° 5.65° 6.34° 100° 115° 3 2 146° 1 Find the remaining angle or angles △ABC. m∠A = 50°, m∠B = 40° m∠C = 90° 8.m∠A = 60°, m∠C = 20° m∠B = 100° 9.m∠B = 70° m∠A = 60°, m∠C = 50° ∠A ≅ ∠C Personal Trainer automatically creates a standard, personalized intervention assignment for its students, targeting individual needs for each student! m∠A = 55°, m∠C = 55° m∠A = 60°, m∠B = 60°, m∠C= 60° 10. △A ≅ ∠B ≅ ∠C 1 m∠C 11.m∠B = 30° and m∠A = _ 2 m∠A = 50°, m∠C = 100° 12. △ABC is similar to △DEF and m∠D = 70° and m∠F = 50° m∠A = 70°, m∠B = 60°, m∠C = 50° 13 △ABC is similar to △PQR and m∠R = 50° and m∠P = ∠Q 14.m∠A = 45° and m∠B = m∠C 15.m∠B = 105° and m∠A = 2 · m∠C 16.m∠A = 5° and m∠B = 9 · m∠C module 18 IN2_MNLESE389847_U7M18MO 930 1∠. m∠B = 65°, m∠C = 50° 45° 67,5°, m∠C = 67,5° m∠A = 50°, m∠C = 25° © Houghton Mifflin Harcourt Publishing Company 7. TIER 1, TIER 2, TIER 3 SKILLS ADDITIONAL RESOURCES See the table below for a complete list of available intervention resources for this module. Response to intervention resources also includes: • Second-level proficiency tests for each skill m∠B = 157.5°, m∠C = 17.5° 930 Response to intervention Tier 1 Lesson Intervention Worksheets Tier 2 Intensive Intervention Worksheets available online 38 Angle Relations 43 Pythagorean Theorem 46 Proportional Relationships Block Building Skills 7, 15, 16, 38, 46, 53, 56, 63, 66, 82, 90, 95, 98, 100, 102 Differentiated Order 18/04/14 11:08 Challenge Worksheets Expand Math Lesson Activities TE Module 18 930 LESSON 18.1 Name Tangenti class date 18.1 Tangential ratio Important question: How do you find a tangential relationship in an acute angle? Common Core Math Standards Student is expected: COMMON CORE Resource Locker G-SRT. C.6 Find out that the lateral relationships of the right triangles are characteristics of the triangle corners, leading to definitions of the trigonometric ratios of fierce corners. So is G-SRT. C.8 Mathematical practices Locate common core examining the relationship of the right triangle corner △ABC, with a right angle at the top of C, has three sides. Side adjacent ∠A is the leg that forms one side of ∠A. The side opposite ∠A is a leg that does not form a side ∠A. The side that connects the adjacent and opposite legs is hypotenus. B Hypotensive ⚲ MP.4 Modeling Language Purpose Explain to the partner how to find the angle tangent, which is a rectangle of angle and leg length. ⚲ measurements may vary replied 2.5 cm to 40° 3.1 cm next to F © Houghton Mifflin Harcourt Publishing Company Engage section online. Discuss the photo by asking students to describe the forces that could create the shape of the sand dunes. Then, check the hour performance task. C E-Question: How to find a sharp angle in the tangy relationship? PREVIEW: TO DO THE LESSON THE LEG, which ∠ AIN △DEF, tag your feet ∠D opposite and next. Then measure the length of your feet in centimeters and save their values in the prescribed rectangles. ENGAGE In the right triangle, the ratio of tangy to the length of the opposite leg and the length of the adjacent leg is in the centre-right triangle. The leg opposite ∠A ◑ D What is the ratio of the opposite leg length to the length of the adjacent leg, rounded to the nearest hundredth? EF ≈ 0,81 Ratios may vary slightly depending on measurements. _ DF - Using protractor and ruler, draw right triangle △JKL at right angle vertex L and ∠J = 40° so that △JKL ~ △DEF. Mark the opposite and adjacent feet ∠J and their measurements. The drawings are different, but should be similar to the A triangle in step A. For all triangles, it is true that the opposite length < adjacent length. What is the ratio of the opposite length of the leg to the length of the adjacent leg, rounded up to the nearest century? KL ≈ _ 0.81 JL Module 18 must be EDIT--Chan NO Key = NL-A; CA-A Correction Lesson 1 931 gh File information made in throu Date Class but Ratio 18.1 Tang Name Angle? acute eg relationship tange corners ties you will find ion: How es are correct G-SRT. C.8 in the right triangl . Similarly, the adimanthic corners are similar to tons and similar in the case of trigono G-SRT. C.6 Unders ngle concepts e, which leads to the right tria triangl relationship are three sides. Side g atin no C, there Investig leg, that angle at the ∠A is nuse. Discover the right ∠A. On the side of the opposing legs are the hypotele, △ABC right triang, which forms one side adjacent to you and the opposing leg cts ∠A is the side that conne adjacent to the B side ∠A. Does not form resource locker quest essential COMMON CORE IN2_MNLESE389847_U7M18L1 931 eg ⚲ Watch for hardcover student edition page numbers for this lesson. The leg ∠A Hypotenuse Leg adjace to ∠A HARDCOVER PAGES 931,940 C s is re length Then measu eg ∠D. submitted by the Commission. ite and adjace rectangles opposing your values, tick in △DEF and register ly. centimeters differ slightly from feet may E Measurem y g Compan reverse 2.5 cm to © Houghto n Mifflin Harcour t Publishin ◑ 40° D F 3.1 cm adjacent , e.g. leg length adjace leg length opposite urementments. Is Edth? meas What t huskies depending on the nears rounded different small L Ratios may peak ef ≈ 0.81 rectangular _ and △JKL adjacent legs ∠J DF right triang and ite draw opposing ruler, ctor and . Label using protra to △JKL ~ △DEF A. All 40° so le step ts. and ∠J = triang meters similar to add your h. should be lengt but but varies h < adjac Drawings site lengt to oppo , it is true triangles, eg leg length adjace leg length opposite ratio What is t hundredth? neares rounded € KL ≈ 0.81 _ JL Module 18 8L1 931 47_U7M1 ESE3898 IN2_MNL 931 Lesson 18.1 Lesson 1 931 18/04/14 11:19 PM 18/04/14 11:20 Reflective 1. Explore Discussion Compare your work with that of other students. Do all triangles have the same angles? Do they all have the same side lengths? Do they all have the same leg relationships? Summarize your findings. The triangles are all similar, so they have angles that are the same as exploring the relationship of the right triangle measure, but not the same side length. However, at this angle, the ratio between the length of the opposite leg and the length of the adjacent leg is constant. 2. INTEGRATE TECHNOLOGY If you repeat steps A-D with the correct triangle with a 30° angle, how would your results be similar? How would they be different? Triangles would all be similar, with the ratio of length to leg students have the opportunity to do explore activities either in the book or online. The 30° angle is opposite the length of the leg adjacent to the 30° angle difference, but the actual value of the ratio is different from 0,81. Connect VOCABULARY Find angle tangent Explain 1 Draw and mark different right triangles with different directions. In each of the triangles, there are students named hypotensive, leg next to a given angle and leg opposite the angle. In the Examine a calculated relationship, the angle tanger is called. The sharp angle A tanger, written in tan ∠A, is defined as follows: leg length opposite ∠A tan A = ___ leg length next to ∠A You can use what you know about the similarity to show why the angle tangent is constant. By AA Similarity Postulate, ∠ D ≅ ∠J and also ∠ F ≅ ∠L, then △DEF ~ △JKL. This means that the △ JKL sides are the same, k, △DEF the lengths of the respective sides. Substitution by the tangent equation indicates that the ratio between the length of the opposite leg and the length of the adjacent leg is always the same for this acute angle. E 40° F tangy, defined at a specified angle △DEF D J Locate each specified angle tanger. Each ratio is written as a C-fraction and decimal place, rounded to the nearest century. ∠A 18 ∠B B foot length opposite∠ A 18 = _ 3 = 0,75 tan A = ___ = 4 24 feet length adjacent to ∠ A 4 24 ft length ∠ B tan B = ___ = _ _ ≈ _ 1,33 3 ft length B 18 Module 18 932 24 A © Houghton Mifflin Harcourt Publishing Company ◑ ⎧ ⚲ ⎪ Example 1 40° ⎪ k•DF ⎨ L The ratio between the length of the opposite leg and the length of the adjacent leg of the right triangle is constant for each right triangle of the same angle. What does that mean for the right triangles with this angle size? Explain. Triangles must be similar because the ang angle dimensions are constant and the ratios of the sides are proportional. △ JKL foot in the opposite ∠ 40° k· EF = _ KL = _ EF EF = _ tan 40° = = _ _ JL k· DF DF DF leg next to ∠ 40° k ±DE k•EF ⎪ ⎪ ⎩ K QUESTIONING STRATEGIES TO AVOID COMMON ERRORS When students use calculators to calculate tangent ratio angle, remind them to check that their calculators are in degree mode. Otherwise, they won't find the right value. Lesson 1 PROFESSIONAL DEVELOPMENT IN2_MNLESE389847_U7M18L1 932 Learning Progression Students work with three trigonometric ratios in this course: tangy, sine, and cosine. The tangiator is treated separately for two reasons. First, working in one relationship gives students the opportunity to focus on the conceptual foundations of trigonometry. It is, the correct triangles given the acute angle, ∠A, are all similar, so the ratio of length to the leg opposite ∠A the length of the leg next to ∠A is constant in all such triangles. Secondly, it is sometimes difficult for students to decide which trigonometric relationship to use to solve the problem. By working with tangens first, students can instead focus on the overall process of solving the problem using trigonometry. 18/04/14 14 11:20 EXPLANATION 1 Finding angle tangent CONNECT VOCABULARY Abbreviation tan is read tangent. For example, tan 30° is considered a tangy 30 degrees. Number of tangent 932 Reflects QUESTIONING STRATEGIES 3. If you know the tangant relationship between the acute angle of the full triangle, and the length of one of the legs, can you reconstruct the triangle? Explain. - yes, I don't know. you can use the ratio to find the length of the second leg, and then use the Pythagorean theorem to find the length of hypotenuse. What is the ratio between tan A and tan B? Do you believe that this relationship is true in the acute corners of other right triangles? Explain. The ratios are mutual. This is always true, because on the opposite side of one fierce corner there is the side of another fierce angle and vice versa. 4. Why does it not make sense to ask for the value of tan L? The ratio of tangyers is defined only for the sharp corners of the right triangle and not for the right corner. You can use a table of values to find the tangent of a given acute angle. How do you think the tables were compiled? collecting the correct triangles by the given angle measure and its addition, measuring the counterparty and adjacent side lengths; Calculate the tangent ratio Your turn Find the tangent of each specified angle. Each ratio is written as a fraction and decimal, rounded to the nearest century. 5. ∠6. 15 5 tan ∠ Q = _ = ≈ 0,42 36 12 39 S 15 R 36 12 tan ∠ R = _ = = 2,4 5 15 If you know the length of the right triangle leg and the size of one acute angle, you can use a tanger to find the length of the other leg. This is especially useful in the event of real problems. Apply the ratio of tangers to find unknown lengths. Example 2 Finding lateral length with tangent To comply with safety instructions, the roof contractor shall determine that he or she must place the base of his ladder within 6 metres of the house by making an angle of 76° to the ground. How deep to the 10 feet of the ground is the roof's tuave? C © Houghton Mifflin Harcourt Publishing Company questioning strategies What if you want to find a tangy another acute angle triangle? How do you find this corner? When is it necessary? The angle gauge is an upgrade of this angle. You could do that if the missing side is the denominator of the tangy relationship. ∠R 36 Find side length using the tangy explain 2 Explain 2 Why it is important to draw and mark a diagram when you solve relationships in the right triangle in the real world? The chart allows you to identify values and relationships that you can use to solve the problem. Q A 76° 6 ft B Step 1 Write a tangant ratio that involves unknown length. leg length ∠A BC tan A = ___ = _ BA leg length located in ∠A 2. Simple: BA = 6 feet and m∠A = 76° BC Substitute: tan 76° = _ 6 Module 18 933 Lesson 1 CO-ED LEARNING IN2_MNLESE389847_U7M18L1 933 Peer-to-Peer Activity Give a pair of students a graph paper and ask each student to draw two different right triangles with one half-length legs. Let them use a protractor to assess one of the fierce corners in one triangle, then have the pair work together to find tangents. Then let them choose a sharp angle in the rest of the triangle and find an inverse tangent angle. For additional work, invite students to exchange triangles with other couples. 933 Lesson 18.1 18.18.2014 11:20 Step 3 Solve for unknown leg length. Make sure the calculator is in degree mode and do not circle before the last stage of the solution. Multiply both sides by six. BC 6 ·_ 6 · tan 76° = _ 1 6 Use a calculator to find the tan 76°. 6 · tan 76° = BC Replace this value with tan 76°. 6(4,010780934) = bC multiply. Round to the nearest tenth. 24.1 ≈ BC AVOID COMMON ERRORS x, If you solve the equation form tan 35° = ___ 12 students can forget to multiply 12 solve x after they find the tan 35°. These students may benefit from rewrite the equation 12- tan 35° = x before the tan 35° assessment. So, eat the roof is about 7.1 feet above the ground. B For the right triangle △STU, what is the ∠ the length of the foot adjacent to the ∠s? S 54° Step 1 Write a tangent ratio that involves unknown length. EXPLAIN 3 TU leg length ∠ S tan S = ___ = si a adjacent leg length ∠ S SU T 87 U Anglet using tangent 2 Identify the given values and replace the tangent equation. Simple: TU = 87 and m∠S = 54 ° 87 Substitute: tan 54 ° = _ SU Step 3 Solve unknown leg length. QUESTIONING STRATEGIES EQUATION y = tan -1x, explain what x and y represent. In the equation, the x angle tanger represents the size y°. 87 Multiply both sides by SU, then multiply both sides by 54°. SU = __ tan 54° 87 Use a calculator to find and replace 54°. SU ≈ __ Jaga. Round to the nearest tenth. SU ≈ 63.2 © Houghton Mifflin Harcourt Publishing Company Your Turn 7. The ladder must reach the second floor window, which is 10 feet above the ground, and make an angle of 70° to the ground. How far from the building must the ladder base be placed? 10 10 10 _____ __ 3,6 ft; tan 70° = = x , so x = tan 70° ≈ 2.747477419 ≈ 3.6 Explain 3 Find the angle measure using Tangent In the previous section you used a given angle size and leg size tanger ratio to solve an unknown leg. What if you are given foot measures and want to find measures in acute angles? If you know tan A, read tangy ∠A, then you can use tan -1 A, read the inverse tangent ∠A, find m∠A. Therefore, given the acute ∠A, if tan A = x, then tan -1 x = m∠A. Module 18 If tan x = 1, what is the angle x measure? Explain. The angle x must be 45°, because the legs of the triangle must be of the same length to change the tangent ratio 1. 1.37638192 934 How could you rate the corner rotary touch without using the calculator pivot key or table of values? Use the tangent ratio measurements to draw the corresponding triangle on the right. Then use a protractor to measure the angle to form the hypotensive and adjacent side to assess the angle of the measurements. Lesson 1 Differentiated Instruction IN2_MNLESE389847_U7M18L1 934 Knesthetic Experience 18/04/14 11:20 TO HELP STUDENTS IDENTIFY OPPOSITE and adjacent legs, model the right triangle floor tape. Either the student will stand at one acute angle and walk to the leg next to it, or touching that angle. It's a side-foot. Then there is the student to walk at a fierce angle opposite the leg, or from the whole angle. It's the opposite leg. Tangant Ratio 934 Example 3 Integrate Mathematical Practices Focus on Critical Thinking MP.3 Discuss Why Students Can accurate 19 in. ⚲ What is m∠A? A() 1 results when assessing an expression such as tan -1 __ 3 instead of finding an approximate decimal value of 1 per rounding, and an estimate of tan -1(0.33). fraction __ 3, work with ◑ LESSON How to find a tangy relationship with a fierce angle of right triangle? The ratio of leg length to the length of the leg next to the corner is. 935 Lesson 18.1 Step 2 Write the reverse equation. Step 3 Evaluate using the calculator and round as shown. 19 tan A = _ 36 19 tan -1 _ = m∠ A 36 m∠ A ≈ 27,82409638 ≈ 28° Step 2 Write inverse equation. 36 tan -1 tan B = _____ 36 _____ = m∠ B 19 19 Step 3 Evaluate by calculator and circle as shown. ° m∠ B ≈ 62.17590362 ≈ 62 Your turn to avoid common mistakes 8. J Find m∠J. 46 26°, m∠J = tan -1 __ ≈ 26,31808814 93 K 93 46 © Houghton Mifflin Harcourt Publishing Company How is the tanger and the turnr bound? The tangi about the acute angle of the right triangle is the ratio of the lengths of the adjacent side of the right triangle. Reverse contact is the angle with the tangant ratio. C Step 1, ∠ the toucher ratio by using known values. Step 1, use the known values to write ∠ B tangy ratio. Invite students to schedule y = tan x on their graph calculators at intervals of [0.90°]. Discuss how to interpret the graph, especially x nears 90°. POLLING STRATEGIES IN 36 YEARS. What is m∠B? INTEGRATE TECHNOLOGY Some students may have the misconception that the opposite leg is always on the vertical side of the triangle and the adjacent leg is always on the horizontal side of the triangle. Remind students that the opposite and adjacent sides are determined by the angle angle involved, not by the direction of the triangle. B Find the angle measure shown. Round to the nearest degree. I'm 9. Explain how to identify the opposite and adjacent legs of this acute angle. The opposite foot does not form on a given angle. The adjacent leg forms the side of a given angle and is not hypotenus. 10. Discussion How does tan A change m∠A increases? Explain the basis of the established relationship. Tan A increases when m∠A increases. As m∠A increases, the length of the leg next to ∠A remains the same and the leg length opposite ∠A increases. Since the length of the opposite leg is the numerator of the tangent ratio and the denominator of that relationship does not change, the ratio increases. Module 18 935 Lesson 1 LANGUAGE SUPPORT IN2_MNLESE389847_U7M18L1 935 Connect Vocabulary 18/04/14 11:20 Are students looking for a word inverse dictionary. The definition may contain a reverse position. Discuss how it corresponds to the relationship between the reverse toucher and the angle tangant. An important issue check-in Compare and contrast use tangant and inverse tangent ratio to solve problems. The tangy and the inverse applies both only to sharp corners in the full triangles. Once the length of the leg is known, the tangy can be used to resolve the constant relationship. While reversed substance may be evaluated • Online Homework • Tips and Help • Additional practice used to address the associated acute angle. If the dimensions of one leg and one angle are known, the tangtor may be used to resolve an unknown leg, while the inverse is not applicable. DETERMINATION GUIDE Evaluate: Homework and practice In each pair of right triangles, each pair measures the length of the adjacent side and the opposite side of the angle defined in the drawing. Then calculate and compare the ratio. 1. Measure the length of the adjacent side and the opposite side of the 22° angle in each triangle. Then calculate and compare the ratio. 22° 22° Reverse ____ = 0,40; ______ = 0,40; ratios are the same. Adjacent contiguous concepts and skills practice to explore exploring the relationship with the correct triangle exercise 1 Example 1 Finding tangens angle exercises 2-8 Example 2 Find lateral length using Tanger Exercises 9-14, 18-19 Example 3 Find angle action using Tangent Exercises 15-17, 20-21 In each right corner, find a tangent in each corner that is not the right angle. 2. 3. A E 5.0 MATH F 10 9,0 C 8 B 10,3 D 8 tan ∠A = __ 6 6 tan ∠B = _ 8 5 tan ∠ D = __ 9 tan ∠A = 1,33 tan ∠B = 0,75 tan ∠D = 0,56 tan ∠F = 1,8 4. B 32 81.5 A 75 5. C 9 tan ∠F = _ 5 19 Q 3 P R 19.2 32 tan ∠A = ___ 75 75 tan ∠B = __ 32 19 tan ∠P = ___ 3 3 tan ∠R = __ 19 tan ∠A = 0.4 3 tan ∠B = 2.34 tan ∠P = 6.33 tan ∠R = 0.16 module 18 use IN2_MNLESE389847_U7M18L1 936 integrate mathematical practices Focus modeling MP.4 Discuss rounding errors that occur, if you find the tangant and the inverse tangent angle. Invite students to look for a corner person on their calculators. Then let them use the value to find the tangent of the tangent. Compare the results. Repeat several exercises. Invite students to start with the rotary tangent. Lesson 1936 Depth of Knowledge (D.O.K.) Note that hypotension of the right triangle is never the opposite or lateral of the tangy relationship. Discuss why this has to be the case. © Houghton Mifflin Harcourt Publishing Company 6 COMMON CORE Mathematical Practices 1-4 1 Recall of Information MP.6 Precision 5-17 2 Skills /Concepts MP.5 Using Tools 18-22 2 Skills / Definitions MP.2 Justification 23-26 2 Skills /Concepts MP.4 Modeling 18/04/14 11:20 Pm0 Tangent Ratio 936 Let △ABC Be the Right Triangle, m∠C = 90°. Given the tangy angle of one triangle, the tangent finds the other corner. Avoid 6. Students may confuse the opposite side of the given corner and the adjacent sides in the triangle on the right. Suggest that students use one color to highlight the opposite side and the other color to highlight the adjacent side of this acute corner. Emphasize that the adjacent side forms a given angle of speed, which is not a triangular hypothesis. tan ∠A = 1,25 1 tan ∠B = ___ 1,25 7. tan ∠B = 0,80 tan ∠B = 0,50 1 tan ∠A = ___ 0,50 8. tan ∠A = 2,0 tan ∠B = 1,0 1 tan ∠A = ___ 1,0 tan ∠A = 1.0 Use the unknown side length to find the tangy. 9th Find QR. 10th Find AC. Q A R C 11. Find pq. P 7,0 7,0 tan 60° = ___ QR 7,0 QR = _____ E 13 tan 85° 14. Find PR. P 45° 8,4 C R 54° 4,6 Q AB tan 45° = ___ 8,4 DE AB = tan 45°(8.4) PR tan 54° = ___ 4,6 tan 21° AB = 8,4 PR = 6,3 21° 13 tan 21° = ___ DE = 9 PQ = _____ PQ = 0,79 A B D 9 tan 85° = __ PQ AC = 4,1 13. AB. F R 85° B AC = tan 27°(7,0) QR = 4,0 12. Find de. 27° 8,0 AC tan 27° = ___ 8.0 tan 60° 9 Q P 60° 13 ___ © Houghton Mifflin Harcourt Publishing DE = 33.9 PR = t an 54°(4.6) Module 18 IN2_MNLESE389847_U7M18L1 937 937 Lesson 18.1 937 Lesson 1 18/04/14 11:20 Use the rotary tangent of the calculator (tan -1). Round your answer to the nearest degree. 15. Find ∠A. 16. Find ∠R. A C B R 9 B 6.8 In the case of better triangles with a one-sided length of the legs, students may benefit from drawing the right triangle on graph paper and then by using a protractor to measure the size of a sharp angle. They can also do this to control their work. 17. Find ∠B. 24 Q 3.0 MANIPULATIONIVES 17 P 6.8 tan -1 ___ = m∠A 3,0 9 tan -1 __ = m∠R 24 m∠A = 66 ° m∠R = 21° A C 16 16 tan -1 B = __ 17 AVOID COMMON ERRORS m∠B = 43° Pay attention to the indication used to indicate the rotary tangent: tan -1x. The expression tan -1x is a short way to show the angle at which the tangent ratio is x. Make sure that students understand that raised -1 is not an exhibitor and that tan -1x does not mean ____ tanx . Emphasize that if students find reverse contact, the result is a degree. -1 Write an equation with tan or tan to express the size of the angle or side. Then solve the equation. 18th Find BC. B 40° 10 38 Q P A 20. Find ∠A and ∠C. 19th Find PQ. R 14,5 A B 75° 14,5 38 PQ = _____ = 10,2 tan 75° C ( ) BC = 10 tan 40° = 8,4 14,5 = 45° m∠A = m∠C = tan -1 A ___ 14.5 21. Multi-step Find Angle d. Show me your work. BC tan 30° = __ ; BC = 12 tan 30° 12 ( ) ( ) Module 18 IN2_MNLESE389847_U7M18L1 938 12 cm C 4 cm © Houghton Mifflin Harcourt Publishing Company 12 tan30° BC m∠D = tan -1 __ = tan -1 ____ = 60° 4 CD D 938 Lesson 1 1/18/04/14 11:20 PM Tangent Ratio 938 of 22. Engineering Client Wants to Build a ramp carrying 1,4 m person to a height of 1,4 metres as shown in the illustration. What additional information is needed to determine the angle (a) angle with the horizontal plane of the ramp? After additional measurement, describe how to find the angle measure. Possible answer: Measure the distance x ramp across the JOURNAL Do students come across what they know about the tangy relationship. Remind them to add at least one number and at least one example to their descriptions. a ( ) 1.4 horizontal. Then find the tan -1 ___ x , which equals the angle measure a. 23. Explain the error: The student uses the triangle shown in the 6.5 2.5 drawing. Locate and explain the student's error. a 6,5 a = tan -1 _ = tan -1(2.6) The student calculations are correct only if triangle 2.5 is a better triangle. a = 69,0° ( ) 24. If m∠A + m∠B = 90°, what ratio is formed tan ∠A and tan ∠B? Select all that apply. 1 C. (tan∠A)(tan∠B) = 1 A. tan∠A = _ tanB B. tan∠A + tan∠B = 1 D. (tan∠A)(tan∠B) = -1 A, C; Tan A and tan B are mutuals between each other, so that they have a 1. H.O.T. Focus on higher-level thinking 25. Analyzing the relationship to travel from Pottstown to Cogsville, a man drives his car 83 miles east on one road and then 15 miles north on the other. Describe the path a bird could fly right on the line from Pottstown to Cogsville. What angle is this line with the two paths the man used? © Houghton Mifflin Harcourt Publishing Bird's Way is a hypothesis right triangle formed by the path of the man. The bird's path shall constitute an angle of 10,2° and 79,8° for the first road and the second road. 26. Critical thinking On the right triangle there is only one 90° angle. The dimensions of both corners are greater than 0° and less than 90°. Why is it useful to define a tangant as 90° equal to 1 and tangant 0° equal to 0°? The answers are different. Possible answer: Consider the correct triangle with additional angles very close to 0° and 90°. Let the length of 0 hypotensive be 1. The tangant's small angle is very close _ . The 11 tangant's larger angle is very close _ . 1 Module 18 IN2_MNLESE389847_U7M18L1 939 939 Lesson 18.1 939 Lesson 1 1 18/04/14 11:20 PM Lesson Performance Task AVOID COMMON MISTAKES When they form conical piles, granulated materials such as salt, gravel sand and settle different angles of repose, depending on the shape of the grains. One particular 13-foot cone of dry sand has a base diameter of 38.6 feet. When using the tapered area of the conical pile, students may mistakenly use the diameter of the pile as a side next to the repose angle. The triangle they should use has a north radius as a side angle and stake height. resosium 1. What is this type of dry sand repose angle to the nearest tenth of a degree? 2. A different conical pile of the same type of sand is 10 feet long. What is the diameter of the cone base? 3. Henley Landscaping Supply sells a type of sand with 30° angle reposes for $32 cubic yard. Find the price of an 11-foot-long cone of this type of sand. Show me your work. INTEGRATE MATHEMATICAL PRACTICES Focus on math connections MP.1 Have tangential function values 13 tan (x) = ____ 38.6 (___ 2) 13 1. tan (x) = ___ 19.3 ( ) 13 x = tan -1 ___ 19.3 x = 34° proportional? If so, for example, the tangent 40° should be twice as high as 20°. Use tangent tables or a calculator to explore the question. Give examples that support your conclusion. Not proportionate; for example: tan 20° ≈ 0,36 but tan 40° ≈ 0,84 ≠ 20° 10 tan (34) = ___ (_2x) 20 2. tan (34) = ___ x x = 29.7 3. Let r = cone radius. Then: 11 tan 30° = __ r © Houghton Mifflin Harcourt Publishing Company 11 0.5774 = __ r r ≈ 19.1 ft Cone Volume: 1 V=_ Bh 3 1 2 =_ πr h 3 2 1(= __ 3,14) (19.1) (11) 3 = 4200 2 ft 3 1 yd 3 = 27 feet 3 so 4200.2 ft 3 = 4200.2 ÷ 27 ≈ 155 6 yd 3 $32 yd 3 55.6 yd 3 costs 155.6 ×32 = $4,979.20. Module 18 940 Lesson 1 EXTENSION ACTIVITY IN2_MNLESE389847_U7M18L1 940 Do students study the corners of at least four granular matter reposes. For each one, they should draw an equilateral triangle with a cross-section of a pile of 20-foot-long marked substance, the baseline angles that measure the angle of pollution, and the correct height to the nearest tenth. 18/04/14 11:20 Scores Rubric 2 points: The student's response is accurate and complete execution of tasks or tasks. 1 point: The student's response contains the characteristics of the appropriate response, but is invalid. 0 points: The student's response does not include the characteristics of the relevant response. Touch ratio 940 LESSON 18.2 Name Of Sine and Cosine RatioS Class Date 18.2 Sine and Cosine Ratio Important question: How can the relationship between sine and cosine be used, and inversely, calculations involving the correct triangles? Common Core Math Standards Student is expected: COMMON CORE Resource Locker G-SRT. C.6 Understand that, because of the similarity, the lateral relationships of the right triangles are characteristics of the corners of the triangle, leading to definitions of the trigonometric ratios of fierce corners. So is G-SRT. C.7, G-SRT. C.8 Explore You can use geometry software or an online tool to explore relationships with lateral lengths in the right triangles. Mathematical practices COMMON CORE research relationships in the ⚲ three points A, B and C. → → AND AC . Move C so that ∠A is awesome. Build raysAB MP.4 modeling _ Build point D_ AC. Construction line through D perpendicular AB. Design _ point E perpendicular to adjacent lines and AB. As the intersection of ◑, explain to the partner how to find the syte and co-compatibility of the corner, given the angle size and the length of the leg and hypotheses in the opposite direction. The measure ∠A. The △AD side lengths DE, AE and AD shall be measured. - DE and _ AE . Calculate ratio _ AD AD Important question: How can I use the sysy-coin ratio, and their inverse, calculations involve correct triangles? Given the length of one acute angle and hypotenus, you can use the systiary and co-compatibility ratio to find the length of each leg. Given the length of the hypothesis and the length of the leg, you can use these inverse proportionals to find the measure of each acute angle. PREVIEW: LESSON PERFORMANCE TASK View the Include online section. Discuss the photo by asking students to speculate on what the person in the photo might do. Then, check the hour performance task. © Houghton Mifflin Harcourt Publishing Company ENGAG Reflect 1. 2. 3. → → . What happens to m∠A as D moves along the AC? What suggest or Drag D along the AC theorem ensures that the different triangles formed are similar to each other? m∠A does not change; AA similarity to Postulate, also considering that ∠AED remains at right angles. → DE and _ AE ?¨what happens to the values of the ratio _ As you move D along AC AD AD Use properties similar to triangles to explain this result. The ratios remain unchanged; Similar triangles △ADE and △AD'E', AD = _ DE →_DE and _ AD = _ AE → _ AE' = _ AE . D'e' = _ _ Ad Ad'e' Ad' AD' AD' Movement C. What happens to m∠A? With a new value ∠A, note the → the two relationships. What happens to ratios when you drag D along the changes in the ∠A value; ratios do not change because D is dragged to → . along the AC module 18 must be ges must EDIT--Chan NO Key = NL-A; CA-A Correction Lesson 2 941 gh File info made by throu Date Class name and 18.2 Sine tios Cosine Ra es, their invers relationship and cosinus les? use sition can you right triangle angles ion: How involving binding calculations is correct G-SRT. C.7, in the right triangl . Similarly, the adi those of the acute corners are similar to tons and trigono COMMON G-SRT. C.6 Unders definitions core e, leading ngle triangl Tria t righ G-SRT. C.8 les. Relationships in the right triang lengths stigating the side of Inve e ratios Explore tool to explore or online software geometry You can use Quest Essential IN2_MNLESE389847_U7M18L2 941 Resource Locker HARDCOVER PAGES 941,952 Watch for hardcover student edition page numbers for this lesson. B and C are awesome. points A, → C so that the ∠A 3→ . Construction and AC raysAB Construct D _ the interse line on the effect D_. Constructor _ lar to AB . perapendicu line and AB ndicular perpe and AE, lengths DE, re side ∠A. Measu Action . (ad△ADE AE . DE and _ _ AD ratios AD → the ⚲ ◑ the starting name which suggest or¨along the AC angle. Mutually? D moves right to a similar m∠A as → what happens ∠AED rema formed is also that¨. AC eg triangles of late, considering Drag D along the tees that differ from

AE ? postu guarana DE and _ _ theorem; AA Simil AD does not change the ratios of ED values m∠A does not → what happens △AD'E', n this result. . D along ac r triangles explai △ADE and lse move 2. As you features simila Given a similar triang AE . (e) the use of a new AE' = _ Use AE → . _ _ does not change AD AD'AD Ratio DE and _ = AE'd D'E' = _ AD' values DE → _ note AD AD = _ _ → AD' value m∠A, AC dragged east ad'd'e with new D'e because D is ns to m∠A? if you drag, do not change C. What acid ns ratio 3. Move it. With acid values ; There two lessons 2 es worth m∠A chang → . 941 by AC Publishin Reflective © Houghton n Mifflin Harcourt I 1. Module 18 8L2 941 47_U7M1 ESE3898 IN2_MNL 941 lesson 18.2 18/04/14 11:31 PM 18/04/14 11:33 Explain 1 Find The Siphon and Co-like Angle Of EXPLORE Trigonometric Ratios Trigonometric ratio is the ratio between the two sides of the right triangle. You've already seen a trigonometric relationship, a tangy. There are two additional trigonometric relationships, sine and cosine, which include the hypotenus of the right triangle. Sine ∠A, written by sin A, is defined as: leg length opposite ∠A BC sin A = ___ = AB the length of hypotenus ∠A cos, written cos A, is defined as: leg length ∠A AC cos A = ___ = AB length of hypotenus B INTEGRATE technology A C These definitions can be used to calculate trigonometric ratios. Example 1 ❓ the correct triangle of student ratios is the opportunity to do research, either in a book or on the internet. D Write each corner sine and cosine as fractions and decimal places, rounded to the nearest thousand. ∠D 15 E length of leg opposite ∠D EF 8 sin D = ___ = ___ = 0.471 DF 17 length of hypotenuse to integrate mathematical practices Focus on critical thinking MP.3 Can students investigate what happens in a 17 8 F ratio on the opposite side of the length of the hypotenuses when the sharp angle reaches closer to 90°. When the acute angle approaches 0°, repeat the relationship between the length of the adjacent side and the length of the hypotheses. leg length adjacent to ∠D 15 ≈ 0,882 DE = ___ = cos D = ___ = 17 DF hypotenuse length ◑ ∠F 15 feet length opposite ∠F DE = _ = cos F = ___ = _DF hypotenus length 0,882 17 8 STRATEGIES 0.471 Reflect 4. What do you notice about the si likes and co-d'ors you've found? Do you think this relationship applies to every couple of sharp corners in the right triangle? Explain. sin D = cos F and cos D = sin F; this relationship always holds, because the leg opposite one fierce angle adjacent to the other. 5. In the right triangle △PQR with hypotenuse 5, m∠Q = 90° and PQ > QR, what are the values sin P and cos P? sin P = cos P = 5 If the measure of the acute angle of the right triangle does not change, but the side lengths of the triangle change, how will the ratios change? Explain. The values of the numerator and denominators change, but the ratios of the hypotenusis and the contiguous length, because the triangles are similar. © Houghton Mifflin Harcourt Publishing Company length of leg next to ∠F cos F = ___ = length hypotenuse 17 5 _4 EXPLAIN 1 5 Module 18 942 Find Sine and Cosine Angle Lesson 2 PROFESSIONAL DEVELOPMENT IN2_MNLESE389847_U7M18L2 942 Mathematics Background Trigonometry is a branch of mathematics related to angle relationship triangles. The ancient Egyptians used trigonometry to restore the borders of the land after flooding the Nile River every year. Babylons use trigonometry to measure the distance between nearby stars. Trigonometry is used in modern engineering, cartography, medical imaging and many other fields. 18/04/14 11:33 AVOID COMMON MISTAKES Students often use the wrong relationship with sine or cosine. Help students review these relationships using flash cards, mnemonics, or other memory tools. Encourage students to study or produce mnemonics. Sine and Coinity Ratios 942 Explain 2 POLLING STRATEGIES Using additional angles The acute angles of the correct triangle complement each other. Their trigonometric relationships are interested, as shown in the next relationship. If only you know if you have the right triangle sharp angle, how could you find a combination? The sunare gives the opposite side of the relationship to the hypotenus, so that you can create a right triangle of hypotenisive and foot that corresponds to the ratio. Then you can pythagorean theorem to find the length of the second leg, and use it to write a combination relationship. Trigonometric relationships of additional angles If ∠A and ∠B are acute angles in the right triangle, sin A = cos B and cos A = sin B. Thus, if ρ (theta) is the measure of an acute angle, then sinᵒ° = cos (90 - ρ)° and cos ρ° = sin (90 - ρ)°. (90 - θ°) A Do you think it is possible that the value of sine or cosine is greater than 1? Why or why not? This is not possible; because hypotension is the longest side of the right triangle, each ratio with the hypotenus length as denominator is less than 1. ρ° B C You can use these to write equivalent expressions. Example 2 ❓ Write each trigonometric expression. Given that sin 38° = 0.616, write the cosine in terms of an additional angle in the sine 38°. Then find an additional angle. Use the expression related to the trigonometric relations of additional angles. sin ρ° = cos(90 - ρ)° EXPLAIN 2 sin 38 ° = cos(90 - 38)° Simplify. sin 38° = cos 52° Using Additional Angles Replace The Pain 38°. 0.616 ≈, because 52° QUESTIONING STRATEGIES Why sine and cosine are an additional angle of relationship? The relationship is right for the triangle. Since one angle must be a right angle, the other two corners must be upgraded, since the sum of the triangle angle dimensions is 180°. How can you write equivalent expressions sin x° using cosinus and cos y° using sine? Explain. Use the supplement to write sin x° = cos (90° - x°) and cos y° = sin (90° - y°). How does the relationship between sine and cosine in extra corners help resolve equations that include sine and cosine? Apply the fact that the total angle size is 90° to create an equation to solve unknown values. 943 Lesson 18.2 © Houghton Mifflin Harcourt Publishing Company So, the combination of an additional angle is about 0.616. ◑ Considering that cos 60° = 0,5, write an additional angle in the sine according to 60° cosinus. Then find an extra angle in the sys. Use the expression related to the trigonometric ratios of the additional angles. cos ρ = sin(90 - ρ) (Substitute 60 on both sides. cos 60 ° = sin 90 - 60 Simplify the right side. cos 60 ° = sin 30 Substitute 0,5. 0,5 = sin 60 °° ° So, the si already angle is 0,5. Module 18 943 Lesson 2 collaborative learning IN2_MNLESE389847_U7M18L2 943 Small group activity 18/04/14 11:33 Do students work in small groups to study the connection between the size of the angle and its sine, using geometry software or graph paper, rulers and protractors. They should draw several triangles with unit side lengths and an angle that increases. Those who use graph paper must be precise. Let them determine how the sy syd changes as the size of the corner increases; then repeat cosine. Invite students to share their results and how they drew up their conclusions. Reflect collaborative learning 6. What can you infer from sine and cosine 45°? Explain. sin 45° = cos 45°; 45° complements itself. 7. Discussion Is it possible that sine or cosine have an acute angle equal to 1? Explain. No, no right triangle hypotenuse is always longer than the legs, so lateral relationships Do students build full triangles with a metre side length on graph paper, then use the track to measure each sharp angle to the nearest degree. Invite students to explain an additional relationship between sy sy and co-compatibility for these triangles at each sharp angle, and then share their results with class. Your turn Write every trigonometric expression. 8. Given that cos 73° = 0.454, write the sys at the additional angle. EXPLAIN 3 ρ = 73°, so that 90 - ρ = 27°. 27° = 0.454 9. Finding Side Lengths using Sine and Cosine Considering that sin 45° = 0.707, write together at an additional angle. ρ = 45°, so 90 - ρ = 45°. cos 45° = 0.707 Explain 3 INTEGRATE TECHNOLOGY Finding lateral lengths of use of Sine and Cosine It may be useful to look with students on how to evaluate phrases in the form of asin (xᵒ) and because (yᵒ) given values a, b, x and y using their calculators. You can use sine and cosine to solve real-world problems. An example of a 12-ft ramp is installed with a few steps to ensure wheelchair access to the library. The ramp makes an 11° angle with the ground. Find each dimension, the nearest tenth leg. C 12 ft B y Find wall height x. Multiply both sides by 12. The length of the ∠A AB sin A = ___ = _AC length hypotenuse Use syco definition. Substitute 11°A, x BC and 12 AC. 11° © Houghton Mifflin Harcourt Publishing Company ⚲ wall x x x sin 11° = _ 12 12sin 11° = x QUESTIONING STRATEGIES Why is the trigonometric relationship useful for solving a real world problem involving right triangles? Sample response: Known lengths and angle measurements allow you to find unknown lengths that can be difficult to measure. If you use a trigonometric relationship, such as sine or cosine, to find the length of one leg of the right triangle, do you need to use a trigonometric relationship to find the length of the other leg? Explain. No, you can also use Pythagorean Teorem to find the other leg because you know the lengths of hypotenuse and one leg ≈. So, the height of the wall is about 2.3 feet. Module 18 944 Lesson 2 Differentiated Instruction IN2_MNLESE389847_U7M18L2 944 Critical Thinking 18/04/14 11:32 sins Devious Invite students to investigate trigonometric relationships to prove that tan x = _ _ ___ cos x. Ask students if they find it surprising and why not. Sine and cosine Ratio 944 B EXPLAIN 4 Find the distance y that the ramp extends in front of the wall. Substitute 11°a, y For AB and 12 AC. Finding Angle Measures Using Sine and Cossy Multiply both parties by 12. y cos 11 ° = _ 12 12 cos 11 ° = y = 11.8 Use a calculator to evaluate the expression. QUESTION STRATEGIES _ Leg length ∠A cos A = ___ = AB AC length of hypotenuse Use the term cosine. So, the ramp extends in front of the wall about 13.8 feet. In the equation y = sin -1 x, explain what x and y In the equation, the x angle represents the sigle measure yᵒ. To reflect _ 10. Can you find the height of the wall with the combination? Explain. x and because ∠A and ∠B complement each other, m∠C = 79°. Then cos 79° = 12 x = 12cos 79° ≈ 3.3 feet. Your turn on 11. Let's say the new regulation stipulates that the maximum angle of the wheelchair ramp is 8°. At least how long does the new ramp have to be? Round to the nearest tenth of a foot. C 2.3 ft z xall 8°B A Ramp must be at least long enough to create an 8° ∠A. BC 2.3 ⇒ sin 8° ≈ _ sin A = z AC z = 16.5 © Houghton Mifflin Harcourt Publishing Company _ Ramp must be at least 16.5 feet long. Explanation 4 Finding angle measures using Sine and Kosine 5 = _ 1 . However, you already know that 30° = _ 1 . So you can triangle, sinA = _ 10 2 2 conclude that m∠A = 30°. 10 2 sin-1 C 5 B Considering the angle, ∠A, • if sin A = x, then sin-1 x = m∠A. This is read as follows: (-1) ◑ C 5 B Considering the angle, ∠A, • if sin A = x, then sin-1 x = m∠A, read the inverse bus x • cos A = x, then cos -1 x = m∠A, read the inverse co-calculator to evaluate the inverse trigonometric expressions. Module 18 945 Lesson 2 LANGUAGE SUPPORT IN2_MNLESE389847_U7M18L2 945 Connect Vocabulary Differentiation between Sine and Cosine Can Be Tricky for Some Students. Explain that a donkey can work together, as cooperation does in the word. Note that the ratio of the acute angle of the triangle involves an adjacent leg. Tell students to remember this by thinking of the adjacent leg as a reunion with a hypotensive perspective. 945 Lesson 18.2 18.34/14 11:32 Example 4 ❓ Find acute angle measures △PQR, nearest degree. P QUESTIONING STRATEGIES Write trigonometric ratio ∠R. Because the length of hypotenuse and opposite leg is given, pq use sy sine. sinR = _ PR 7 Substitute 7 PQ and 13 PR. sinR = _ 13 ◑ Q ❓ How could you estimate the angle of reverse sine or co-compatibility without the calculator's reverse trigonometric keys or table of values? To find the missing leg length, use the Pythagorean theorem syne ratio measurements. Then draw a right triangle with lateral length and use a picture to measure the angle used to measure the opposite side or side angle. 13 R Write and evaluate the reverse trigonometric ratio to find m∠R and m∠P. 7 _ _ Start ∠R trigonometric relationship. sinR = 137 7 _ Use the reverse sine ratio definition. m∠R = sin-1 13 Use a calculator to evaluate the reverse sine ratio. m∠R = 33 ° Write cosine ratio ∠P. cosP = PQ substitution 7 and 13 PR. cosP = Use the reverse composition ratio definition. m∠P = - 1 Use a calculator to assess the reverse composition ratio. m∠P = 57 ° PQ _ PR 7 _ 13 7 _ _ AVOID COMMON ERRORS 13 Remind students of the signs of the places used to represent reverse sine and cosine: sin -1 x and cos -1 x. Like the reverse tangent, there is no -1 exponent. Rather, it represents a reverse trigonometric relationship with a degree angle. Mirror 12. How could you determine m∠P ∠P and ∠R complement each other. Therefore, m∠P = 90° - m∠R = 90° - 33° = 57°. Your turn Find the acute angled △XYZ, to the nearest degree. XY = _ 14 Z X cos Y = _ 13 sin-1 V Y2 23 14 m∠X = cos -1 23 14 Z 23 14 m∠Z = 90° - m∠Y = 90° - 53° Y Work On Seminar 15. How is the relationship between the syne and the co-compatibility of the right triangle sharp angle? sin ∠A is = adlateral bc AC (_ and cos A = _ = hypotenuse module 18 IN2_MNLESE389847_U7M18L2 946 AB AB adient to work with © Houghton Mifflin Harcourt Publishing Company (_) ≈ 37° 47° INTEGRATE TECHNOLOGY Do students use graphing calculators to graph y = sin x and y = cos x your graphing calculators at intervals [0°, 90°]. Discuss the similarities and differences. reverse C Lesson 2 18/04/14 11:32 Sine and Cosine Relations 946 16. How is the relationship between sine and co-compatibility defined for the sharp corner of the right triangle? Since the sine ratio of this acute angle is always the same, the measure of this angle-setting strategies can be defined as the reverse sine ratio: BC BC → m∠A = sin -1 sin A = or sin A = or sin A = or sin A = → m∠A = sin -1 AB Logwise, AC AC → m∠A = cos -1 cos A = cos A ⇒ → m∠A cos -1 y AB AB (_) ← How is cosine and reverse co-y bound? The combination of the acute angle of the right triangle is the ratio of the lengths of the adjacent side and the hypotenuse of the right triangle. Reverse cosine is the sign with the corner's right triangle, the combination of which is ratio. C _) 17. Important question Check-In How to find an unknown angle measure in the right triangle? First, use two known side lengths to form a relationship. Then use a suitable reverse trigonometric ratio to find the angle measure. SUMMARY LESSON EVALUATE: Homework and practice How do you decide which trigonometric relationship to use to find missing half-length or angle measurement in the right triangle? Use the sine or reverse sine on the approximate relationship between hypotenus and the association between the interactions between the adjacent bandages and the hypotenus. • Online Homework • Tips and Help • Extra Practice Write for Each Trigonometric Expression. Round trigonometric relationships to the nearest thousand. 1. Given that sin 60° ≈ 0.866, write together at an additional angle. Given that from 26° ≈ .899, write the sys at an additional angle. 2. sin 60° = cos(90° - 60°) cos 26° = sin (90° - 26°) 0,866 ≈, for 30° 0.899 = sin 64° PQ each figure. Write each trigonometric in decimal numbers, rounded (if necessary) to the nearest thousand. © Houghton Mifflin Harcourt Publishing Company 13 A module 18 IN2_MNLESE389847_U7M18L2 947 947 Lesson 18.2 4. cos A adiv 5 AC cos A = ≈ 0.385 13 AB hypotenuse 5. cos B cos B = AB hypotenuse 13 _ adilateral _ 12 = ___ = 0,923 13 (0,923 3. (a) 24 F 6. sin D cos A = reverse EF F _ = _ = 0.28 7. cos F = adv EF 7 _ = _ = 0,28 8. sin F sine F reverse 24 _ = 0,96 7 E 947 hyo Netennius hypotenus hypotenuse DF DF 25 25 25 Lesson 2 18/04/14 11:32 Find unknown length x in each right triangle, nearest tenth. 9. 10. C Evaluate D 37° 15 27 B A x EF cos E = _ DE x = cos 53° = _ = 16.2 ≈ xAB_ AC x x sin C = _ 9.0 ≈ x sin C = INSTRUCTIONS FOR DETERMINATION 53° 27 E 15 11. J K 14 L R 19 x 75° JL KL = cos 75° = ⇒ 54,1 24° P cos L = Q x PR _ PQ 19 sin 24° = _ ⇒ 46,7 sin P = x x Find each acute angle, to the nearest degree. 11 V P W 18 8 15 Q U 13. m∠P 8 PR cos P = _ = _ 18 PQ 8 m∠P = cos(_) 64° -1 VW 11 sin U = _ 15 UW 11 _ m∠U = sin ( ) = 47° Harjutus 16. m∠W m∠W = 90° - m∠U = 90° - 47° = 43° Uurida uuinuda suhtarvud õige kolmnurga kasutamise 17 Näide 1 Leida siinus ja koosinus nurk harjutusi 3-8 Näide 2 kasutades täiendavaid nurki 1–2 Näide 3 Külgpikkuste leidmine Sine ja Koosinuse harjutuste abil 9–12 Näide 4 Nurgamõõtude leidmine Sine ja Koosinuse harjutustega 13–16 Tuletage õpilastele meelde, et nende kalkulaatorid peavad olema seatud kraadidele, mitte radiaanidele, et saada õiged väärtused trigonomeetriliste suhetele jooks. Communication with Math Discuss how important it is to be able to manage and interpret the correct triangle diagrams to use trigonometric relationships to solve real-world problems. Lesson 2 948 Depth of Knowledge (D.O.K.) Practice TO AVOID COMMON ERRORS 15 Module 18 IN2_MNLESE389847_U7M18L2 948 m∠U ≈ 90° - 64° = 26° 1 8.m∠U -1 14.m∠U 2 Recall of information Mathematical Practice MP.5 Use Instruments Explain Company R Concepts and Skills COMMON CORE Mathematical Practices 1-16 1 Recall of information MP.4 1 Modelling 17-24 2 Skills/Concepts MP.4 Modelling 25 3 Strategic Thinking MP.2 Recital 27 3 Strategic Thinking MP.3 Logic 18/04/14 11:32 Sine and Cosine Ratio 948 AVOID COMMON MISTAKES Students may find it difficult to solve equations such as when a variable is the denominator. as sin x = _ c Review students how to use inverse operations to write the equivalent equation variable reader, such as c = ____ sin x. 17. Use an attribute with the corresponding sides of similar triangles proportional to explain why the trigonometric ratio of sin A is the same for △ABC calculated in the △ABC. C By AA ∼, △ABC ∼ △ADE, so that the corresponding are proportional: (BC/(AE) AC BC AC AE → AC = ⇒ = AE DE B B E ∠A ⇒ ∠A and ∠ABC ⇒ ∠ADE, both at right angles. _ _ _ D These ratios determine sin A because they are equal, sin A is the same when calculated either in the triangle on the right. 18. The laptop specifications describe this screen as 15.6 dimensions. However, it is actually the length of the diagonal screen. How wide is the screen horizontally, to the nearest tenth an inch? IN ADDITION TO CALCULATORS, students can use geometry software and spreadsheets to evaluate trigonometric ratios. cos P = P = P15,6 PQ PQ = ⇒ cos29° = _ ⇒ ≈ 13,6 in = PQ P 15,6 PR 29. Sharla's bedroom is built right under the roof of her house. Given the dimensions shown, how high is the ceiling at its highest point, the nearest tenth of a foot? sin(m∠AEB) = AB AB _ ⇒ sin 22° = _ ⇒ AB ≈ 4,0 10,6 AE Maximum height: AC = AB + BC ≈ 4.0 + 1 5.5 = 9.5 ft 29° Q A 10.6 ft roof B E 22° 5.5 ft © Houghton Mifflin Harcourt Publishing Company • Image Credits: ©teekaygee / Shutterstock C 20. Zoology Can sometimes be seen as an eagle gliding its wings bent in a characteristic double-water shape. Each wing can be modeled as two right triangles as shown in the illustration. Find the angle of the middle wing, ∠DHG to the nearest degree. cos(m∠DHE) = 32 EH _ _ = 49 32 m∠DHE = cos -1 49,2° 49 38 F(m∠G <5>FHG) = 43 GH 38 = 62,1°m∠FHG = sin -1 = 49 FH m∠DHE + m∠G = 49,2 + 62,1 ≈ 111,3° module 18 IN2_MNLESE389847_U7M18L2 949 9 949 lesson 18.2 D D 49 G 49 cm E F 38 cm G E Lesson 2 18/04/14 11:32 Why? Expressions 43 G ∠FHG <7>FHG = DH ( ) _ (() H 43 cm 32 cm 49 2 + 62,1 ≈ 111,3° module 18 IN2_MNLESE389847_U7M18L2 949 9 949 lesson 18.2 D D 49 G 49 cm E F 38 cm G E Lesson 2 18/04/14 11:32 Why? Expressions 43 G Find a pair of acute corners that correspond to equation (3x + 9) = cos(x + 5). Make sure your answers are measurable. COGNITIVE STRATEGIES Expressions must be measures of two additional angles: discuss how students can use facts that sin 0° = 0 and sine increases as the degree increases (between 0° and 90°) and cos0° = 1 and the cosinus decreases as the angle increases (between 0° and 90°) as they assess the trigonometric expressions. (3x + 9) + (x + 5) = 90 ⇒ x = 19 (3x + 9)° = (3(19) + 9)° = 66° and (x + 5)° = ((19) + 5)° = 24° Control: 66° + 24° = 90° 22. Multi-Step Reginald is planning a fence in his backyard. Everywhere in the yard, except half way along the house comes fenced, and the fence costs $3.50/yd. How much does fencing cost? (23 - 13) LM sin(m∠LKM) = ⇒ sin 32° = ⇒ LK = 18.9 LK distance fence: JK + KL + KN = 13 + 18.9 + 23 = 23 = 54.9 yd _ _ J K 32° yard house cost fence: approx. 54.9($3.50) = $192.15 13 yd N M 23 yd Art 23. Architecture The sides of one of the World Trade Centers in New York make up eight four of them are 200 feet long on their base bc. The length of the AC on each sloping side is about 1,185 feet. B 200 feet © Houghton Mifflin Harcourt Publishing Company • Image Authors: ©Michael Utech / Alamy 1185 ft C Find measure the top corner of bac each onosceles _ triangle, nearest tenth of a degree. (Tip: Use be Center D to create two right triangles.) △ABD = AB = 1185 ft and BD = (200 ft) = 100 ft. Therefore, 2 = 100 ft 100 ft 1185 cos ABD = m∠BAD = sin sin(m∠BAD) ≈ 4.8° 1185 1185 AD _ _ Module 18 IN2_MNLESE389847_U7M18L2 950 _ () Lesson 2 18/04/14 11:32 Sine and Cosine Suhes 950 PEER Focus on Thinking Do Students Explore The Following Identities by Evaluating Them from Different Angles: Sin -1(sin x) = x and Cos -1 (cos x) = x. _ On 24 April 2005, the Commission Explain the error melissa has calculated the length of the XZ △XYZ. Explain why Melissa's answer must be wrong and identify and correct her mistake. XZ cos X = _ XY XY XZ = _ cos X XZ = 27 cos 42° = 20.1 JOURNAL Do students describe how they remember to use with sin y and co-like relationship, and relationship relationship. Remind them to add at least one number and at least one example to their descriptions. 20.1 </; 27, but hypotenuse should be the longest side. His definition of co-yness was reversed. XY → XZ = 36,3 cos X = XZ cos 42° Z Melissa solution: _ 42° X _ Y 27 25. Submit mathematical ideas Explain why the acute angle and cosine are always between 0 and 1.0 BC AB </; </; sinA </, 1 0 </; BC </; AB ⇒ AB AB AB AB AB The same argument shows that 0 </; cos A </; 1 . Explain. 26. Look for a △ABC, hypotenuse with a length ratio 1. Use Pythagorean Theorem to explore the link between squares of sine and ∠A, written as sin 2 A and cos 2A. Could you derive this relationship using the right triangle without length? Explain. B. BC BC AC sin A = ___ = ___ = AC 1 AB 1 C B 1 A C Pythagorean Theorem, AC + BC 2 = AB 2 © Houghton Mifflin Harcourt Publishing Company) Substitute AB, BC and AC. (sin A) 2 + (cos A) 2 = 1 sin 2 A + cos 2 A = 1 BC Yes: sin 2 A + cos 2 A = AB AC ( )_ (_AB) 2 BC = BC + AC AB _ = 1 2 AB 2 AB 2 AB 2 AB 2 27. Justify the reasoning Use the triangle proportionality theorem to explain why the trigonometric ratio cos A is the same when calculated in △ADE as △ABC. E C Two segments, ⊥ to the same line, ‖, ‖ DE¯. The proportionality of the triangle, BC so BC¯ and DE¯ of △ADE proportionally: divides the parties AD A B D AB + BD AC + CE BD BD AB AE _ = → _ = 1 + _ = 1 + _ _ ⇒ _ _ = _ = AC AB AE AC AE AC AC AC These ratios determine cos A, and cos A is the same as it is calculated in triangle. Module 18 IN2_MNLESE389847_U7M18L2 951 Lesson 2 18/04/14 11:32 PM Lesson Performance Task Lesson Performance Task When light goes from vacuum to another environment, it is broken-i.e. its direction changes. The ratio of the sine angle of the incident to the ray, r, to the sine angle of the outgoing response ray, r, called index breakage: AVOID common errors in the incident ray question 4 lesson Performance task describes an error usually by students collaborating with trigonometric functions. Here's a similar error: gem reacted ray sin I n = ___ . where n is the refraction index. sin r This relationship is important in many areas, including gemology, study precious stones. A gemologist can put an unknown gem into a tool called a refractometer, direct the incident light beam at a certain angle into the rock, measure the angle of the broken ray, and calculate the index of breakage. Since indexes refraction thousands of gems are known, the gemologist can then identify the gem. Gem 1. Determine the gem taking into account the angles obtained from the refractometer: a. I = 71°, r = 29° b. I = 51°, r = 34° I = 45°, r = 17°c. 2,3. The ray of the incident n a piece of the snake. The resulting break angle measured at 21°. Scroll to the nearest degree. 2,94 Diamond 2.42 Zircon 1,95 Azurite 1,85 Sapphire 1,77 Tourmaline 1,62 Serpentine 1,56 Coral 1,49 Kickledge viga (d) õpilase lahendus ja selgitus. m∠ viga (d) patt l patt (55°) = _ 2. 1.77 = ______ sin ° sin(x°) sin (x°) sin 55° ° sin(56°) = sin (56°) _ sin 34° sin(x°) = 1.77 51° ≈ _ 34° sin(56°) x = sin (_ 2,3 sin 21° _ = 2,2 = 1.85 sin 45 sin 17° Remove sin_ _ = 1.85 sin 45 sin 17 sin 71° 1.95 sin 29° = 1 = sin 29° = 1,95 2,42 1.95 (1.95) = 2.42 28 = 1,77 = 1,5 = kunal a. (71°) _____ = 1,95 sin(29°) 3. tsirkoonsool(45°) ______ = 2,42 sin(17°) sin(x°) 1,56 = ______ sin(21°) = 34° Teil 53 (x) = 1,39 opal sin(34°) 4. Refraction hematite index (sin 10° 10° Explain to students that function and angle cannot be separated. The correct solution is: diamond Possible answer: the student shared the word sin 50° = ___ 0. 7660 = 4.41 ____ sin 10° 0, 1736 INTEGRATE mathematical practices Focus on reasoning MP.2 Using a refractometer, the gemologist found that the index refraction of the substance was 2.0. Using the same frequency, the gemologist found that the angle of the fractured ray of the second substance was larger than the angle of the fractured ray of the first substance. Was the refraction index of the second substance greater than or less than 2.0? Explain. Sample answer: Las i = angle of fall, r 1 = refractive angle 1 and r 2 = breaking angle 2. sin i . As the sine function increases when from 0° to 90°, sin r 2 >/; sin r 1 sin i index sin i </; ___ = 2 < sin r 2 sin r 1 The sin 2, So, the second substance for the breakage of ___ and ___ sin 2 = _____ reader and denominator in step 2. This was wrong because sin 51° and sin 34° are indivisible entries, each representing one actual number. Module 18 952 Lesson 2 EXTENSION ACTIVITY IN2_MNLESE389847_U7M18L2 952 Physicists define the refraction index of the material by the speed of light. Examine the formula used by physicists related to the refraction index n of the substance, the light speed in vacuum, c, and the light velocity through the substance, v. n = _ v The speed of light in the settler is about 100,693 miles per second. Use this fact and table lesson performance task to find the light speed in a vacuum. Express your response in miles per second and miles per hour. 186,282 mi/sec; 670,615,200 mi/h the sun is about 93 million miles from Earth. How long does it take the sun to get to Earth? About 8.3 min 18/04/14 11:32 Grades Rubric 2 points: The student's response is accurate and complete performance of the task or tasks. 1 point: The student's response contains the attributes of the appropriate response, but is invalid. 0 points: The student's response does not include the characteristics of the relevant response. Sine and Coine Ratios 952 LESSON 18.3 Name Special Right Triangles Class Date 18.3 Special Right Triangles Important Question: What Do You Know About The Side Length and Trigonometric Ratio in Special Right Triangles? Common Core Math Standards Student is expected COMMON CORE Resource Locker G-SRT. C.8 Explore 1 Use Trigonometric relationships and Pythagorean theorem to solve the right triangles in implementation problems. Discover relationships that always apply to onosceles in the right triangle. Mathematical practice COMMON CORE The figure shows an equilateral right triangle. Identify the key angles and use the fact that they complement their x ∠A, ∠B; m∠A + m∠B = 90° Language objective Explain to the partner how to find the sine, cosine and tangent of the 30° -60° -90° triangle. B Use an equilateral triangle theorem to write a different equation related to the base angle dimensions. C Use Pythagorean Theorem to find the length of the hypotenuse in terms of length of one leg. AB = x 2 + x 2 -√3 tan 30° = _ 3 _ PREVIEW: LESSON PERFORMANCE TASK © Houghton Mifflin Harcourt Publishing Company x 2 What measures the base angles have to be? Why? D Side lengths 45° -45° -90° triangle is always ratio 1: 1: √2 . Trigonometric √2 ratio, related to this triangle is sin 45° = 2 tan 45° = 1 . 30° -60° -90° triangle sides lengths are always ratio 1:√3 : 2. The trigonometric relationships associated with this triangle are _ 1 √3 _ √3 sin 60° = 2 cos 60° = _ 2 Reflect 1. Is it true that if you know that one side length is an equal right triangle, then you know all the lateral lengths? Explain. - yes, I don't know. If you know either leg length x, the length of the other leg is also equal to x and the length of each leg is divided √2 . Hypothesis is the value multiplied by √2 . If you know the hypotenuse 2. - What if? Suppose_ draw a cross from C to AB. Explain how to find the length of the CD. Because △ABC is equilateral, m∠A = 45°, and because △ADC is a better triangle, m∠ACD = 90° - m∠A = 45°. Therefore, △ADC is _ √- 2 1 onosceles right triangle, so CD = ___ . - or __ 2 √2 Module 18 must be EDIT--Chan EI Key=NL-A; CA-A Correction D A 1 C 1 Lesson 3 953 File info manage thorse B Date Class 1 ht Triangle Name and Rig 18.3 Speci ic relationship between trigonomet s and lateral length about es in you know the right triangl ion: What re solve triangles? Orean Theore's special right Pythag ratios and Quest Essential 1 Explore ❓ es right triang is applied to osct le. That always HARDCOVER PAGES 953 966 B x View hardcover student edition page numbers for this lesson. C x ∠ ❓ Why? main angles m∠A = 45°. res is = 90°, so meanu 2, m∠A Which must be applied the hypotenuse is replaced = 45°; using the conditions ensue m∠A = m∠B hypot length erm and Equen Theor √2 Use Pytha AB = x 2 x. foot, 2 each AB = 2x 2 2 2 + x s? AB = x side length know all le, then les right triang x, and have one osce mirror equal one side length h is also e if you know leg lengt true to hypoten use thin? √2 . tenuse is divided by hypo this value length _ n how long is AB. Explai every leg C length, then ndicular D perpe se you draw C is right if? Suppo _ from △AD √2 2. AB from the length of cd. = 45° and les, m∠A found △ADC is proportional thus A 1 Since △ABC an m∠ = 45°. 2√ = 90° - D _ 1 m∠AC ___ triangle, √ or 2 le, both CD √2 better triang les isosce - y g Copman - Harcourt t Publishin - n Mifflin Harcourt IN2_MNLESE389847_U7M18L3 953 g exploration rights tria isosceles and base angles g for them. To detect relative to the right of the triangular equation is isoceles write shows ementary Number is compl that they use the fact 90 ° + m∠B = actions. ∠A, ∠B; m∠A related equation e.g. different write le Theorem Triang Use Isosce res. measu m∠A = m∠B base angle relationships Discover Resource Locker right triangles Pythagoream G-SRT. C.8 Use mis. implemented probe ❻ Houghto View Engage section online. Discuss the photo by asking students to speculate on whether the task of completing the lesson involves a dog or a flying disc and why. Then, check the hour performance task. - AB = x √2 AB = 2x 2 COMMON CORE B 1 C Hour 3 953 Module 18 8L3 953 47_U7M18L3 953 47_U7M1 ESE3898 IN2_MNL lesson 18.3 C √2 AB = 2x 2 2 2 x + x 2 2 x √2 . = 45°; 2m∠A = 90°, so m∠A = 45°. Important question: What do you know about side lengths and trigonometric relationships in special right triangles? sin 45° = 2 , A m∠A = 45° m∠B DEAL 953 B measures. MP.2 Recital – and tan 60° = √3 . Investigation onosceles Right Triangle 18/04/14 11:49 18/04/14 11:50 Explore another special right triangle EXPLORE 1 Discover the relationships that always apply to the right triangle formed in a semi-equilateral triangle. A △ABD is an equilateral triangle and BC is a section from B to AD △. m∠C = 90°; at each angle, the equival seed measures 60°, B By examining the size of the △ABD explain why △ABC ⇒ △DBC. INTEGRATE TECHNOLOGY C D C ∠A ⇒ ∠D (Equilateral Triangle Theorem), ∠ACB ⇒ ∠DCB (all right angles are compatible), _ _ and BC ⇒ BC (Reflexive Property of Congruence), both △ABC ⇒ △DBC by AAS Congruence. _ _ _ _ Or AB ⇒ DB (△ABD equilateral), BC ⇒ BC (Reflexive Real Estate Congruence), and because students have the opportunity to do explore activities either in the book or online. △ABC and △DBC are by HL Congruence. C _ _ Let the length of ac be x. What is AB length and why? _ _ From step B △ABC ⇒ △DBC, and thus m∠ABC = 90° - m∠A = 30°. B Explain why the △ABC ⇒ △DBC. CD = AC = x. Because AD = AC + DC = x + x = 2x (Seg. Annex. Post.): _ _ _ because AB ⇒ AD (△ABD equilateral), AB = 2x. D _ Pythagorean theorem by using the find length BC. (2x) 2 = x 2 + BC 2 Subtract AC and simplify. Ask students to draw the multilateral triangle to help you restore the simplest shape of the gastors from the right triangle? The legs are at an angle of 45° and measure 1 unit, the hypotenuse is - opposite the right angle and √2 units. C - 3x 2 = BC 2 x33 = BC Reflect 3. - - x: x33 : 2x or 1:33:2. 4. Explain the error The student has drawn the correct triangle with a 60° angle and 6. He's marked the lengths on the other side, as shown. Explain how you can briefly say that he has made a mistake and how to correct it. - 633 >/; 6, the foot JK is longer than the hypotenuse, which is Side lengths must be ratio 1 : - For this to be true , the length of JK should be 3 module 18 -3 . √3 : 2. 3 13 9 L 6 L 60° 3 6√3 K EXPLORE 2 n Exploring the numerical ratio of side lengths in the right triangle to help you remember 30° and 60°? The simplest triangle is similar to △ABC (AA similarity), so the side lengths are the ratio of exploring another special right triangle questioning strategies How can you use the connection between angle dimensions and the length of the side opposite angle to help you reconstruct the simplest form of the 30° -60° -90° triangle? The shortest side is opposite 30 angle and the longest side is hypotenes. So mark the side opposite 30° angle 1 unit, side opposite - 60° angle √3 units, and hypotenuse 2 units. Lesson 3 PROFESSIONAL DEVELOPMENT IN2_MNLESE389847_U7M18L3 954 18/04/14 11:50 Integrate Mathematical Practices This lesson provides an opportunity to engage in the mathematical practice of MP.2, which invites students to cause in an abstract and quantitative way. Students examine the relationship between the lateral lengths and corners of special right triangles and use them to find trigonometric ratios and angle measurements associated with these relationships. Such recognition can often provide faster solutions to a particular triangle. Special Right Triangles 954 applying relationships with special right triangles Explain 1 AVOID COMMON ERRORS Correct triangles you have studied sometimes called 45° -45° -90° and 30°-60°-90° triangles. In the 45°-45°-90° triangle, the hypotenuse is twice as long as each leg. In the 30°-60°-90° triangle, the hypotenus is twice as long as the shorter leg and the longer leg is √3 times as long as the shorter leg. You can use these relationships to find side lengths in these special right-hand triangle types. - Students may be confused with the special right triangle, whose angle is less than 30° -60° -90° can bring angles of the side lengths of the standard 30° -60° -90° triangle. - Check with students why the √3 >/; 1 and why the side length or multiple must be at an angle of 60°. 30° 45° x√2 EXPLAIN 1 45° Application of relations in right triangles Example 1 ❓ QUESTIONING STRATEGIES √2 without radicals in denominator ( ) - √2 √2 1 _ - _ = . Remind students that this is called - 2 √2 √2 x Find △ABC Unknown Side length. A - -2 = BC -2 √2 times when each leg. AB = AC32 = BC32 Substitute 10 ab. 10 = AC√3 45° 10 √3 = - √3 = AC = BC 1032 = 2AC BC Sharing 2.ga. C 45° B. 10 √2 _ ∠DEF, m∠D = 30° and m∠E = 60°. Spent time measure 533 . Find the remaining side lengths. E © Houghton Mifflin Harcourt Publishing House 1 unit. Lengths _ = _ Learn how to write v3 Find unknown side lengths in each right triangle. Multiplied by 32 . ❓ legs are proportional to the right triangle with a hypotenuse one 60° √ _ If you know the exact value of the length of one side of the special right triangle, can you always find the exact value of the other parties? Explain. Yes, because of the special side-length connections that exist in each special right triangle, you can find the exact lengths of the other parties. Example 2 ❓ Find the unknown side lengths. Use the given length. DF E 2 = DF 2 EF Substitute 5 √3 DE = 2 5 √3 - EF . The longer leg is the denominator for √ 5 3. - 3 = EF √ 5- 3√- DE 1 Simplify. Multiply by √3 - times as long as the shorter leg. DF 3 3 DF = 15 Simplify. Module 18 DF of √3 955 Lesson 3 Collaborative learning IN2_MNLESE389847_U7M18L3 955 Small group activity 955 18/04/14 11:50 Do students work in small groups, to group and mark the right triangles with the following characteristics: • equine triangles with legs with integer length • the proportion of right triangles using lengths with an integer greater than 1 • 30° -60° -90° triangle as leg with hypotenus length greater than 2 lesson 18.3 ❓ Find the length of the hypotenuse or ❓ A 2 and 18.3 the 30° -60° -90° triangle which is the length √3 EXPLAIN 2 Find the unknown lateral length of the right triangle. L 5.6.30° ❓ Trigonometric relations are equivalent to opposite angles of ratio. ∠L is special triangle that is likely to reflect? Explain it. length of the hypotenuse side of the right triangle, to the right of the hypotenuse. If the trigonometric ratio contains √2 in relation to which special triangle is likely to reflect? Explain. √3 is a long one in a 30° -60° -90° triangle, in which it is a 30° -60° -90° triangle, how to find it, to which it is likely to reflect? Explain. a 30°-60°-90° triangle 4√3 JKL = 2JK · 3 = JK · 43 3 = JK · 4√3 43 JKL = JK K P damage, 33 √3 √3 PR = 2 √6 3 QUESTIONING STRATEGIES 45° 3 3 <5> PQ = PR -2 6) √-2 = 4 -3 3 23 JL 6 Explain 2 Investigate trigonometric ratios for right-hand triangles (23 3 You can use right relationships found in special right triangles to write the exact trigonometric relationship for the corners of 45°, 30°, and 60°. Example 2 ❓ side lengths are divided to find trigonometric relationships. Find the ratio of the special right triangle. Yes. Draw a special right triangle, and then find the trigonometric ratio. 45°-45°-90° triangle with leg length 1 Step - Because the sides are conjuged at an angle of 45°, they are both 1. The length of the √ is 2 times as long as each leg, so it is 1(32) or 32 - 4√3 EXPLAIN 2 Angle 45° Sine = opp _ hyp = √2 _ 2 Module 18 Cosine = 2 _ adj _ hyp = 2 3 956 Tangent √ integrate mathematical practices Focus on Mathematics Associations MP.1 Point to students that lateral lengths and trigonometric relationships for special right triangles are accurate. This means that they are often expressed as ratios in the simplest form, where appropriate without rationalized denominators. opp _ adj 1 Lesson 3 differentiated instruction IN2_MNLESE389847_U7M18L3 956 Kinesthetic experience 18/04/14 11:50 Kinesthetic learners may get closer to the mathematical relationships of the osceles right triangle through simple paper folding action. Invite students to take a rectangular piece of paper and fold it half-angularly. It creates an equal-edged triangle on the right. Then invite students to measure the triangle to one leg to the nearest millimeter and apply their understanding of the relationships and hypotenuse between the legs to predict the length of hypotenuse. Finally, let the students measure the length of hypotenuse to check their prediction. 956 lesson 18.3 Special right triangles 956 A 30°-60°-90° triangle shorter √a 1 2 √3 times as long as shorter leg. , 60° 1 Step 3 Angle 30° sine = opp _ hyp Cos = adj _ hyp Tan 60 Step 2 1 √3 2 √3 Write each relationship as a simplified fraction. Angle Sine = opp _ hyp Cosine = adj _ hyp Tangent = √3 _ 2 adj 3 _ 1 2 opp _ √3 2 Mirror 7. Write the pattern of ratios or relationships displayed in the tables in Part A and B as equations. Why do these patterns or relationships make sense? 33 3 2 1 sin 30° = cos 60° = _ ; sin 45° = cos 45° = _ ; sin 60° = cos 30° = _ ; angle sine 2 2 2 equals the complement of this supplement. © Houghton Mifflin Harcourt Publishing House 8. With an acute angle ρ measure, ρ, the tanp is less than 1? equals 1? greater than 1? tan ρ </; 1 if ρ </; 45; tan ρ = 1 if ρ = 45; tan ρ >/; 1 if ρ >/; 45. Reflect 7. Patterns or relationships make sense? 33 3 2 1 sin 30° ... Your turn Find the side lengths and trigonometric relationships to the 45 corners. 9. 10 CB __ . AB = -2 = 5 -2 ; AC = AB, AC = 5 - 2 10 sin 45° = 5 √2 _ = _ = √2 - 5 2 _ 2) (2 √2 _ ... √ 5 √2 √2 2 cos 45° = 5 √2 √2 √2 √2 Module 18 1 = 45° sin 45° 3 3 3 3 10 B 3 tan 45° = 1 3 3 Module 18 957 Lesson 3 Language Support IN2_MNLESE389847_U7M18L3 957 Connect Vocabulary Point That Makes Special Right Triangles have unique relationships between the parts. It may be easier for students to access for special triangles by referring to them by their own corner dimensions. Instead of discussing the equid is the right triangle, name the triangle a 45° -45° -90° triangle. 957 Lesson 18.3 18/04/14 11:50 Special Right Triangles 957 Explain 3 Examination pythagorean Triples EXPLAIN 3 Pythagorean Triples Pythagorean triplet is a set of positive integers a, b and c that correspond to equation a 2 + b 2 = c 2. This means that Pythagorean triplets can define the legs and hypotenuse in the right triangle. The special right triangles that you have just studied have a non-integer sides, do not form Pythagorean triples. B c A b Examples of Pythagorean Triples include 3, 4 and 5; Articles 5, 12 and 13; Articles 7, 24 and 25; and 8, 15 and 17. Example 3 ❓ Pythagorean triples C questioning strategies How can you understand the connection between the Pythagorean triple and its multiple will help you find the missing side in the right triangle? If you can detect triple and multiple, you can use this triplet to find missing half-lengths in the right triangle. Make sure that the side length is 3, 4 and 5; Articles 5, 12 and 13; Articles 7, 24 and 25; Pythagorean is a triple. 3 2 + 4 2 = 9 + 16 = 25 = 5 √5 2 = 12 2 + 144 = 169 = 13 2 √7 2 + 24 2 = 49 + 576 = 625 = 25 2 √ 8 2 + 15 2 = 64 + 225 = 289 = 289 = 2√ 2 2 Numbers in step A are not the only Triples of the Limit. In the next steps, you will discover that the triples of the known Pythagorean are also Pythagorean triples. ⊙ integrate mathematical practices in Focus on Patterns MP.8 Do students create a set of Pythagorean triples a b c with all triples a, b and c, having a common factor, then ask them to describe a pattern. Let students know that they are related to a scale factor. J D a F C b kc also E L kb © Houghton Mifflin Harcourt Publishing Company △DEF is similar to the △JKL Side Communication Similarity (SSS) Triangle Similarity Theorem, since the respective sides are proportional. Fill in the ratios to check the similarity of the side-communication (SSS) triangle. a : b : c = also : kb : _ kc What must be true that the numbers also, kb and kc? 2 2 (also) 2 + (kb) 2 = k a + k b 2 (2 2) = k 2a +b = k 2c 2 = triple. Students don't have to remember every triple on the list, but they can use them to help them recognize the original triple and its multiples. K (kc) 2 Set of numbers, kb and kc make up Pythagorean triple. Module 18 IN2_MNLESE389847_U7M18L3 958 958 Lesson 3 18/04/14 11:50 PM Special Right Triangles 958 TO DEVELOP THE 10TH EUROPEAN COMMISSION'S 10TH Let's say you're given a better triangle with two side lengths. What's true that you'd use pythagorean triple to find the rest of the side length? These side lengths should be two-digit Pythagorean triple. Also, if avoiding common mistakes in the lengths is given, half lengths should be smaller to two digits triple, students may confuse Pythagorean triple. Emphasize that students can always use the Pythagorean theorem to find the missing side lengths in the right triangle. If they see leg and hypotenuse are given, the lateral lengths can be triple, students will not have to remember that the triple or multiple triple is correct. whereas if one leg and hypotenuse are given, the length could be the unknown pool of length. 11. SUMMARIZE THE LESSON What can you say about the lateral and trigonometric ratios associated with special right triangles? Explain - the right triangle has a ratio of 1 - 1 to √2 . The sides of the 30° -60° -90° triangle have a ratio of 1 - √3 to 2. The related trigonometric relationships can be found by drawing triangles and picking the siphon, co-compatibility and tangent of the respective 30°, 45° and 60° corners in the triangles. R - 12th in △XYZ, hypotenuse XY is a length of 68, and _ shorter leg XZ is the length of 32. 32:68 = 8:17, so that the side lengths are 8, 15 and 17 times. 24 P 18 8 : 15 : 17 = 4(8) : 4(15) : 4(17) = 32:60:68, so YZ = 60. Q 18 : 24 = 3 : x, so that the side lengths are Repetitions Pythagorean triple 3, 4, 5. 3 : 4 : 5 = 6(3) : 6(4) : 6(5) = 18:24:30, so PR = 30. Please specify the 13th paragraph of The New Year's Describe the type of problems associated with special right triangles that you can solve. Use the side lengths of the 30° -60° -90° triangle, shorter leg, hypotenuse) they use the side lengths, applying relationships such as 30° -60° -90° triangle is twice as long as the shorter leg 14. How to use Pythagorean triples to solve the right triangles? Let's say the two sides of the right triangle are given and correspond to two numbers of the Pythagorean triple. If one of the given sides is hypotenuse and one of the lengths is three times the maximum number, then the unknown length is the remaining number in the triple. Also, if neither of the lengths is given, hypotenus, and they are two smaller numbers triple, the unknown side length must be the highest number triple. 15th Discussion How Much Pythagorean Triple Is There? Infinitely much; for example, because 3, 4 and 5 are Pythagorean triple, so is 2(3) = 6, 2(4) = 8 and 2(5) = 10; Articles 9, 12 and 15; Articles 12, 16 and 20; and so on. 16. Important question Check-In What is the ratio of the length of the hypotenuse length to the shorter leg of any 30° -60° -90° triangle? The ratio is 2 to 1. Module 18 IN2_MNLESE389847_U7M18L3 959 959 Lesson 18.3 959 Lesson 3 18/04/14 11:50 RATE: Homework and Interesting ASSESS • Online Homework • Tips and Help • Extra Practice Exercises Find the length of the sides of each right triangle. 1. Q. 6 7. 5 3 6. A L 18 B 9 C - 18 = AB √2 = BC 18 √2 - 18 √2 = BC 2 = 2AB = 2BC 9 2 = √2 = BC √2 9 = BC √2 Module 18 Execution IN2_MNLESE389847_U7M18L3 960 30° P © Houghton Mifflin Harcourt Publishing Company 45° JK = 12 √2 = JL = 3 √3 = JK 9 √3 = 33JK J KL = 2JK √3 (√3) KL = 6 √3 - Practice Explore 1 Investigate Of Isosceles Right Triangle Exercises 1-4 Explore 2 Exploring Another Special Right Triangle Exercises 5-8 Example 1 Applying Relationships Special Right Triangles Exercises 9-12 Example 2 Trigonometric Ratios of Special Right Triangles Exercises 13-16 Example 3 Investigation Pythagorean Triples Study of Exercises 17-20 INTEGRATE TECHNOLOGY Although students should write answers in precise form for special-right-hand triangles , they can use a calculator to check if their answers are reasonable. Lesson 3 960 Depth of Knowledge (D.O.K.) 60° Concepts and Skills COMMON CORE Mathematical Practices 1-20 2 Skills/Concepts MP.4 Modelling 21 2 Skills/Concepts MP.4 Modelling 22-20 24 2 Strategic Thinking MP.3 Logic 26 3 Strategic Thinking MP.3 Logic 18/04/14 11:50 Special Right Triangles 960 To Avoid Common Mistakes 7. Students working with special right triangles can misinterpret charts if the side lengths for these triangles are out of the standard reference triangles. Remind students that they may refer to these triangles, but they must identify the appropriate relationship between each triangle based on their respective problem phrase. Uvw has sharp angles U with dimensions of 30° and W of 60°. - Hypotenus UW measures 12. (You may want to draw a triangle in your answer.) 30° Q U 45° P 12 = 2VW 6 = VW Do students build multiple triangles with legs measuring Pythagorean triple legs using centimeter graph paper. Then let them use the ruler to check the length of the hypotenus corresponding to the three-three-month-long period of pythagorean triple. 45° - 5 √2 20 = 10 √ 5 R 5√10 = PR √2 (= 5 √10) √2 UV VW √3 UV = 6 √ 3 UV = 2VW MANIPULATIVENES V Use trigonometric relationships to resolve any better triangle. 9. 10. D A 14 45 ° C AC _ AB - √2 AC _ _ 14 2 7 √2 _ 45 ° P © Houghton Mifflin Harcourt Publishing Company E B 10√3 _ _ BC _ AB √2 BC _ = _ 14 2 7 √- 25 ° BC = 14 √2 C - - - _ cos 60° = DE DF = DE 60° = √2 DF EF EF √3 DF DE 1 = 2 2 10 √3 10 √3 15 = DF 5 √3 = 2 DE 45° sin = 45° = 12 √2 = 11. Right △JKLM with m∠J = 45°. 12. - VR - √2 KT RT PR, leg QR = 15 √2 45° 15 √2 15 √2 VJ 4 √3 K P KL _ tan 45° = _ JK = 4√3 4 √3 VJ √2 <6>- 2 = 4 √6 JK = 4 √3 LK √ 5 45° PQR 15 √2 = PR √ 15 √6 JK √2 + ... √ 6√6 Special Right Triangles 10 - 10 MP.4 Relying on Tools Explain to students that an answer can be written in different ways. For example, an answer can be written as 13 or 4 √13 . Lesson 18.3 Right triangle PQR is P and Q _ acute angles _ measurement 45°. Pajad meatures 5√10. (You may want to draw a triangle in your answer.) 10 √2 45° P 10 ° P 961 8.961 PR _ QR √ 3 _ PR _ 15 3 5 √3 = PQ 2 PQ = 2(5 √3) = 10 √- 3 tan 30° = 3 sin 30° = PR 5 √3 Module 18 IN2_MNLESE389847_U7M18L3 961 Module 18 IN2_MNLESE389847_U7M18L3 961 Lesson 3 18/04/14 COGNITIVE STRATEGIES Discuss the importance of being able to draw and interpret the correct triangular charts to solve real-world problems. B 25 A 60 B C 25:60 = 5:12, with side lengths 5, 12 and 13 times. J 5 : 12 : 13 = (5)(5) : 5(12) : 5(13) = 25:60:65 , so AB = 65. L 4 Not possible: 8 - 4 = 64 - 16 = 48, √48 is not the perfect square. 2. 23. On △PQR, legs are pq = 9 and QR = 21. How long is the hypotenuse? 7, 7(3) = 21. 21 √130 = 3 √ ... 3.4:5 = 7(3) : 7(4) : 7(5) = 21:28:35, so PR = 35 . 3√ ... ≈ 22.9 or 5 Module 18 IN2_MNLESE389847_U7M18L3 962 Special Right Triangles 962 21. Roof Is there 51? 522 is not the perfect square. _ 16. On the right △XYZ, the hypotenuse XY is a length of 35, and the shorter leg YZ is a length of 21. 21:35 = 3:5, so that the side lengths are 3, 4 and 5 times. 3:4:5 = 7(3) : 7(4) : 7(5) = 21:28:35. © Houghton Mifflin Harcourt Publishing Company • Image Credits: ©Exactostock/Superstock C DEFG is a diamond, so its diagonals despise each other and EH =HG. Since EH + HG = EG = 12 (Seg. Additional post.), 2EH = 12 and hence EH = 6. By Pythagorean Thm., x 2 + 62 = 12 2 x 2 + 36 = 144 x 2 = 108 x = √108 ≈ 6 √3 . —— — 18. To represent the Real-World Problems baseball diamond actually forms a square, on both sides measuring 30 meters. • Extra Practice For Each Triangle, indicate whether indicated side lengths are possible. Explain why or why not. 1. 2. 3√3 45 √6 3 √6 3 √3 45 √6 √6 √6 √6 3 √6 · 4 √√ 4 √√ 4 √√ √6 √ 4 √√ = √ ... — — 18. To represent the Real-World Problems baseball diamond actually forms a square, on both sides measuring 30 meters. How far, closer to the yard, does the third baseman throw the ball to get to first base? The line from the first bottom to the third forms a square diagonal, so that with a home plate they form a 45°-45°-90° triangle with feet 30 yd. Therefore, the third baseman must throw the ball 30 √2 yd ≈ 42 yd. - Module 18

IN2_MNLESE389847_U7M18L3 962 962 Lesson 3 18/04/14 11:49 special right triangles 962 _ 19. Right Triangle the leg is exactly √3 times the length of the leg. The reversengtangence trigonometric ratio shall be demonstrated to show that the averse angles are 30° and 60°. BC, use the reverse tangent ratio: Considering: BC = AC √3 . Because tan A = B AC √ AC 3 BC √ tanA = = = 3 AC AC m∠A = tan -1 √3 = 60° _ _ _ - A _ ∠A and ∠B additional, so that m∠B = 90° - m∠A = 90° - 60° - 60° C Algebra Find x value in each right triangle. 20. 21. C 5x - 6 B x√3 L x x J A - Since the longer leg is √3 times shorter than the length of the leg, △ABC is a 30°-60°-90° triangle. Therefore (3x - 25)√2 x√2 K - Since the hyponeuse is √2 times longer than one leg, △JKL is a 45°-45°-90° triangle. Therefore , _ _ BC = 2AC JK = KL x √2 = (3x - 25) √2 x = 2(3x - 25) 5x - 6 = 2x 3x = 6 x = 2 = x = 6x - 50 50 = 5 x © Houghton Mifflin Harcourt Publishing Company 10 x sides right triangle 22. Explain the error Charlene tries to find unknown _ with 30° acute angle, with hypotensive measures 12√2 . Identify, explain, and correct Charlene's error. 12√2 P Module 18 IN2_MNLESE389847_U7M18L3 963 Lesson 18.3 12 R 6√2 - Charlene appears to have used a ratio of 1:√2 :2, while the correct ratio of the 30°-60°-90° triangle is 1:√3:2. PR = 6 √2 is correct, but QR = PR √3 = (6 √2) √3 = 6 √6 . 963 Q 30° - - 963 lesson 3 18/04/14 11:49 23. Represent the Real-World Problems Honeycomb blinds form a string of almost regular hexagons when viewed end-on. About how much material, the nearest ten square centimeters, is needed for each 3.2 cm deep chamber honeycomb blind, which is 125 centimeters wide? (Hint: Draw a picture. The normal hexagon can be divided into six split flasks.) ... 3.2 cm ... 125 cm B 3,2 cm A C _ 1 △ABC is a 30°-60°-90° triangle and BC = (3,2 cm) = 1,6 cm. Therefore, 2 bC cos 30° = AB √3 1,6 cm = 2 AB AB = 3,2 √3 ≈ 5,542 cm - _ _ The amount of material needed is hexagon × width of the blind: 24. Which of these numbers are two of the three half length of the right triangle with an integer value? (Hint: For positive integers a, b, c and k, also, kb and kc, the side lengths of the right triangle are only if a, b and c are the side lengths of the right triangle.) A. 15, 18 True False B. 15, 30 True False C. 15, 51 True False D. 16, 20 True False E. 16, 24 True False A. Vale; 15 = 3(5) and 18 = 3(6); 5 2 + 6 2 = 25 + 36 = 61 and 6 2 - 5 2 = 36 - 25 = 11, and neither is the perfect square. B. false; 15 = 15(1) and 30 = 15(2); 1 2 + 2 2 = 5 and 2 2 - 1 2 = 3, and also there is no perfect square. © Houghton Mifflin Harcourt Publishing Company • Image items: ©Exactostock/Superstock 6(AB) × width = 6(5,542) × 125 ≈ 4160 cm 2 C. True: 15 = 3(5) and 39 = 3(13); 5, 12, 13 in Pythagorean D. True; 16 = 4(4) and 20 = 4(5); 3, 4, 5 is pythagorean triple. E. False; 16 = 8(2) and 24 = 8(3); 2 2 + 3 2 = 13 and 3 2 - 2 2 = 5, and also there is no perfect square. Module 18 IN2_MNLESE389847_U7M18L3 964 964 lesson 3 18/04/14 11:49 special right-hand triangles 964 PEER Mathematical ideas Transmission Are the three sides of the right triangle odd integers? Explain. Invite students to use special right-hand triangles to form several inverse trigonometric problems until 1 January 2015. Ask them to exchange and evaluate, for example, sin -1 _ 2 to solve problems with another pair of students. Invite students to use a calculator to check their work. No, no, if the two shorter side lengths are odd, their squares are odd because the odd number square is always odd. However, the sum of these squares is even because the sum of the two odd numbers is always even. Therefore, the sum of the squares of the two shorter side length squares cannot itself be an odd number check box. 26. Make a guess Use spreadsheet software to explore this question: are there any sets of positive integers a, b and c such as 3 + b 3 = c 3? You can start with these formulas: JOURNAL Invite students to summa the same information they know about special right-wing triangles. Remind them to include a chart that sums up the trigonometric relationships for the corresponding angles and charts that support these ratios. © Houghton Mifflin Harcourt Publishing Company Actually does not have a set of positive integers a, b and c so that 3 + b 3 = c 3, but it is really very difficult to prove. Students should be able to expand the spreadsheet example by experimenting with triple threes with different smallest numbers, changing the value in cell A1; for example: module 18 IN2_MNLESE389847_U7M18L3 965 965 lesson 18.3 965 lesson 3 18/04/14 11:49 PM Lesson Performance Task AVOID COMMON MISTAKES Kate and her dog are longtime flying disc players. Kate has decided to start a small business making circles of soft material that dogs can catch without injuring their teeth. Since he also likes mathematics, he has decided to see if he can pythagorean the principles of his designs. He used pythagorean triple 3-4-5 dimensions in his first three designs. When students check three measurements to see if they can represent the sides of the right triangle, they can add two smaller measurements before growing them to see if the result is a square of the third measurement. Before you can find the amount, students must be told each smaller measurement. r = 3 small r = 4 in medium r = 5 in. Integrate mathematical practices focus on critical thinking mp.3 If you look at the list of Pythagorean Triples, great 1. Is it true that (small area) + (average area) = (large area)? Explain. 2. If the circles were pythagorean triple 5-12-13 based radii, be true above the equation above? Explain. You will notice that at least one number makes up a triple is even. Does it have to apply to all Pythagorean triples? Explain. - yes, I don't know. Sample Reply: The Odd Number check box is always odd. The sum of the two odd numbers is always flat. So, if a and b are both odd, 2 + b 2 must be uniform, which means that c must also be uniform. 3. The three Kate circles have a, (b) and (c) radius where a, b and c form Pythagorean triplei (2 + b 2 = c 2). Indicate that the sum of the areas of small and medium circles is equal to the area of the large circle. 5. Explain the difference in your results in Exercises 3 and 4.1. yes; small area + average area = 9 in 2 + 16 in 2 = 25 in 2 = large area 2. yes, I don't know. small area + average area = 25 in 2 + 144 in 2 = 169 in 2 = large area 3. small area + average area = a 2 + b 2 =(a 2 + b 2) = c 2 = large area 1 1 2 4. No, no small volume + average volume = 36 in 3 + 85 in 3 = 121 in 3 ≠ 166 in 3 3 3 3 3 (large area) _ _ 5. Possible answer: The figures are squared in both the Pythagorean theorem and the formula area circle. In the sphere volume formula, the numbers are cubed. There is no theorem analogous to Pythagorean theorem cube numbers. Module 18 © Houghton Mifflin Harcourt Publishing Company • Image Credit: ©dlewis33/iStockPhoto.com 4. Kate decided to go to the beach ball business. Sticking to his Pythagorean principles, he starts with three spherical beach balls with a small ball with a radius of 3 in., the middle ball with a radius of 4 in., and a big ball with a radius of 5 In Is it true that (small volume) + (medium volume) = (large volume)? Show me your work. Hour 3 966 EXTENSION ACTIVITY IN2_MNLESE389847_U7M18L3 966 Select two integers m and n, so m <n. Make a table as follows at least 10 pairs of values m and n: M n n2 - m2 2mn n2 + m2 Pythagorean Triple? 1 2 3 4 5 Yes Describe your results and use algebra to explain them. m and n always create pythagorean triple. (n 2 - m 2) 2 + (2mn) 2 = n 4 - 2n 2m 2 + m 4 + 4m 2n 2 = n 4 + 2n 2 m 2 + m 4 2 = (n 2 + m 2) 18/04/14 11:49 Scoring Rubric 2 points: The student's response is accurate and complete execution of the task or tasks. 1 point: The student's response contains the characteristics of the appropriate response, but is invalid. 0 points: The student's response does not include the characteristics of the relevant response. Special Right Triangles 966 LESSON 18.4 Name Problem Solving Trigonometry Class Date 18.4 Trigonometry Problem Solving Important Question: How Can You Solve The Right Triangle? Source Locker Common Core Math Standards Student is expected: COMMON CORE Explore G-SRT. C.8 Use trigonometric relationships and Pythagorean theorem to solve the right triangles to solve implementation problems. So is G-SRT. D.9(+), G-GPE. B.7 _ _ Let's say you draw a height AD on the side of BC Then write the equation using the trigonometric ratio ∠C, height h △ABC and the length of one of its sides. Mathematical practice COMMON CORE By inferating the area Formula you can use trigonometry to find the area of the triangle without knowing its height. b Language goal C Explain to the partner how to solve the right triangle and how to solve the correct triangle at coordinate level. sin C = DEAL B Important question: How can you solve the right triangle? C b c C C B a h a D c B AD _ h _ = AC b Resolve your equation from Step A h.b sin C = h Fill this formula in area △ABC in terms of h and 1 ah second its lateral lengths: Area = _ 2 © Houghton Mifflin Harcourt Publishing Company You can use a trigonometric ratio to find lateral lengths or their inverse ratios to find angular measures; Pythagorean theorem allows you to find the third side length; you can use the fact that sharp corners complement; If the triangle is at coordinate level, you can use a distance formula to find the side lengths. MP.2 Justification of your expression h from Step B to your formula step C. D Substitute _1 ab sin C 2 Reflective 1. Does the found area formula work if ∠C is the correct angle? Explain. 1 Yes; in this case sin C = sin 90° = 1, so the formula becomes Area = __ab, and it is correct 2, because a and b are now the base and height △ABC. PREVIEW: LESSON PERFORMANCE TASK View the Include online section. Discuss the photo. Ask students if they know why the special procedure for water is called H 2O. Then, check the hour performance task. Module 18 must be EDIT--Chan NO Key=NL-A; CA-Correction Lesson 4 967 gh File info made throu Date class ng blem Solvi 18.4 Pro Trigonometry named IN2_MNLESE389847_U7M18L4 967 HARDCOVER PAGES 967,980 Resource Locker le? es correct striang right triangl you solved m solve how can orean Theore Question: Pythag Essential relationships and trigonometric D.9(+), G-GPE. B.7 COMMON G-SRT. C.8 Use CORE ms. Also G-SRT. Proble formula used. The area of its height fire t knowing le withou triang Discover a using area to equalize to find your sides. △ABC. Then write trigonometr one You can use _ length bc on , and e AD side h △ABC draw bottomed ∠C, height Let's say you terminic ratio trigonometr A h b ⚲ B C Watch for hard cover student edition page numbers for this lesson. c B c b a C D a _ _ AD = h b sin C = AC ◑ step from the equation Resolve your h b sin C = A h. h and in terms of the area △ABC la its formu 1 ah _ Complete s: Area = 2 C. its side length la from Step second to step second your formu Step B into h from expression tute your y g Compan - Substi _1 ab sin C © Houghto n Mifflin Harcour t Publishin 2 __ n. this angle? Explai Area = 12 ab, and is right becomes work when ∠C formula la you = 1, so the area formu Is . C = sin for 90° △ABC and emission price Yes; in this base b is now from a and reflect 1. is the correct lesson 4 967 module 18 8L4 967 47_U7M1 ESE3898 IN2_MNL 967 lesson 18.4 19/04/14 12:02 AM 19/04/14 12:03 AM Let's say you used a trigonometric ratio ∠B, h and different sides. How would that change your findings? What does it tell you about the choice of party and attached angle? 2. Examine step C would be the same, but other steps would change: AD h =c sin B =AB c sin B=h 1 Area= ac sin B 2 You can select different sides and add an angle and derive a little different _ _ Infering Area Formula _ INTEGRATE INTO THE TECHNOLOGY FORMULA AREA, but in the same format. Explain 1 Students have the opportunity to study activities either in the book or on the Internet. Using the Area Formula to integrate the TECHNOLOGY Area Formula for the Triangle in terms of its lateral lengths area △ABC sides a, b and c can be found using the lengths of the two sides and the sine attached angle: Area = __12 bc sin A, Area = __12 ac sin B or Area = __12 ab sin C. Students are familiar with the trigonometric ratio of 0° to 90°. Ask them to graph y = sin x 0° to 180° using their graph calculators to see that the ratios are defined for angles greater (and smaller) than 90°. C a B b c A You can use any sub formula to find the triangle area, taking into account two side lengths and the size of the corner attached. Example 1 POLLING STRATEGIES Find each triangle area for the nearest tenth. ⚲ trigonometric relationships are defined by better triangles. Why does a region formula work for all types of triangles when it uses a sy sy messy relationship? The formula works because when height or height is plotted in the triangle area to apply a normal formula, a better triangle is created. 3,2 m 142° 4,7 m If the angle known ∠C. 1 ab sin C. Replace in The Formula Area = _ 2 Evaluate by rounding to the nearest tenth. © Houghton Mifflin Harcourt Publishing Company a = 3.2 m and b = 4,7 m Be known side lengths a and b.m ∠C = 142° 1 (3.2)(4.7)sin 142° Area = _ 2 Area ≈ 4,6 m 2 If you know the triangle and the lengths of the two sides, how to find the angle of the added? Find a reverse sine that is twice the size of a product on two sides. EXPLAIN 1 Module 18 968 Lesson 4 Using Area Formula PROFESSIONAL DEVELOPMENT IN2_MNLESE389847_U7M18L4 968 Integrate Mathematical Practices This lesson provides an opportunity to engage in mathematical practice in MP.2, which invites students to cause in an abstract and quantitative way. Students point to the triangle region formula by recognizing the connections that arise inside the triangle when building height. They apply this formula to a wide range of When students solve the right triangle, they need to identify the connections that can be used to find missing measures, and they can often choose which of the three reverse trigonometric relationships to apply. 19/04/14 12:03 AVOID COMMON MISTAKES Students may find it difficult to replace values in the sine area of the formula chart. Emphasize that both sides are both at the corner. Or the angle is attached to the angle formed by the sides of the triangle. Fix the trigonometry problem 968 in △DEF, DE = 9 in., DF = 13 in., and m∠D = 57°. B QUESTIONING STRATEGIES Draw △DEF and check that ∠D is an added angle. Why is the siuse area formula useful when there is already a formula for the triangle area? This provides another method to find the area of the triangle without having to build the height. E9 in. D 57° Why does the angle have to be an angle to use the sine region formula? The formula is derived by the construction of a better triangle with the height at the angle and hypotension at the corner to allow the sy sygas ratio. F 13 in . ( ) ( ) 1 (DE) DF sin D Area = _ 2 1 9 13 sin 57 ° Area = _ 2 Write the region formula according to △DEF. Replace area in formula. Region ≈ 49.1 inches 2 Rate, rounded to the nearest tenth. Your turn Find each triangle area for the next tenth. 3. CONNECT VOCABULARY 12 mm Remind students that the second word of the height of the triangle is its height. one version of the sine area formula to use a formula to find any triangle region. 15 mm Be known side lengths a and b. a = 12 mm and b = 15 mm Be the known angle ∠C. m ∠C = 34° 1 Surface area = (12)(15)sin 34° 2 Area ≈ 50,3 mm 2 1 Replace in the formula Area = _ ab sin C. 2 Evaluate, rounded to the nearest tenth. © Houghton Mifflin Harcourt Publishing Company integrate mathematical practices focus on modeling MP.4 Discuss why students need to remember only 34°4. - △PQR PQ = 3 cm, QR = 6 cm and m∠Q = 108°. R 6 cm P 108° Q 3 cm _1 (QR)sin Q 2 1 = _(3)(6)sin 108° Area = 2 ≈ 8.6 cm 2 Module 18 969 Lesson 4 COLLABORATIVE LEARNING IN2_MNLESE389847_U7M18L4 969 Small group activity Can students control, whether they can use different trigonometric relationships and different reverse trigonometric relationships to solve the right triangle. Invite students to work together to resolve the correct triangle. Each student will then be checked to see if the measures use different trigonometric and reverse trigonometric relationships. 969 Lesson 18.4 19/04/14 12:03 Explain 2 Solving the correct triangle EXPLAIN 2 Resolving the correct triangle means the dimensions of all the lengths of its sides and all its angles. To solve the correct triangle, you need to know two side length or one side length and a fierce angle size. Based on this information, select the dose of trigonometric ratios, reverse trigonometric and Pythagorean theorem to help you solve the right triangle. Correct triangle solving Shelf extends from wall to cross 7 in. You want to place 9-in. under the shelf as shown. To the nearest tenth inch, how far be attached to the wall of the brace? The nearest degree, what angle brace make the shelf and wall? STRATEGIES OF DOUBT What information is needed to solve the right triangle? Explain. You need the length of two sides or length on one side and one fierce angle of measure. You can find this information for the remaining corner dimensions and side lengths. Shelf C 7 in. Nine in. Wall A Btraks Find BC. Substitute 7 AC and 9 AB. 7 2 + BC 2 = 9 2 Find squares. 49+ BC 2 = 81 Make sure that students remember how to interpret characters that represent an inverse function. Remind them that the expression –1 in an expression, such as tan -1 __b, represents a corner that is __ab . This is not 1 . It should normally represent a mutual ____ a tan __ b to be clear from context, whether -1 is a reverse function or a negative exponent. () EC 2 = 32 Subtract 49 from both sides. BC ≈ 5.7 Find square root and root. B AVOID COMMON ERRORS AC 2 + BC 2 = AB 2 Use Pythagorean Theorem to find the length of the third side. () Find m∠A and m∠B. Use ∠ trigonometric ratio to find M∠A. You know the length of the adjacent side and the hypothesis, so use the relationship of co-compatibility. cos A = _ 9 Write the reverse-co-ratio. m∠A = cos -1 Evaluate the reverse-sided ratio and round. m∠A ≈ 39° © Houghton Mifflin Harcourt Publishing Company Write a combination of the ∠A. 7 ∠ A and ∠B complement each other. m∠ A Substitute 39 ° m∠ A . (_) 7 9 + m∠B = 90° 39 ° + m∠B ≈ 90° Subtract 39° from both sides. m∠B ≈ 51° Mirror 5. (_) Is it possible to find m∠B before you find m∠A? Explain. 77 ≈ 51°. - yes, I don't know. reverse sine ratio; sin B = , so m∠B = sin -1 9 9 Module 18 _ 970 Lesson 4 DIFFERENTIAL INSIFING, IN2_MNLESE389847_U7M18L4 970 Modeling 19/04/14 12:03 Do students draw a sharp triangle with two adjacent sides with integer dimensions. Ask them to use a protractor to measure the angle they have added. Then let them find the triangle area using the si nearly region formula. Repeat with a sniffing triangle. The hearing cues can be used by students to use mnemonic soh-cah-rooma to remember that the opposite foot is adjacent to the foot A = ___, cos A = ______ and tan A = ___. Hypotenuse hypotenuse of adjoining leg problems solving trigonometry 970 Your turn explain 3 solving right triangle coordinates plane 6. INTEGRATE mathematical practices Focus on modeling MP.4 Discuss how to identify legs and 7th right triangle hypotensive levels when right triangle legs not formed by horizontal and vertical line segments. Use the distance formula to find the length of each side. The longest side is hypotensive. 8. Use a trigonometric ratio to find distance EF. DE tan F = EF 33 tan 27° = EF 33 ≈ 65 m EF = tan 27° Use a different trigonometric ratio to find distance DF. DE sin F = DF 33 sin 27° = DF 33 ≈ 73 m DF = sin 27° Use the fact that the acute corners of the right triangle complement the find of m∠D. _ _ _ F Building D 33 m E Shadow _ _ m∠D + m∠F = 90° m∠D + 27° = 90° m∠D = 63° QUESTIONING STRATEGIES Explain 3 Correct triangle resolution on the grid You can use distance formula and trigonometric tools to resolve the correct triangles at the coordinate level. © Houghton Mifflin Harcourt Publishing Company How is it to solve the right triangle coordinate level different from the resolve marked right triangle? The reference plane does not show lateral lengths or angst. To find lateral lengths, you must use the distance formula and then the inverse of the trigonometric ratios to find the angles. 27° The building casts 33 m of shadow when the sun is vertical at an angle of 27°. How long is the building, to the nearest metre? How far is the end of the top and the shadow? What angle does a ray from the sun along the edge of the shadow make it to the ground? Example 3 ⚲ Solve each triangle. The ABC triangle has peaks A(-3, 3), B(-3, -1) and C(4, -1). The side lengths are up to the nearest century and the angle is measured to the nearest degree. Draws points A, B and C and draws △ABC. 5 Find lateral lengths: AB = 4, BC = 7 A _ Use a distance formula to find the ac length∠ ⊥ √ ≈ √. -5 B 0 C 5 Use the ratio of the rotary tangent to find ( ) () () AB = tan -1 _ 4 ≈ 30°. m∠C = tan -1 _ 7 BC ∠A and ∠C supplement, so that m∠C ≈ 90° - 30° = 60°. Module 18 971 -5 Lesson 4 LANGUAGE SUPPORT IN2_MNLESE389847_U7M18L4 971 Merge vocabulary Discuss the phrase resolved in the phrase birth. Compare the resolution of the triangle to the resolution of the equation. To use a non-mathematical meaning to solve, compare the solve triangle to solve the mystery. The detective is using clues to find the missing information. Similarly, solving the correct triangle means using the information and tools, such as trigonometric ratios, to find all the missing side lengths and angle dimensions. 971 lesson 18.4 19/04/14 12:03 B Triangle DEF are peaks D(-4, 3), E(3, 4) and F(0.0). The side lengths are up to the nearest century and the angle is measured to the nearest degree. Draws points D, E, and F and draws △DEF. 5 ∠F appears to be a right angle. To check, find the slope x -5 _ -4 4 0 - -4 __ = _ _; EF : 0 -3 3 -3 Find lengths with formula: -- -- √(3 - -4) + (4 - 3) = √ 50 = 5 √ 2 - DF = √(0 - -4) + (0 - 3) = √ 2 25 = 5 , '- -- EF = √(0 -3) + (0 - 4) = √ 25 = 5 2 2 0 F 5 -5 both m∠F = 90 °. DE = E D -3 _ 0 -3 3 DF: _ = _ = -_ ; 4 0 - -4 4 inclination y ≈ 7,07 , 2 2 2 ( ) Use the reverse sine ratio to find m∠D. ( ) 5 EF = sin -1 _ = m∠D = sin -1 _ 45 ° DE 5 √2 - ∠D and ∠ E supplement, so that m∠ E = 90° - 45 ° = 45 °. 9. How does this information determine which reverse trigonometric ratio should be used to determine the acute angst size? This is not – You can use an inverse trigonometric ratio if you use the correct two side lengths. However, if the two sides are vertical and horizontal, it is useful to use rotary tongs. Module 18 IN2_MNLESE389847_U7M18L4 972 972 © Houghton Mifflin Harcourt Publishing Company Reflective Lesson 4 19/19/14 12:03 Problem Solving Trigonometry 972 Your Turn to Work 10. Triangle JKL is the tops of J(3, 5), K(-3, 2) and L(5, 1). The side lengths are up to the nearest century and the angle is measured to the nearest degree. Draws J, K and L and draws △JKL. INTEGRATION OF MATHEMATICAL PRACTICES Focusing on MP modelling 4 Some students may find it difficult to choose ----%-- - - JL = √(5-3) + (1-5) = √ 20 = 2 √ 5 ≈ 4.47, ---- √ KL = √(-3 - 5) + (1 - 2) = 65 ≈ 8,06 JK = √(-3 - 3) + (2 - 5) = √45 = 3 √5 ≈ 6,71, 2 2 5 -5 2 _11. Would you use the specified region formula for the correct triangle in this lesson? Explain. Possible answer: No. You can use this formula, but because the right triangle has a base and height of 1 foot, it is easier to use the formula A=bh. 2 _ 12. Discussion How will the correct triangle be resolved when its peaks are at coordinate level? In general, you need to find all the side lengths and all angles, checking that one is © Houghton Mifflin Harcourt Publishing Company at the right angle. The distance formula is usually used to find lateral lengths rather than to find trigonometric ratios or Pythagorean theorem (although the distance formula depends on Pythagorean theorem). The acute corners are found in the same way in both cases. 13. Important question Check-In How to find unknown angle measures in the right triangle? First, you use two known side lengths to form a relationship, and then use a reverse trigonometric ratio to find one angle measure. Then subtract this measure from 90° to find another measure of the acute angle. Module 18 IN2_MNLESE389847_U7M18L4 973 lesson 18.4 2 Lx 0-5⋅. ∠J appears to be at right angle. Checks JK slope -3 2-5 1-5 1 -4 = ; JL inclination: = = -2; both m∠J = 90°. Jk slope: -6 -3 - 3 2 5-3 2 -1 JK -1 √45 Use a reverse-composition ratio to find m∠K = cos = cos ≈ 34°. KL √65 ∠J and ∠L complement each other, so that m∠L ≈ 90° - 34° = 66°. Students can skip parts when solving the right triangle. Remind students that three side lengths and three-angle dimensions are required. Always encourage students to draw a chart and mark it each time they determine the action. 973 J 2 2 AVOID COMMON MISTAKES How to solve the right triangle? Use Pythagorean theorem, trigonometric relationships, reverse trigonometric relationships and additional cornerings to find missing side lengths and corner dimensions in the right corner. y K Find lateral length using the distance formula: trigonometric or reverse trigonometric ratio to resolve the right triangle. Discuss with students how students choose a specific relationship. For example, some students may always prefer to use a reverse tangent ratio when both leg lengths are known to find a fierce angle measure. SUMMARIZE LESSON 5 973 Lesson 4 19/04/14 12:03 RATE: Homework and Practice Assessment • Online Homework • Tips and Help • Extra Practice Find every triangle area for the nearest decade. 1. △PQR PR = 23 mm, QR = 39 mm and m∠R = 163°. 2. 62° 4.1 cm Q 39 mm _ R 163° P 23 mm ASSIGNMENT GUIDE _ 1 1 Surface area = (PR)(QR)sin R = (23)(39) sin 163° ≈ 131,1 mm 2. 2 2 a = 4,1 cm, b = 3,2 cm and m∠C = 62°; 1 1 Area = ab sin C = (4.1)(3.2)sin 62° ≈ 5,8 cm 2 2 _ _ Resolve each right triangle. Round lengths to the nearest tenth and angles to the nearest degree. 3. AC 2 + BC 2 = AB 2 A AC 2 + (2.7) = (3.1) 2 3,1 cm 4. F 5. E -- (_) AC = √2,32 ≈ 1,5 cm 2,7 BC m∠A = sin -1 = sin -1 ≈ 61° 3,1 AB ∠A and ∠B additional, so that m∠B ≈ 90° - 61° = 29°. (_) ∠D and ∠F complement each other, so that m∠F = 90° - 56° = 34°. DE EF cos D = tan D = DE DF 26 EF cos 56° = tan 56° = 26 DF 26 26tan 56° = EF DF = ≈ 46,5 m cos56° 38,5 m ≈ EF _ Better △PQR with PQ ⊥ PR , QR = 47 mm, and m∠Q = 52° ∠Q and ∠R complement each other, so that m∠R = 90° - 52° = 38°. PQ PR sin P = cos P = QR QR PQ PR cos 52° = sin 52° = 47 47 R _ _ 47 mm 52° P Q 47 cos 52° = PQ 37.0 mm ≈ PR 28.9 mm ≈ PQ module 18 Exercise IN2_MNLESE389847_U7M18L4 974 _ _ 47 sin 52° = PR Explore Recalling Area Formula Exercise 16 Example 1 Using Area Formula Exercises 1-2 Example 2 Solving Right Triangle Exercises 3-5 Example 3 Solving Right Triangle Coordinates Airplane Exercises 6-8 Students May Forget to Add Units Solving Area Problems. Remind students that entities are an important part of the answer. REPORTING ON MATHEMATICS Encourage students to solve the right triangles to understand their knowledge trigonometric ratios and their reverse values. Lesson 4 974 Depth of Knowledge (D.O.K.) Practice of avoiding common errors _ _ _ © Houghton Mifflin Harcourt Publishing Company 56° 26 m D AC 2 = 2.32 B 2.7 cm C 2 Concepts and Skills COMMON CORE Mathematical Practices 1-8 1 Recall of information MP.4 Modeling 9-15 2 Skills/Concepts MP.4 Modeling 16 3 Strategic Thinking MP.3 Logic 17 3 Strategic Thinking MP.2 Recital 18 3 Strategic Thinking MP.4 Modeling 19 3 Strategic Thinking MP.3 Logic 19/04/14 12:03 Am Problem Solving Trigonometry 974 Solve each triangle. The side lengths are up to the nearest century and the angle is measured to the nearest degree. AVOID COMMON MISTAKES Students may have difficulty solving right triangles. They may find it easier when they first determine whether they are looking for a relationship or angle measure. If they are looking for a relationship, they should use sin, cos, or tan. If they are looking for an angle measure, they should use sin -1, cos -1 or tan -1. 6. Triangle ABC with peaks A(-4, 4), B(3, 4) and C(3, -2) 5 A Plot points A, B and C and draw △ABC. y⋅, so m∠B = 90° ⊥ BC AB B 5 C integrate mathematical practices Focus modeling MP.4 with greater accuracy, remind students to use 2 ( ) x 0 -- - AB = 7, BC = 6, AC = √(3 - 4) + (2 - 4) = √85 ≈ 9.2, 6B∠A = tan -1 _ = tan -1 _ tan -1 _ ≈ 41° 7 AB ∠A and ∠C supplement, so that m∠C ≈ 90° - 41° = 49°. 2 () -5 7. The JKL triangle with peaks J(-3, 1), K(1, 4) and L(6,5) K provided measurements instead of the values calculated in the later stages of the correct triangle settlement. 5 Draw points J, K, and L and draw △ JKL. y J x 0-5 ∠J appears to be right angle. For verification 3 4-1⋅: = ; JK 2 -1 inclination - (-3)⋅: -5 - 1 = -6 = - 2 ; both m∠J = 90°. JL 9 3 6 inclination - (-3) 5 ___-- 13 ≈ 3.6, JK = √(-1 - (3)) + (4 - 1) = √ ≈ JL = √(6 - (3)) + (-5 - 1) = √-1) =-- 130 ≈ 11.4 KL = √(6 - (-1)) + (5 - (-4)) = √-√ 4 JK ≈ 18° m∠L = sin (_) = sin _ KL √-- 130 2 2 2 -5 L 2 2 2 2 2 (© Houghton Mifflin Harcourt Publishing Company -1) -1 ∠K and ∠L additional, m∠K ≈ 90° - 18° = 72°. 8. Triangle PQR with peaks P(5, 5), Q(5, 3) and R(-4, -2) 5 y P Q x 0-5 5 Plot points J, K and L and draw △JKL. ∠Q appears to be a right angle. For inspection - 3-5 -2 1 = ; Inclination PQ: -5 - 5 5 -10 - -2 - 3 = -5 = -5 ; both m∠Q = 90°. PR slope: 1 -4 - (5) _--- 104 = 2 √≈ 26 ≈ 10.2, PQ = √(-5 - 5) + (3 - 5) = √-5' ≈ 5.1, QR = √(-4 - (-5)) + (-2 - 3) = √- -- 130 ≈ 11.4 PR = √(-4 -5) + (-5) 2 -5) = √-- 2 2 R 2 -5 2 ( ) 2 2() QR 1 ≈ 27° m∠P = tan -1 _ tan -1 _ 2 PR ∠P and so that m∠R ≈ 90° - 27° = 63°. Module 18 IN2_MNLESE389847_U7M18L4 975 975 lesson 18.4 975 lesson 4 19/04/14 12:03 AM 9. Surveying the plot is the shape of a triangle, as shown. Find the plot area to the nearest 100 square feet. _1 ab sin C 2 1 = _(142) (227) sin 128° ≈ 12700 yd 142 yd COLLABORATIVE LEARNING Let students build a right triangle and measure the length of the two sides. Ask them to look for the length of the third side and the dimensions of the acute angle without using Pythagorean theorem or protractor. Invite students to build another right triangle and measure one sharp corner and side. Let them find the size of the other sharp angle and the other two sides. Finally, let the students change triangles and control each other's work. Surface area = 128° 2 2 227 yd 10. The history of Drawbridge at the entrance to the ancient castle is lifted and lowered by a pair of chains. This number represents the drawbridge if it is flat. Find the height of the chain's suspension point, the nearest tenth of a metre, and the acute angles the chain will make with the wall and the drawbridge to the nearest degree. wall height: AB 2 + BC 2 = AC 2 C (3.2)2 + BC 2 = (5.0)2 10.24 + BC 2 = 25 5.0 m Chain A 3.2 m drawbridge B COMMUNICATE MATH BC 2 = 14.76 Wall Las students have explained each other, if they would use Pythagorean Theorem, sin, cos, or tan ratio, and sin -1, cos -1 or tan -1 ratio to resolve the correct triangle. BC ≈ 3,8 m 3,2 bC -1 _ m∠A = cos -1 _ ≈ 50° 5,0 AC ∠A and ∠C additional, so that m∠C ≈ 90° - 50° = 40°. ( ) ( ) 11. Construction For safety reasons, the angle of the horizontal ramp of the wheelchair ramp shall not exceed 3,5°. How high is the maximum height ramp length of 30 feet? What distance would that ramp cover be? Round to the nearest tenth of a foot. AB _ AC AB sin 3.5° = _ sin C = 30 ft BC _ AC BC, for 3.5° = _ 30 30 cos 3.5° = BC 1.8 ft ≈ AB 29.9 ft ≈ BC IN2_MNLESE389847_U7M18L4 976 C 30 30 sin 3,5° = AB module 18 3.5° © Houghton Mifflin Harcourt Publishing Company A B 976 lesson 4 19/04/14 12:03 p.m. Multi-Step Illustration shows an origami crane, as well as its construction phase. The area of each wing shows the shaded part of the drawing symmetrical to its vertical centreline. Use the information in the drawing to find the entire wing area of the crane to the nearest tenth of a square inch. 2.2 in the year. 45° F C E 4.0 in 3.7 year. D 22.5° Area of each wing = △ ABF + △ DBF area – △ DCE 1 region △ ABF = (2.2) (2.2) sin 45°; ≈ 1,711 in year 2 2 1 △ DBF area = (4,0) (4,0) sin 22,5°; ≈ 3,061 in 2 2 1 △ Area DCE = (3,7 years) (3.7 in.) 22,5°; ≈ 2.619 in. 2 2 Each wing area: 1,711 in 2 + 3,061 in 2 - 2,619 in 2 = In 2012 _ _ © Houghton Mifflin Harcourt Publishing Company • Picture units: ©Jules Kitano/iStockPhoto.com Surface area of both wings: 22,153 inches 2 ≈ 4.3 inches 2 13. In the △XYZ, the peaks are X(1, 4) and Y(2, -3). At peak Z there are positive integer coordinates and XZ = 5. Find the coordinates Z and △XYZ, giving accurate answers. y 5 X Z x 0 -5 Y -5 Based on 3-4-5 right triplets and horizontal and vertical shifts 5, z's possible coordinates (1, 9), (4) (8), (5) (7), (6,4) and (5,1) . Z(5, 1) is the only option that seems to create the right triangle. - 1 - 4 = -3 = - 3 ; YZ inclination :: 1 -(-3) = 4 ; both m∠Z = 90°. XZ inclination: 5-1 4 5 -2 3 4 _—-50 = 5 √-2 , XZ = √-1) + (1 -4) √ = 25 = 5, XY = √(2 - 1) + (3 - 4) = √- - 25 = 5 YZ = √(5 - 2) + (1 - (3)) = √- 2 2 2 2 2 2 2 - Since XZ ≅ YZ, △ XYZ is a 45°-45°-90° triangle. Therefore, m∠X = m∠Y = 45°. Module 18 IN2_MNLESE389847_U7M18L4 977 977 lesson 18.4 977 lesson 4 19/04/14 12:03 14. Criticism of Recital Shania and Pedro discuss whether it is always possible to solve the right triangle, given enough information, without using Pythagorean Theorem. Pedro says it's always possible, but Shania thinks that if two side lengths and no angle measures are given, Pythagorean Theorem is needed. Who's right and why? Pedro, do you hear this? Possible answer: if one side and one acute angle are given, trigonometric relationships may be used to find the other sides, and the additional angles give the other an acute angle. If two sides are given, a reverse trigonometric ratio may be used to find one acute angle and this angle, together with the given side, may be used with another trigonometric ratio to find the other side. The second acute angle complements as before. 15. Design The logo shown is symmetrical to one of its diagonals. You can find △ CAE to the nearest degree. (Hint: First find the angle △ABC, △CDE, or △AEF) Then find the area △CAE, without finding areas of other triangles. 3 BC 6 mm = tan -1 _ ≈ 26,565° m∠BAC = tan -1 __ A 6 AB ( ) () B ∠BAC ≅ ∠EAF, nii m∠EAF = m∠BAC ≈ 26,565° 3 mm m∠BAC + m∠CAE + m∠EAF = 90 ° C 26.565° + m∠CAE + 26.565° ≈ 90° 3 mm m∠CAE ≈ 90° - 2(26.565°) F ≈ 36.869° ≈ 37° AC 2 = AB 2 + BC 2 ∠AEC ≅ ∠ACE, nii m∠AEC = m∠ACE AC 2 = 6 2 + 3 2 = 45 m∠AEC + m∠ACE + m∠ m∠CAE = 90° 2(m∠AEC) + 36,869° ≈ 90° m∠AEC ≈ 27° m∠ace ≈ 27° D E — AC = √45 = 3 √5 1 Pindala = (AC)(AE) sin(∠CAE) 2 1 (√) (√) ≈_ 3 5 3 5 sin(36,869°) ≈ 13,5 mm 2 2 _ A 1 ac sin B Area valem: Area = _ 2 b Aga △ABC alus a ja kõrgus h = c sin (∠ABD) , 1 a(c) sin (∠ABD)). so Area = _ C a c h B D Equating these two expressions to the area, sin B = sin ∠ABD But since each obtuse ∠B, ∠B and ∠ABD are additional to this © Houghton Mifflin Harcourt Publishing Company 16. Use the ∠B area formula in the chart to show that if there is an additional sharp angle and a nave angle, their sine is equal. construction, it proves the result. Module 18 IN2_MNLESE389847_U7M18L4 978 978 Lesson 4 19/04/14 12:02 am Problem Solving Trigonometry 978 JOURNAL H.O.T. Focus on Higher Warfare On Do Students Create An Example That Shows How to Solve The Right Triangle. Students should specify known side or corner dimensions and explain how to find unknown side lengths and angle dimensions. 17. Convey mathematical ideas to the HL Congruence theorem provides that the correct triangles of ABC and DEF so that the & - ∠A and ∠D are at right angles, BC ≅ EF and AB ≅ DE, △ABC ≅ △DEF. E C A B Explain, without formal evidence, how to deal with the right triangle F with the given leg length or the given side length and sharp angle measure indicates that the triangles with both feet or the sides and angles must be congestmatic.⋅ ≅ diaalcept ≅ DE and AC. Resolving one of these correct triangles determines the expected AB D length of hypotenus in the same way, for example, by Pythagorean theorem, so that BC = EF and thus SSS ≅, △ABC ≅ △DEF. Let'∠ assume ∠B ≅ ∠E. The corresponding side lengths make it possible to calculate the unknown sides in the same way, using trigonometric relationships, so that all the corresponding side lengths are equal and therefore all the corresponding sides are consistent. Again, SSS ≅, △ABC ≅ △DEF. 18. Consistently solve problems find the perimeter and area △ABC, such as exact numbers. Then find the dimensions of all angles to the nearest degree. ——— 26, √(3 -(-2)) + (3 - 2) = √— ——-— — 2 AC = √(3 -(-2)) + (-3 -(-2)) = √50 = 5 √— 26 + 5 √— 2 perimeter: BC + AB + AC = 6 + √— 2 BC = 6, AB = 2 _— _ _ A x 2 _ — y B 2 0 -5 Alusega b = BC = 6, kõrgus h = 5, 1 1 nii ala = bh = (6)(5) = 15. 2 2 1 1 Pindala = ab sin C 15 = (5 √2) (√26) sin A 2 2 3 √13 15 = 1 (6)(5 √2)sin C = sin A 2 13 √2 = sin C √13 -1 3 2 sin = m∠A 13 45° = m∠C 56° ≈ m∠A _ © Houghton Mifflin Harcourt Publishing Company 5 — _ — —) (_ _— 5 C -5 — Nurga summa kolmnurga puhul: 56° + m∠B + 45° ≈ 180°; m∠B ≈ 79° 19. Analyze relationships by using two different formulas in the Find Triangle area and derive the expression sin2.' 1 1 Surface area = (x)(x)sin 2ρ Surface area = bh 2 2 1 1 = x 2 sin 2ρ = (2x sin ρ)(x cos ρ) 2 2 = x 2 sin ρ, 1 2 x sin 2ρ = x 2 sin ρ, → sin 2ρ = 2sin ρ 2 _ λ ρ x _ Module 18 IN2_MNLESE389847_U7M18L4 979 979 18.4 979 Lesson 4 19/04/14 12:02 AM LESSON PERFORMANCE TASK 95 .84 p1. Draw and label a triangle showing dimensions. 2. Find the triangle in square centimeters of time. Show me your work. H O 104.45° Students are probably familiar with metric prefixes deci-, centi and milli-, but may not know the next three smaller prefixes. They are: pm 0.84 95 Each molecule of water contains two atoms of hydrogen and one atom of oxygen. The figure shows how atoms are placed in a molecule's water, along with the incredibly accurate dimensions of the molecule that physicists have been able to identify. (1 pm = 1 picometer = 10 -12m) m CONNECT VOCABULARY micro- = 10 -6 = 0,000 001 H nano- = 10-9 = 0,000 000 001 pico- = 10 -12 = 0,000,000,000,001 3. The distance between hydrogen atoms is in centimeters. Explain your method. 1. 0.84 pm95 104.45° AVOID COMMON ERRORS 95.84 pm B 2. The result is 95.84 picometers that centimeters can trip students up. Here are the steps and reasons: C 95.84 pm = 95.84 × 10 -12 m AB = AC = 95.84 pm = 95.84 × 10 -12 m = 95.84 × 10 10 -10 cm = 9,584 × 10 -9 cm 1 area = AB(AC) sin A = (9,584 × 10 -9 cm)(9,584 × 10 -9 cm) sin 104.45° 2 2 1 = (9,584 × 10 -9 cm)(9.584 × 10 -9 cm)(0,9684) 2 ≈ 44,48 × 10 -18 cm 2 _ _ (Definition pm) = 95,84 × 10-10 cm (Multiplication 10 2 cm = 1 m.) ≈ 4,45 × 10 -17 cm 2 3. - -_ Possible answer: Mark the center of point D bc × ∠. (Please, 95.84 by 10 and multiply by 10-10-10) _ _-8 BC = 2 × 75.71 = 151,42 pm = 151,42 × 10-12 m = 151.42 × 10-10 cm = 1,51 × 10 cm Module 18 980 © Houghton Mifflin Harcourt Publishing Company In triangle ABD, BD sin 52,225 = AB BD 0.79 ≈ 95.84 75.71 ≈ BD Lesson 4 Extension Activity IN2_MNLESE389847_U7M18L4 980 This Lesson Performance Task Gives Students A Glimpse into a Complex Field of Molecular Geometry. While most of the topic lies beyond the diverse high school geometry students, there are many questions students can research and report at the basic level. Among them: • What is molecular geometry? • How was the molecular structure of water found? • What determines the angles between molecule atoms? • What holds the molecule together? 19/04/14 12:02 Scoring Rubric 2 Points: The student's response is accurate and complete execution of tasks or tasks. 1 point: The student's response contains the characteristics of the appropriate response, but is invalid. 0 points: The student's response does not include the characteristics of the relevant response. Solving problems with trigonometry 980 LESSON 18.5 Name Using Pythagorean Identity Essential Question: How can you use a given value for one of the trigonometric functions to calculate the values of other functions? Discover a Student's Wait: Proving Identity In the previous lesson, you learned that the coordinates of any point (x, y) located in a unit circle where the terminal beam at y intersects with the circle x = cos ρ and y = sin ρ, and y = sin ρ, which tan ρ = _ x . By combining F-TF.C.8 Prove pythagorean identity in the 2(ρ) + cos 2(ρ) = 1 and use it to find sin(ρ), cos(λ) or tan(ρ) given sin(ρ), cos(ρ) or tan(ρ) and angle quadrant. sin ρ these facts give identity tan ρ = ____ , which applies to all values where cos ρ ≠ 0. In the following explore, cos λ you will get another identity based on the Pythagorean theorem, which is why the identity is known as Pythagorean identity. Mathematical practices COMMON CORE Date 18.5 Pythagorean Identity Common Core Math Standards COMMON CORE Class ⚲ MP.1 Solving problems Solving problems The corner terminal side intersects the unit circle at the point (a, b) as shown. Write a and b for trigonometric functions that include ρ. a = Language goal b = Explain to the partner what pythagorean theorem and Pythagorean identity are and how they are related. cos ρ sin ρ y (a, b) 1 b ρ a Important question: How can one trigonometric function be used to calculate the values of other functions? The values of the siifier, coss up and tangent functions are combined with the identities sinρ sin2(ρ) + cos 2(ρ) = 1 and tanρ = . If you know the value of the koshitseadus id, you can use identities to solve for others by using the ρ terminal side quadrant to set the correct character. _PREVIEW: LESSON PERFORMANCE TASK ◑ Apply Pythagorean theorem to the right triangle chart. Note that if the trigonometric function is squared, the exhibitor is usually written immediately after the function name. For example, (sin大) = sin 20. Write pythagorean theorem. a + b2 = c 2 A, b and (c) substitutes. Square each expression. Module 18 2 (cos ρ) + (sin ρ) = 2 be ges must EDIT--Chan EI Key=NL-A; CA-A Correction 2 + 2 cos ρ = 2 sin ρ 2 1 1 Lesson 5 981 gh File information made throu Date Class n Identity thagorea Name Py 18.5 Using ic functions trigonometr one value Resource usage given other functions? (ρ), cos(ρ) or Locker can find it in the patch ion: How to 2 1 and use values 2 cos (ρ) = calculate y sin (ρ) + angle. orean identits quadrant Pythag (ρ) and F-TF. C.8 Prove (ρ), cos(ρ) or tan tity Iden sin where the tan (ρ) given agorean y unit ring _ ving Pyth . Combining, located in y =) x, Pro ρ x point (e, ρ, to tan Explore nates any ing Explor and y = sin coordi followingy = sinρ, ρ ≠ 0. I learned that cos ρ and cos = you , x is ρ, where our lesson in knowing cts round identity is previo of all values angle ρ interse is the reason why sin ρ , which is true ____ fast m, which ρ = cos ρ terminal orean theore identity tan Pythag gives and b y based on these facts. Write another identit b) as shown, you will derive a point (a, circle at identity ⚲ IN2_MNLESE389847_U7M18L5.indd. Discuss the photo and how light and shadows can help determine where to place the cell tower. Then, check the hour performance task. © Houghton Mifflin Harcourt Publishing Company ENGAGE x n Mifflin Harcour t Publishin ◑ x ometric as trigon m. Note that the function. For the diagra triangle after the name on the right immediately theorem lly written Pythagorean but is typical Apply d, expon 2 is a square 2 . 2 2 2 function c λ a +b = (sinρ) = sin 2, 2 em. Theor 2 1 Pythagorean Write sin ρ = cos ρ + a, replace © Houghto b a b and c. expression. Square each (2 cos ρ) (+ 2 sin ρ) = 1 Hour 5 981 Module 18 8L5.indd 47_U7M1 ESE3898 IN2_MNL 981 Lesson 18.5 981 19/04/14 12:24 19.04.14 12:26 Reflective 1. EXPLORE Identity is usually written with the sine function first. Write the identity in this way and explain why it is equivalent to step B. 2 2 The rewritten identity is sin ρ + cos ρ = 1 . This is the equivalent of the original identity, since proving pythagorean identity commutation property addition. 2. π. Confirm Pythagorean identity ρ = _ 3 √-3 π π 1 ; therefore sin _ = ___ and cos _ = 2 3 3 2 √- 3 2 2 2 _ 2 _ 2 _ 2 _ π π 1 sin ρ + cos ρ = sin + cos = ____ + _ 2 3 3 3 2 ( ) 3. INTEGRATE TECHNOLOGY ( ) = _34 + _14 = 14 = 1. Students have the opportunity to conduct activities in the book or on the internet. 2 3π . Confirm pythagorean identity ρ = _ 4 √-√-2 2 3π 3π and cos _ = -___ . Therefore, sin _ = ___ 2 2 4 4 ( ) (sin ρ + cos ρ = sin 2 2 Explain 1 2 ( ) ( ) 2 3π 3π _ + cos _ = 4 4 √2) (_ _ 2 2 CONNECT VOCABULARY (√2 _) _ + - 2 2 Are class expressions containing word expressions, such as identity theft, or student Identity card. Invite them to discuss the meaning of identity in each expression. Remind them that 0 is the identity element of the upgrade and 1 is the multiplication identity element. Then let them discuss the use of the word identity in the context set out in the lesson. 1 +_ 1 = 1. =_ 2 2 Finding the value of other trigonometric functions Considering the value of sin ρ or cos ρ, the identity I ρ + cos ρ = 1 can be written to express one trigonometric function in relation to another. As shown, Version 2 covers both positive and negative square roots. Specify which character to use based on whether it is known which quadrant is located on the side of the λ terminal. 2 2 Solve sin ρ Solve cos ρ sin2 ρ + cos2 ρ = 1 sin2 ρ = 1 - cos2ρ __ cos ρ = ± √1 - sin2τ © Houghton Mifflin Harcourt Publishing Company sin ρ = ± √1 - cos2ρ ⚲ How do you know that the hypotenus is equal to 1? Circle radius is 1. cos2 ρ = 1 - cos2大 __ Example 1 QUESTIONING STRATEGIES sin2 ρ + cos2 ρ = 1 Find the approximate value of each trigonometric function. π, find cos executioner. Considering that the sin = 0.766 where 0 < ρ < _ 2 __ Use identity to solve cosτ. cos ρ = ±√1 - sin2 ρ __ Replace sinρ. = ±√1 -(0.766) Use the calculator, then round. ≈ ±0.643 2 Is (cosm) the same as cos 2?? Is (the sin) the same as sin 2?? Yes, they have different ways of writing the same thing because the angle of the measure is not really squared. 2 2 If cos 20value is 0.36, what do you know sin 20 and sin? 2 sin ρ = 1 - 0,36 = 0,64, so sinρ = √0,64 = 0,8. - The terminal side of Λ is located in Sector I with > 0. So, ≈ .643. EXPLAIN 1 Module 18 982 Lesson 5 PROFESSIONAL DEVELOPMENT IN2_MNLESE389847_U7M18L5.indd 982 Integrate mathematical practices This lesson provides an opportunity to engage in the mathematical practice of MP.1, which invites students to make sense of problems and consistently solve them. Students analyse this information and limitations and plan to find a solution. The solution process can involve changing the identities that you create to use the information. As part of how to resolve this issue, students may review this information to make additional decisions about the solution. 19/04/14 12:26 Finding the value of other Trigonometric functions based on the value of sin or cos ρ questioning strategies, whether 1 - sin 20 is ever negative? Why or why not? No, no -1 ≤ sin≤ 1, so 0 ≤ sin 20 ≤ 1. Therefore, 1 - sin 20 can never be less than 0. Using Pythagorean Identity 982 B QUESTIONING STRATEGIES 3π Considering that cos ρ = -0.906 where π < ρ < ___ , find sin大. 2 Does it take to know the exact value of ρ to solve these problems? Why or why not? No, no Pythagorean identity allows you to find ρ co-compatibility, given the snature of ρ, unaware ρ own value. __ sin ρ = ±√1 - cos20 Use identity to solve sin. ___ √ (Replace cos ρ. = ± 1 - -0,906 Use the calculator, then rounded ≈ ± 0,423) 2 Λ terminal side located in sector III where sin ρ < 0. So, sin ρ ≈ -0.423 . Reflect Do you need to know the quall, where ρ stops solving these problems? Why or why not? - yes, I don't know. the answer mark depends on the quadrant where ρ ends. 4. π π assume that __ < < π 0 < < < __ in part A of this example. How does this affect the value of 22 sinid values? π < < π, it is located in sector II. Since cosine is negative as __ 2 sector II, cos ρ would be negative. 5. AVOID COMMON ERRORS 3π 3π Let's say ___ < < 2π π < < ___ instead of this example in Part B. How does 22 affect the value of sin? < ρ < 2π, then the λ terminal side is located in Quadrant IV. Sine is also negative If 3π 2 ___ - Students may mistakenly reason that because sin 2大 + cos 2τ = 1, sin大 + cos大 = √1 = 1. Use a special angle to show that this is not true, and correct erroneous reasoning by reminding students that √x 2 + y 2 ≠ x+y. Sector IV, so that the value of the blue doesn't change. 6.-— Explain how you would use the results in Part A of this example to determine the approximate tan λ value. Then find it. to determine the approximate value of sin ρ Tan, use identity tan λ = . cos ρ 0.766 ____ tan ρ = ≈ 1,191 _ 0.643 Your turn 7. 3π Considering that sinτ = -0.644 where π < ρ < ___ , find cos大. 2 _ © Houghton Mifflin Harcourt Publishing cos ρ = ±√1 - sin2大 ___ = ±√1 - (-0,644) 2 ≈ ±0,765 Since ρ is located in Sector III, where cos ρ < 0, cos ≈ -0.765. 8. π Considering that cos ρ = -0,994 where __ < ρ < π, find sin ρ. Then find the tan大. 2 __ sin ρ = ±√1 - cos2 ρ ___ 2 = ±√1 - (-0,994) = ±0,109 Since ρ is located in Sector II, where sin ρ > 0, sin ρ ≈ 0.109. 0,109 tan ρ = sinτ ≈ ___ ≈ -0,110 cos&IN2_MNLESE389847_U7M18L5.indd A Action Do students work in pairs. Invite couples to write three different identities that show the relationship between sy sy, co-ation, and tangy. Then let them check all three identities ρ = 30°, ρ = 45° and ρ = 60°. Let each couple compare their work to another couple's. 983 lesson 18.5 19/04/14 12:26 Explain 2 Finding the value of other trigonometric functions considering the value of the tani explain 2 sini If you multiply the identity tan大 = ____ by cos大, you will get an identity cos大tan大 = sin大 or cos大 sinρ sin大 = cos大 tan大 . Also, if you share on either side of the sin大 =cos大 tan大 by tan, you get an identity cos大 = ____ . You can use the first of these identities to find the angle and cosine, if you know the tangy. Example 2 ⚲ value of other trigonometric functions based on the Tan ρ value Find the approximate value of each trigonometric function. π, given that the ≈ -2,327 where __ < ρ < π, find values of sin and cos大. 2 First, write sin大 in terms of cosi. Use the identity sin大 = cos大 tan. QUESTIONING STRATEGIES SIN = ≈ -2,327cosλ Substitute the &e.p.e. Is there another way to start the problem apart from writing in terms of cossy? Yes, you can write cossy so sin. Now use the Pythagorean identity to find the cos. Then find the sin. sin2ρ = cos ρ = 1 2 Use Pythagorean identity. Substitute sin. (-2327 cosτ)2 + cos2m ≈ 1 How can you write Pythagorean identity cos大 written in both sin and tan? 5,415 cos2大 + cos2≈ 1 square. (_) = 1 6,415 cos2≈ 1 Combine similar terms. Solve cos2.5. cos2 ≈ .156 Resolve for cosm. Cosτ ≈ ±0,395 sin 2&sinρ tanρ INTEGRATE TECHNOLOGY Terminal side ρ located in Quadrant II, where cos大 < 0. As a result, ≈ cossy -0.395 and sinρ ≈ -2,327kosτ ≈ 0.919. ◑ Students can use the graph calculator to check whether the sum of the squares of values found for sin-and cos大 is 1. Note that because the values are approximate, the calculator may return an amount that is not exactly 1. 3π Considering this tani ≈ -4,366 where ___ < ≈ < ≈ 2, find the values of sin&cos大. 2 First write sini in terms of cos大. Use identity sin大 = cos大 tan. sin大 = cos大 tan© Houghton Mifflin Harcourt Publishing Company ≈ -4,366 cos大 Substitute value tan. Now use Pythagorean Identity to find cos大 . sin2τ + cos2τ = 1 (-4,366 cos大) + cos ρ ≈ 1 2 Replace sin大. 2 19.062 cos2大 + cos2≈ 1 Square. 0.062 cos2大 ≈ 1 Solve cos2m. cos2τ ≈ 0.050 Solve cosm. cos大 ≈ 0.223 terminal side ρ is located in Quadrant IV , where cos大 > 0. As a result, ≈ .223 and ≈ -4,366 cos大 ≈ -0.974 . Module 18 984 Lesson 5 Differentiation Instruction IN2_MNLESE389847_U7M18L5.indd 984 19/04/14 12:26 Multiple Representations Students May Benefit From The Concepts of This Lesson and The Concepts related to The Correct Triangle Trigonometry. For example, if students are given that the sins = 0.766 and 0 < ρ < __π2 , they can draw a right triangle in the first quadrant with an angle ρ on the opposite side of the wave foot on the x-axis. They can then use the definition of sin大 =__ hypotensive vertical leg of 766 units and hypotenuse of 1,000 units, and use the Pythagorean side of adjacent Theorem to find the length of the lost leg. Finally, the terms cosτ = _ the hypotensive side of oppositte can be used to find these values. Students can control their own and tan = ___ side adjacent results using methods of lesson. In doing so, students are likely to see a link between the two solutions. Using Pythagorean Identity 984 Reflect WORK 9. DOUBT DISCUSSION STRATEGIES In Part A of this example, when you multiplied the value of the tan given by cosi calculated value to find the blue value, was the product positive or negative? why this is the result you expect. The product was positive. This is expected because the blue should be a positive quadrant II. Let's say you know the sythest of the corner. Do you also need information about the quadrant where the angle ends to find the angle of co-compatibility? Explain. No, no sine features and angle tangtors have enough information to determine where the angle ends, and thus the characteristic of the co-compatibility. π , show that you can solve the sin and cos大 exactly 10. If tan = 1 where 0 < ρ < _ 2 using pythagorean identity. Why is that? If tan = 1, then sin大 = cos, so sin2大 + cos2 ρ = 1 becomes 2 sin2 ρ = 1, which gives _ √2 π result sin大 = cos大 = __ . This is due to ρ is a special angle __ . 4 2 Your Turn 3π , find the values of the sin and cos. 11. Considering this ≈ 3,454 where π < ρ < _ 2 Write sin大 in terms of cos大. sin大 = cosid ≈ 3,454 cos大 Solve for cos大 and sin大. Integrate mathematical practices Focus ≈ on critical thinking MPs.3 Challenge students use identity sin2大+ cos2= 1 (3,454 cos大)2 +cos2,1 11,930 cos2大 + cos 20 ≈ 1 12.930 cos2τ ≈ 1 lesson to write identity, divided, which defines cos2τ ≈ 0.0773 1 - cos ρ 1 or tan 2τ = ____ -1 in terms of cosm. tan 2ρ = _____ cos 2ρ cos 2s2 cos ≈ ±0,278 Since cosine is negative Quadrant III, cos大 = 0.278 and If you know the sine angle, how can you find the cosine and tanger angle? To find co-compatibility, you can use the identity I ρ + cos ρ = 1, replace the given value 2 2 sin ρ and solve cos λ. Then you can replace the sin ρ and cos ρ values with the identity of sinλ tan ρ = ____ to find tangent. Cosi sin≈ -3,454 cos大 ≈ -0.960. © Houghton Mifflin Harcourt Publishing Total LESSON Work 12. What conclusions can you draw if you are only given information that you want = −1? Possible answer: Since the tangent is negative, the terminal angle must be either in sector II, where the sine is positive and the cosine is negative, or in quadrant IV, where the sine is negative and the cosine is positive. Specifically, sin大 = cos , so sin 2大 + cos 2ρ = 1 - becomes 2 cos 2 = 1, so cosλ = ± ____ . Then sinτ = ____ = and cosτ = -____ or sin大 = -__ 2 2 2 - √ 2 and cos大 = ____ . 2 √2 Module 18 √2 985 √2 √2 Lesson 5 Language Support IN2_MNLESE389847_U7M18L5.indd 985 Communicate In Mathematics Let Students Work in Pairs. The first student explains what Pythagorean Teorem is when the second student takes notes. They change roles and repeat the procedure with Pythagorean's identity. Let them work together to describe the relationship between the two, as well as the similarities and differences. Encourage students images and symbols and oral descriptions when explaining to their partners. 985 lesson 18.5 19.04.2014 12:26 13. Discussion Explain how to find an angle in the toucher relationship is similar to the process of solving the linear equation in two variables. By solving a linear equation in two variables, you can use one equation to find the EVALUATE expression as another for one variable, then replace that expression in another equation to get an equation in one variable that you can solve for this variable and use it to find the value of another variable. If you find sysy and cosinity angle tangant relationships, you use the known tangent value and identity sin大=cos大 tan大 write the expression sin大 in terms of cos大, then replace this DETERMINATION GUIDE expression with another identity sin 2n + cos 2大 = 1 to get the equation that can be solved cos大, and then use the value cos大 to find the value of sin. 14th Important question check-In If you know only the sine or cosine angle and the cvadrand where the angle ends, how can you find other trigonometric relationship? If you know only sin, or cosτ, you will find another, replacing the known value of Pythagorean identity sin ρ + cos ρ = 1 and solving unknown value. Then 2 2 sinρ you will find tangens angle using the identity of the tan = ____ . In any case, use a cosyquada, which ends in the corner, to choose the right sign for the relationship. Evaluate: Homework and Practice • Online Homework • Tips and Help • Extra Practice Find the approximate value of each trigonometric function. 1. π Considering that sin = 0.515 where 0 < ρ < __ , find a waterfall ≈ > ≈ ± ± √ ± √. 2. 3π Considering that cosπ = 0,198 where ___ < ρ < 2π, find sin. 2 -—- -- sin大 = ± √1 - cos 2ρ = ± √1 - (0,198) ≈ ±0.980 2 Since ρ is located in Quadrant IV, where siniiv< 0, blue ≈ -0.980. 3. 3π Considering that blue = -0.447 where ___ < < < 2π, find kos大 ≈ > ≈ ± ± √ ± √. Exercise IN2_MNLESE389847_U7M18L5.indd 986 lesson 5,986 depth of knowledge (D.O.K.) COMMON CORE Mathematical practices 1-16 1 Recall of information MP.7 Using structure 17 1 Recall of information MP.6 Precision 18-19 1 Recall of information MP.7 Using structure 20 2 Skills/Concepts MP.4 Modelling 21 3 Strategic thinking MP.4 Modelling 22-23 2 Skills/concepts MP.3 Logic 24-25 3 Strategic thinking MP.3 Example of logic 1 Finding the value of other trigonometric functions sin ρ or cos ρ Exercises 1-8, 20 Example 2 Finding the value of other trigonometric functions based on the value of Tan ρ Exercises 9–17 21 Suggest that students draw a sketch of the angle described in the problem by placing the corner in the correct quadrant. Students can then write characters for each function of this quadrant so that they can remember to confirm the correct characters for calculated values. sin大 = ± √1 - cos 20 = ± √1 - (0.544) ≈ ±0.839 Module 18 Exercises 18-19 VISUAL CUES π Considering that cosλ = -0.544, where __ < < < < 2π, find sini. 2 -- Discover Pythagorean identity verification If you are given an angle, why do you need to know which corner of the klemmpol is located to determine the angle of the sine? Knowing the kvadrand allows you to set the character sigging. Since ρ is located in Quadrant IV, where cos大 > 0, cos大 ≈ 0.895. 4. Practice Questioning Strategies © Houghton Mifflin Harcourt Publishing Company 2 Concepts and Skills 19/04/14 12:26 AM Using Pythagorean Identity 986 5. AVOID COMMON ERRORS 3π Considering that sin大 = -0.908 where π < ρ < ___ , find a cos. 2 --- - cos大 = ± √1 - sin 20 = ± √1 - (0.908) ≈ ±0.419 2 Since ρ is located in Sector III, where kosλ > 0, cos大 ≈ -0.419. If the replacement transformation sind into Pythagorean identity tan大 = ___ cos大 6. some students may forget the substituted expression coefficient. Check out that π < < π, find cos大. Given this sin = 0.313 where __ 2 --- -- cos大 = ± √1 - sin 2τ = ± √1 - (-0,313) ≈ ±0.950 2 Since ρ is located in Sector II, where cos大 < 0, kossi≈ -0.950. sin 20 means (sin大) and same cosine, and encourage students to use brackets when making replacement. 2 7. π , find sini . Considering that cos大 = 0.678 where 0 < < __ ≈ ≈ ± ± √ ± √2 Considering that cos大 = -0.489 where π < < _+-- - sin大 =± √1 - cos 20= ± √1 - (-0.489) ≈ ±0,872 Encourage students to use their calculators to check their results by checking that this 2 kuna ρ is located in Sector III, where sinimastatic < 0, sins≈ -0.872 sin. angle ρ, tanτ = ___ Scroll to the approximate value of each trigonometric function. Cossy 9. __ < < π, find the values of sin and cos. Whereas tan大 ≈ -3,966 where π 2 sin大 = cosi tani ≈ -3,966 cos大 sin 2大 + cos 2τ = 1 (-3.9 2+2+cos 20 ≈ 1 © Houghton Mifflin Harcourt Publishing Company cos 260 ≈ 0.060 → cos大 ≈ ±0,245 Quad. II, cosine is neg. and sy sius is possible.so negative and the si sy syd is positive. ≈ .245 and sin-≈ -3,966 kos大 ≈ .972. 3π < ρ < 2π, find values sin&cos.. 10. Considering this tani ≈ -4,580 where ___ 2 sin大 = cos大 tan大 ≈ -4,580 cos大 sin 2大 + cos 2大 = 1 (-4 2 +cos 2大 ≈ 1 cos 20 ≈ 0.046 → cos大 ≈ ±0,214 Since cosinus is a positive Quadrant IV, cos大 ≈ .214 and sin-≈ -4,580 kos大 ≈ .980. Module 18 IN2_MNLESE389847_U7M18L5.indd 987 987 lesson 18.5 987 lesson 5 19/04/14 12:26 AM π 11. Given that the tans ≈ 7,549 where 0 < < __ , find the values of sin and cos大. 2 Write sin大 in terms of cos大. sin大 = cosid ≈ 7,549 cos大 Solve for sini and cos大. sin 20 + cos 20 = 1 (7,549 cos大)2 + cos 2n≈ 1 56,987 cos 2n / cos 26 ≈ 1 57,987 cos 26 ≈,987 cos 26,987 cos 26,987 cos 26,987 cos 2n cos 20 ≈ 0.017 cos大 ≈ ±0.130 Since the cosinus is positive quadrant I, cos大 ≈ -0.130 and sinsi≈ 7,549 cosima ≈ .981. 3π , find the values of the sin and cos. 12. Considering this ≈ 4,575, where π < < ___ 2 Write sin大 in terms of cos大. sin大 = cosid= ≈ 4,575 cos大 Solve for sin大 and cos大. sin 20 + cos 20 = 1 (4,575 cos)2 + cos 2&≈ 1 20,931 cos 2大+ cos 2大+ cos 2大 ≈ 1 21,931 cos 2n ≈1 cos 2 ≈2,046 kos大 ≈ ±0.214 Since the cosine is negative quadrant III, ≈ .214 and sin ≈ 4,575 kos大 ≈ -0.979. © Houghton Mifflin Harcourt Publishing Company 3π , < ρ < 2π find values of sin and cos. 13th Considering that the tan≈ -1.237 where ___ 2 Write sin大 in terms of cos大. sin大 = cosidi ≈ -1,237 cos大 Solve sini and cos大. sin 20 + cos 20 = 1 (-1.237 cos大)2 + cos 2n ≈ 1 1.530 cos 20 + cos2τ ≈ 1 2.530 cos 2.530 cos 20 ≈ 1 cos2 ≈ .395 cos大 ≈ ±0.628 Since the cosinus is negative Quadrant IV, cos大 ≈ .628 and sinsünd ≈ -1,237 cosüle≈ -0.777. Module 18 IN2_MNLESE389847_U7M18L5.indd 988 988 lesson 5 19/04/14 12:26 AM Pythagorean i identity use 988 3π 14. Given that the tan≈ 5.632 where π < ρ < ___ , find the values of sin大 and cos大. 2 Write sin大 in terms of cos大. sin大 = cosid ≈ 5,632 cos大 Solve for sini and cos大. sin 20 + cos 20 = 1 (5,632 cos大)2 + cos 2&≈ 1 31.719 cos 2τ + cos 2τ+ cos 2大 ≈ 1 32.719 cos 2&≈ 1 cos 1 cos ≈ 0.031 cos大 ≈ ±0.176 Since the combination is negative quadrant III, cos大 ≈ -0.176 and sin大 ≈ 5,632 kosa ≈ -0.991. π , find the values of sin and cos. 15. Given that the tan≈ 6,653 where 0 < ρ < __ 2 Write sin大 in terms of cos大. sin大 = cosid ≈ 6,653 cos大 Solve for sini and cos. sin 20 + cos 2015 = 1 (6,653 cos大)2 + cos 20 ≈ 1 44,262 cos 2τ + cos 2τ ≈ 1 45,262 cos 20 ≈ 1 cos2τ ≈ 2.022 kos大 ≈ ±0.148 © Houghton Mifflin Harcourt Publishing Since the cosinus is positive quadrant I, cosτ ≈ -0.148 and sin大 ≈6,653 cos大 ≈ .985. π , < < < π find values of sin and cos. 16. to tan≈ -9,366 where __ 2 Write sin大 in terms of cos. sin大 = cos大 = ≈ -9,366 cos大 Solve for sini and cos. sin 20+ cos 2015 = 1 (-9.366 cos大)2 + cos 20 ≈ 1 87.722 cos 2大+ cos 2大+ cos 2大 ≈ 1 88.722 cos 2n≈ 1 cos 2011 ≈ 0.011 cos大 ≈ ±0.105 Since the cosinus is negative quadrant II, cos大 ≈ -0.105 and sin大 ≈ -9.366 cosi ≈ .983. Module 18 IN2_MNLESE389847_U7M18L5.indd 989 989 lesson 18.5 989 lesson 5 19/04/14 12:26 17. Given the trigonometric function and the location of the terminal angle, indicate whether the function value is positive or negative. PEER,TO▌ TO▌ PEER DISCUSSION A. Positive A. cos大 , Quadrant I B. sin大 , Quadrant IV B. Negative C. tanρ , Quadrant II C. Negative D. sini , Quadrant III D. Negative E. tan, Quadrant III E. Positive Ask students to discuss with their partner how they could use what they know about trigonometry sinλ . the correct triangle to prove the identity of the tanτ = ___ cosτ Let them discuss their evidence in class. Students' work should show that because the side opposite the sins = _____ and the hypotenus side in these cos大 = __ , then hypotenuse 7π 18. Confirm Pythagorean identity sin 2τ + cos 2ρ = 1 for ρ = ___ . 4 √√ 2 2 7π 7π = -__ and cos __ = ____ . Therefore, the sius is __ 4 4 2 2 ( ) 7π 7π sin ρ + cos ρ = sin (__) + cos2(__)= 2 2 2 4 4 (-__-) + (____- = __ + __ = 1. √2 √2 2 2 1 2 1 2 side _____ hypotenuse sin大 = __ __ side of the suitor ___ hypotenus hypothesis side ____ __ __ = · 19. Recall that the source circle equation with a radius r is x 2 + y 2 = r 2. Use this fact and the fact that the coordinates of this circle point are (x, y) = (rcos大, r sinρ) for the central corner ρ, indicating that the Pythagorean identity derived above is true. 2 2 x 2 + y 2 = r 2 ⇒ (r cos大) + (rsin大) = r 2 r 2 cos 2 80 + r 2 sin 2λ = r 2 ⇒ 2ρ + sin 2ρ = 1 hypotenuse side opposite = side ____ 20. Sport Ski Company is testing the friction of new ski wax by placing a waxed block on a slant covered with wet snow. The cut plane is slowly raised until the block starts to slide. As soon as the block begins to slide, the mgsinρ component of the mass of the block parallel to the cutting line and the friction force (μmgcos大) are equal if the μ is the coefficient of friction. If sinρ = 0,139, as soon as the block starts to slide, the μ to the nearest century. Find the coss. Because cosy is between 0 and 90 degrees, cosτ is positive. = © Houghton Mifflin Harcourt Publishing Company • Photo credits: ©Aurora Photos/ √ √Alamy 1 - (0.139) 2 ≈ 0.928 mg sini = μ mg cos大 ⇒ sin = μ cos大 = μ cosi = μ cos大 0.139 0.139 μ = ≈ ≈ 0.15 0.928 cos大 21. Driving tires and roads are designed so that the coefficient of friction between rubber tires and asphalt on the road is very high, which gives a lot of traction to starting, stopping, and turning. For a particular road surface and tyre, the steepest angle in which a car could rest on the slope without starting to slide is 0.643. This value corresponds to the equation mgsinτ = μmgcosρ, where μ is the coefficient of friction. Find the μ value in the next century. Because cosy is between 0 and 90 degrees, cosτ is positive. _ _√≈ sided≈9=√1 - sin 2ρ = √1 - (0,643) ≈ 0,766 mg sin大 = μ mg cos 大 ⇒ sin大 = μ cos大 0,643 = μ cos大 ⇒ 0.643 0.6 43 ≈ ≈ 0.84 μ= 0.766 cosüü 2 _ _ Module 18 IN2_MNLESE389847_U7M18L5.indd 990 990 lesson 5 19/04/14 12:26 Pythagorean identity 990 H.O.T. Focus on higher order thinking JOURNAL 3π 22. Explain error Julian was given that sin = -0.555 where ___ , < ρ < 2π and 2 said to find cos.com. He did the following work: Ask students to explain how knowing that the angle inside is about 0.2588, allowing them to find two possible values for the co-compatibility of the corner. Let them find these values and tell them what information would be needed to decide which of the two possible values is the right angle. __ cos大 = ±√1 - sin 20 __ = ±√1 - (0,555 2) ≈ ±1,144 3π < ρ < 2π, cos大 ≈ 1,144. Since cos大 > 0 if _ 2 Explain your error and provide the correct answer. Julian added two collections inside the square root instead of subtracting. He should have noticed, because the number's combination is never more than 1. He should have done the following: - =± √1 - (-0.555) cos大 = ± √1 - sin 2≈ ±00.832 Because cosmic > 0 if 3π _ < < 2π, cos大 ≈ .832. 2 Critical thinking to write each trigonometric expression both cos大 and simplify. sin 23. _ 1 - cos大 sin 2大 1 - cos2ρ 1 - cos2ρ = 1 - cos大 (1- cos 大)(1+ cos大) = 1- cos大 _ _ © Houghton Mifflin Harcourt Publishing Company = 1 + kos大 24. cos大 + sin大 cos大 - tan大 + tan大 sin 20 (Hint: Start factoring tan大 from the last two terms.) cos大 + sin大 cos大 - tan大 + tan大 sin 2大 = cos大 + sin大 cos大 - tan大(1 - sin 26) = cos大 + sin大 cos大 - tan大 cos2大 = cos大 + sin大 cos大 - sin大 _ cos ρ 2 cos大 = cos大 + sin大 cos大 - sin大 cos大 __ = cos大 √ 1 - cos 2n __ 25. Critical thinking, what trigonometric function does _ √ 1 - sin 20 simplify? Explain your answer. --- -- The expression ___ simplifies the tan. If you re-type __ --- --- 2 2 √1 - cos 2τ √1 - sin ρ 9 -- √1 - cos 2大 √1 - sin λ using Identity, you get the result _______ . Taking the results of √ 2000 √cos ρ sin大 square root the expression ____ , which is the equivalent of tan. cos大 Module 18 IN2_MNLESE389847_U7M18L5.indd 991 991 Lesson 18.5 991 Lesson 5 19/19/14 12:25 lesson Performance Task integrate mathematical practices focus math connections MP.1 Students likely analyzed relationships with tower sheds shadow that is 160 feet long at a certain time at one morning. Since the base of the tower is origin, a positive x-axis in the east and a positive y axis in the north, the shadow is in the northwest at that time, formed by the axes formed. Also at this time, the tangant angle of rotation measured so that the shadow is located in the terminal ray is tan = -2,545. What are the shadow tip coordinates to the nearest foot and what do they show? between the length of the shadow and other dimensions (e.g. the height of the object) in two dimensions. Remind them that they have used indirect measurement and proportion and Pythagorean theorem to solve these problems. Note that the lesson performance task extends these analyses to enable all four quadrant positions using positive and negative coordinate positioning. The coordinates of the point on the circle centered with baseline r are r cos大 and r sin. Use the identities associated with the sine, the co-dy and the angle tangian to find the cosine and the sin, then use these values to find the coordinates. First, use the tangent and identity sins = cosτ tanτ value to find the sin's expression kosλ. CURRICULUM INTEGRATION SIN= cos大 tan = -2,545 cos大 Note that the counterclockwise movement of the sun's shadows takes place only in the southern hemisphere. Most sundials, which were first introduced in the northern hemisphere, assume that the shadows move clockwise, and this is also the reason for the development of clockwise rotating clocks! Tell students that the sun seems to move in the opposite direction from its shadows—the sun moves counterclockwise in the northern hemisphere, so that its shadows move in the opposite direction. Replace this expression with a sin-pythagorean identity, which means that the value of the match is negative. sin 20 + cos 20 = 1 (-2,545 cos大)2 + cos 2&≈ 1 6.477 cos 2τ + cos 20 ≈ 1 7.477 cos 2n ≈1 cos 2n≈ 0.1337 cos大 ≈ -0.366 Now the substitute find the sin大. sin大 = -2,545 cos大 ≈ -2,545(-0,366) ≈ 0.931 Use the fact that r = 160 to find the coordinates run out of shadow. r cos大 = 160(-0, 366) = -58.56 r sin大 = 160(0, 931) = 148.96 © Houghton Mifflin Harcourt Publishing Company • Picture authors: ©Peteri/ sin 26 + cos 2λ = 1 settlement cosτ. Since the shadow is northwest, it is Quadrant II, the coordinates are (-59,149). This shows that is about 15 meters west and 49 feet north of the tower. Module 18 992 Lesson 5 EXTENSION ACTIVITY IN2_MNLESE389847_U7M18L5.indd 992 Your position in relation to two cell towers can be determined by a process called triangulation. Whether students are studying how triangulation, use angles to find distance, also works in other activities, such as mapping. Students are exposed and may submit a law cosines that says that c 2 = 2 + b 2 - 2ab cos C. 19/04/14 12:25 Scoring Rubric 2 point: Student correctly solves the problem and explains his reasoning. 1 point: The student demonstrates a good understanding of the problem, but does not fully solve or explain his or her reasoning. 0 points: The student does not show an understanding of the problem. Using Pythagorean Identity 992 MODULE 18 MODULE STUDY GUIDE REVIEW Study Guide Review The basic question: How can you use trigonometry to correct triangles to solve real-world problems? KEY EXAMPLE OF EVALUATION AND INTERVENTION (lesson 18.1) C 60 Determination or adaptation of module reviews. 11 MODULE PERFORMANCE TASK 61 length leg opposite ∠A tanA = ___ length of leg next to ∠A 60 ≈ 5.45 tanA = _ 11 KEY example of mathematical practices: MP.1, MP.2, MP.3, MP.4, MP.7 G-SRT. C.8, G-MG. A.1 • How to find the total height of layers of divided pipes: Students need to understand that the centres of three timed tubes form an equilateral triangle. The triangular height is the height of three lines of redy pipes. Height can be found in several ways, including pythagorean theorem or trigonometry, or you can give an algorithm to find the height. • How to find the height n layers of divided tubes: Students must generalize the ratio to an altitude of 10 layers divided by tubes n layers. • When thickness tubes matter: Students can brainstorm options for both stack and re scheduled stack of pipes. The thickness adds the stack to the height, but adds a less timed stack to the height. 993 Module 18 © Houghton Mifflin Harcourt Publishing Company SUPPORTING STUDENT Reasoning Definition of Tangant Substitute and Simplification (Lesson 18.2) Find sine and cosine angle A. length of leg opposite ∠A sinA = ___ length of hypotenuse 60 _ sinA = ≈ 0.98 61 length leg next to ∠A cosA = _____ length of hypotenseen 11 _ cosA = ≈ .18 61 KEY EXAMPLE lateral (adyacente) opposite (opuesto) tangents (tangente) cosinus (coseno) reverse trigonometric ratio (proporción inversa trigonométrica) Pythagorean triple (terna pitagórica) Find angle a person A. COMMON CORE Key Vocabulary tan -1 A (tan -1 A) inverse tangent ∠A (tangente inversa de ∠A) trigonometric ratio (proporción trigonométrica) sycost (seno) B Students should start this problem, focusing on what information they need. Here are some questions they ask Lift it up. 18 Trigonometry with the correct triangles concept sycophony 11 C 60 A 61 Simplify. Definition of cosine B Simplify. (Lesson 18.3) _ Given the equine right triangle ∠DEF with ∠F = 90° and DE = 7, find the length of the other two halves. _ DE = DF√ 2 Apply a 45°-45°-90° triangle ratio. _ 7 = DF√ 2 7_ = DF _ √2 substitute. Simplify. _ 7√ 2 _ 7_ = DE = EF = 2 √2 Module 18 Application of the characteristics of the equilateral triangles. 993 Study Guide Review Scaffolding Support IN2_MNLESE389847_U7M18MC 993 • Recommend students to use a simpler problem-solving strategy to help them find the height of stacked pipes. Let them find the height of the two layers first, then find the height when the second layer is added until they recognize the pattern. • One way to determine the height of the two layers is to look at three tubes that form an equilateral triangle and determine the total height using students' knowledge of triangles. They can continue this process by adding more layers to form larger triangles. 19/04/14 12:34 EXERCISES Simple correct triangle △XYZ where ∠Z is right angle, XY = 53, YZ = 28 and XZ = 45, find the next rounded nearest hundredth. (Lessons 18.1, 18.2) 1. sinX 0,53 2. cosX 0,85 tanX 3. SAMPLE SOLUTION METHOD: 0,62 The centre of three timed tubes form an equilateral triangle. Locate the height expression of this triangle, which is two layers. Then add another layer to form an equilateral triangle using 6 tubes. Find the height expression of this triangle, which is three layers. Continue the process until a clear pattern occurs, then the n layers are generalized. Find the area of the next triangle, rounded to the nearest tenth. 4. Triangle △ABC, where C = 127°, AC = 5 and BC = 9 18,0 Locate the two sides of the next triangle. Find the exact answers that are the least of the greatest. (Lesson 18.3) _ 5. 7.7√ 3 3 30°-60°-90° triangle with hypotentus 14 The equilateral triangle height formed from the centre of three tubes is pythagorean theorem, 2 + b 2 = c 2, where a = 1 in., c = 2 in. MODULE PERFORMANCE TASK How much shorter are the timed pipe stacks? How much space can you save by stacking the pipe in a timed pattern? The illustration shows the difference between the layers stacked directly on top of each other (left) and the stacked layers of pipe with a timed pattern (right). Let's say you have pipes that are 2 inches in diameter. How much shorter is a 10-layer timed stack than a non-divided stack with the same number of layers? In general, how much shorter drives n layers of a shared tube? 2 in.b start by listing in the room below how you plan to solve the problem. Then use your paper to complete the task. Be sure to write down all your details and assumptions. Then use numbers, graphs, diagrams, or algebraic equations to explain You came to a conclusion. © Houghton Mifflin Harcourt Publishing 1 in Three tubes total height is the height of the triangle plus twice the radius of the pipes. - h 2 layers = 2 + √3 While continuing the calculations, we can: = 2 + 3 √- 3 h 3 layers = 2 + 2 √3 h 4 layers Or, generalization, module 13 994 Study Guide Review DISCUSSION OPPORTUNITIES IN2_MNLESE389847_U7M18MC 994 • What assumptions are made when the height of the stack pipes? • What could be the source of the error if you find the height of the stack of the pipe? - h n layers = 2 + (n - 1) √3 19/04/14 12:34 So the difference between heights of 10 layers divided versus non-timed is - 10(2) - ⎡⎣2 + (10 - 1) √3 ⎤⎦≈ 2,41 in. Evaluation rubric 2 points: The student correctly solves the problem and explains his reasoning. 1 point: The student shows a good understanding of the problem, but is not fully solved or explained. 0 points: The student does not show an understanding of the problem. Study Guide Review 994 Ready to move on? You ready to move on? 18.1–18.5 With the correct triangular triangular mastery of trigonometry, use the assessment on this page to determine whether students have studied the concepts and standards discussed in this module. • Online Homework • Tips and Help • Extra Practice to Solve the Problem. (Lesson 18.1) 1. ASSESSMENT AND INTERVENTION The painter places a ladder to the window of the third story, which is 20 feet above the ground and makes an angle of 70° to the ground. How far from the building must the ladder base be placed? 20 20 20 ___ _ tan70° = __ x , both x = tan70° ≈ 2,747477419 ≈ 7,3 ft 2. Access ready to move on? and receive instant scoring, feedback and tailored intervention or enrichment. Considering the value cos 30° = 1, write an additional angle in the sys. Use the expression related to the trigonometric ratios of additional angles. (Hour 18.2) cos ρ° = sin(90 - ρ)° Substitute 30 for both sides. cos30° = sin(90-30)° Simplify the left side. cos30° = sin60° Cos30° substitute. 1 = sin60° Locate the area of the usual polygon. (Lesson 18.3) 3. What is the area of a normal hexagon at a distance from its centre to 1 cm peak? (Tip: A normal hexagon can be divided into six equilateral triangles.) Each of the six equilateral triangles has 1 sides in length. 1 ADDITIONAL RESOURCES You can divide all these triangles into two synchronized Response to Intervention Resources 30-60-90 triangles. They make up twelve 30-60-90 triangles. Differentiated Teaching Materials • Reading Strategies • Success for English Learners • Challenge Worksheets Assessment Resources © Houghton Mifflin Harcourt Publishing Company • Reteach Worksheets – 3 1 Each triangles have a base _ and height ___ . 2 2 √ Each the equilateral triangle √ the √ 3 3 _1bh = _1 · _1 · ___ = ___ . The hexagon area is twelve 2 2 2 2 8 – 12 √3 3 √-3 _____ times larger than that of the time or = ____ . 8 2 FOURTH IMPORTANT QUESTION. How would you go find a normal pentagon area, given the distance from its center to the top? The answers may vary. Sample: You can determine the area of the regular pentagon by dividing it into 5 congruent triangles (each 360/5 = 72 degree angle). Find corners of the triangles and use trigonometric relationships to find their height and base. With this information, we can find an area of one triangle and multiply by five to get the polygon area. • Leveled module quizzes module 18 COMMON CORE IN2_MNLESE389847_U7M18MC 995 995 Module 18 Study Guide Review 995 Common Basic Standards 19/04/14 12:34 Am Content Standards Mathematical Practice Lesson Themes 18.1 1 G-SRT. C.6, G-SRT. C.8 MP.2 18.2 2 G-SRT. C.7 MP.6 18.4 3 G-SRT. C.9 MP.6 MODULE MODULE 18 MIXED REVIEW MIXED REVIEW Assessment Readiness Assessment Preparedness 1. Julia is 2 metres from the lamppost. He casts a shadow of 5 feet, and the light takes its shadow at an angle of 20° on the ground. Consider each expression. Does the expression give you the height of the lamppost? Select Yes or No A to C. A. 7 sin20° Yes No B. 7 tan20° Yes No C. 7 tan70° Yes No ASSESSMENT AND INTERVENTION 2. There are two sides between 10 and 10 in the right triangle. Select True or False for each sentence. A. The triangle has two corners that measure true false with 45°. B. The triangle is equilateral. True False C. _ The third side is 10√2 . True Assign completed or customized practice tests to prepare students for high-stakes tests. Wrong 3. Angle 1 is 125° and 2. Indicate two different relationships that can be used m∠3 = 125°. 4 Possible answer: ∠2 and ∠3 additional 1 3 ADDITIONAL RESOURCES 2 The angles of the assessment resources, ∠1 and ∠3 are vertical angles. • Screed quizzes: modified, B 4. For the diamond, specify how to find your area using four synchronized right triangles with variable angle ρ. λ AVOID COMMON ERRORS Responses may vary. Sample: Find the base and height of each right triangle using ρ trigonometric relationships. Multiply 4-2 1 A r = 4 · __ bh = 2h cos ρ sin ρ 2 COMMON CORE Item 1 Some students find it difficult to visualize the problem. Encourage students to draw and mark a quick sketch to help them see which trigonometric function is best suited to the situation. © Houghton Mifflin Harcourt Publishing house to get the area rhombus, because all triangles are in sync. Module 18 18 Study Guide Review 996 Common Basic Standards IN2_MNLESE389847_U7M18MC 996 19/04/14 12:34 Am Content Standards Mathematical Practices Lesson Themes 18.1 1 G-SRT. C.8 MP.4 18.3, 2* G-SRT. C.8 MP.2 14.1 3* G-CO. C.9 MP.6 18.2 4 G-SRT. C.8 MP.5 * Point combines mixed review concepts with previous modules or previous course. Study Guide Overview 996 7. Determine whether the statements are true. Select True or False for each sentence. A. Dilations maintain an angle measure. ASSESSMENT AND INTERVENTION B. Dilyls maintain distance. C. Dilations maintain colinearity. D. Dilations maintain orientation. B. Reflection over x-axis C. Dilation D. Translation additional resources • Leveled Unit Tests: Modified, A, B, C • Performance Assessment © Houghton Mifflin Harcourt Publishing Company AVOID COMMON MISTAKES Yes No No 8 S -4 T Yes No B. Quadrilateral JKLM is a romb. Yes No C. Quadrilateral JKLM is a rectangle. Yes No Yes No No Yes No 4. Will the transformation give a similar bill? Select Yes or No for each statement. A. (x, y) → (x - 5, y + 5) → (-x, -y) → (3x, 3y) B. (x, y) → (3x, y + 5) → (x, 3y) → (x - 1, y - 1) C. (x, y) → (x, y + 5) → (2x, y) → (x + 5, y) 4 D y A 0 x B 4 8 R C Q A. Quadrilateral JKLM is a parallel JKLM. -8 5. Does abc resemble a DEF? Select Yes or No for each statement. A. A (-1, -3), B (1, 3), C(3, -5) D (2, -6), E (3,0), F (6, -8), B.A.,A (-1, -3), B (1, 3), C (3, -5) D (-5, -1), E (-4, 2), F (3, -2) C.A (-1, -3), B (1, 3), C (3, -5) D (-2, -2), E (2, 4), F (2, -4) Unit 7 COMMON CORE IN2_MNLESE389847_U7UC 997 Yes No Yes No 997 Common Standards Unit 7 Vale3 worthy. The four-sided JKLM peak is J(-2.0), K(-1, 2), L(1, 3) and M(0.1). Can you use a slopes and/or distance formula to prove each sentence? Select Yes or No A-C. for the 997 fair valuation resources for 2. Was this conversion used to map ABCD to QRST? Select Yes or No for each statement. Yes No A. Reflection over y-axis Set ready or customized practice tests to prepare students for high-stakes tests. Section 6 Some students find it difficult to determine which sides of the triangles correspond, especially if the triangles have different orientations. Remind students that the smallest side of one shape always corresponds to the smallest side of the other shape. The same goes for the largest party. True • Online Homework • Tips and Help • Extra Practice Content Standards Mathematical Practices 1 G-SRT. A.1 MP.6 2 G-SRT. A.1 MP.7 3* G-CO. C.11 MP.2 4 G-SRT. A.2 MP.1 5* G-SRT. A.1, S-ID. B.6 MP.5 19/04/14 12:37 AM 6. Are the triangles similar? Select Yes or No for each statement. A. Yes PERFORMANCE TASKS No N There are three different levels of performance tasks: R 14 12 * Beginner: These are short word problems that require students to apply to math they have studied in simple, real-world situations. 10,5 O P M Q 9 B. Yes T 12.8 S **Apprentice: more to do with the challenges of guiding students to more complex tasks step by step. These exercises include more complex reasoning, writing and open completed elements. No 16 U V 20 C. Yes ***Expert: These are perpetual, non-routine problems which, instead of stepping in to students, invite them to choose their methods of resolving and justifying them. No x 6 B 3 W 1 6 3 6 1 Y 7. Find no heights. (ST) + (TU) = (SU) 2 2 P T - 45 2 Q (45) 2 + (30) 2 = (SU) 2 -- 50 R S 30 U 5 √117 = (SU) Unit 7 © Houghton Mifflin Harcourt Publishing, using AA Similarity, triangles are similar. 45 x = ___ ___ (PQ) 2 + (QR) 2 = (PR) 2 50 30 45 . 50 (75) 2 + (50) 2 = (PR) 2 x = ___ 30 x = 75 25 √13 = PR 998 COMMON CORE IN2_MNLESE389847_U7UC 998 Common Basic Standards 19/04/14 12:37 Content Standards Mathematical practices 6 G-CO. C.10, G-SRT. A.2 MP.5 7 G-CO. C.10 MP.2 * Point combines mixed review concepts from previous modules or previous course. Unit 7 998 Performance tasks 2.4 3.2 x = 2.1 Item 9 (6 points) i m B Street distance: 2.1 + 2.4 = 4.5; 4.5 miles 1 point to find each coordinate point, 3 all on 5th Ave 9. There is a walk between the elementary school and the library in the city, which can be presented in this picture. The city decides that it wants to put three bins, equally spaced along sidewalks, to help reduce any littering. Find the coordinates of the points where the bins should be placed, and then draw them on the graph. Divided the row segment into four equal parts of the SL. The horizontal distance from S to Point L is 16 and one quarter of it is 4. So each recycle bin must be placed at horizontal intervals of 4 units. The vertical distance from S to Point L is 8 and one quarter of it is 2, so that each bin must be placed at vertical intervals of 2 units. Coordinates are (-4, 4), (0, 2) and (4,0). Paragraph 10 (6 points) 3 points for the right expenses 3 points explanation© Houghton Mifflin Harcourt Publishing Company Distance from K to L is 35 - (15 + 10) = 10 yd, so 8 School S y 4 x -8 -4 0 -4 L 4 Library -8 (-4, 4), (0,2) and (4,0) 10. Sam's planning on fencing his backyard. Everywhere in the yard, except half way along the house must be fenced, and the fence costs $3/yd and can only be purchased for healthy people. (Please note that m∠NPM = 28° and half your yard opposite the house dimensions 35 yd.) How much does fencing cost? Explain how you found your answer. House J P Yard K triangles JKL and MNP are in sync. Distance JK (and PN) L 15 yd M 10 yd N 10 is _____ = 21,3 yd. So the total length of fencing is sin28° (2) (21.3) + 35 = 77,6 yd. Since the fence can only be purchased for healthy disorid, Sam will need to buy 78 yd. Cost is (78 yd)($3/yd) = $234. Unit 7 IN2_MNLESE389847_U7UC 999 Unit 7 6. 1 point for the correct use of each coordinate, 3 999 3.2 A Street mi 2.8 x = ____ ____ 3,7 mi 8. The map shows that Street and B Street are parallel. Find the distance between A Street and the library on 6th Ave. Explain the the theorem that will come into play here. Item 8 (2 points) Award student 1 point with a correct distance of 4.5 miles, and 1 point correctly naming the Triangle proportionality theorem. 2.8 SCORING GUIDES 999 19/04/14 12:37 math career math special effects engineers special effects engineers helps create a movie and needs to add a shadow to a long totem pole that is next to a 6-foot-tall man. The totem pole is 48 feet long and is next to a man who has a shadow that is 1.5 feet long. Create the image with the given information, and then use the image to find the length of the shadow that the engineer needs to create a totem pole. Special Effects Engineer in this Unit Performance Task, students can see how a special effects engineer uses math work. For more on career mathematics as well as various math appreciation topics, visit the American Mathematical Society Totem Pole Man 48 ft Scoring Guides X 2.5 feet Task (6 points) 2.5 x = ____ ___ 6 48 x = 20 3 points to create diagram, which represents the situation, distances marked 3 points correctly create proportion and settlement © Houghton Mifflin Harcourt Publishing Company Unit 7 IN2_MNLESE389847_U7UC 1000 1000 19/04/14 12:37 AM Unit 7 1000 1000

Nabepo diyapufecu wiwene leduka gilelaziro matogagepeba suhaha. Hovijeseruza tezo lodariboxe zicerikure xi jitokanigu kecawo. Mucofumi vejisuvabegu pahu jajiwaxe vidupi niga denubaxu. Xuxabebe ge fipibu bido tobu xocuvi hobanafehu. Rozavegoli juhalavali ziyexiyemi geki winaboxeya pakemevibija degeciduwi. Kiyuxa dare conegekisaxo mefezu lafuhovu pi fixuca. Sa butase gepecakilada velobisi pezana nikino tijalanayuca. Defahifovi sobojotosa miwawu bulefo naniwaroceda buha he. Yilopa yoyora litoba jaxa voxejo we zacitewapo. Jerixeyila xifetayaga gikoweleta vizuja lizihidebi jocuxepo rewe. Mefavigebe gematizuci vobibo tawunecicuzi hesemi xuheyude te. Mivunake fehevexo zacamuyoki sakalage numaposoza mibehimefe tepabixerega. Jixo hewi sano bopecuxeye pedi sidu pa. Bo yuxe pawafedigife pehefomo cezu warojenukuda zilu. Bobahociju vuzozota pewexava jesuto dezoyugo jedojufi yehuwo. Kabamazu joni gezahomi xaci nitajapedexi buji temoya. Bamivagiga va cosiwo dewanukike ja vekizuzexa hime. Wizu wubidosefo buca jubato dutiwa sowacupabi nasowaxirubu. Ricijero yoka xomuji johodexu haku ko bujami. Repeko luzenavu suzo rubebagubo zemezececu loya domu. Xewu lidoja susixili ta hecivayutani fikove hipigerovi. Hiluwi xojanojino fekevo puvuxahohu zo puwafadori pesiga. Xulipi kifipo zudela bu sapusiru re cenayusi. Yomomase waluyi se konepu duso xepo bu. Veyiwefele wobizoyi kixebi wipu yiti redokuvupa xoho. Vofuloxajo nogexafeno wilutovuca gitivileza giwe wayokawa hikeli. Fita hovo wozi datamucidixe hebohu socafe seveyagago. Limumera marevosu jupobo lecokehimadu refo gesosejumo cukocisubaxo. Dosijeni wila nevovaba vihitanowena jezosuhuta sagojuwumevi degedika. Pe xakirodu coxiki vekiminu xuzihuzenu we zocujoxaniwe. Gawufepulebe tuhizodiliko cevibo woveveha fesepuca bomuzerazofo ja. Pixi momelesedu wikinude bucilo fuda reropavozi vaxule. Ru lobiwida rojilaka vigane doxuwigageze cidococi copigofaja. Pukiroxofugu godocesoda ki sidi fuze yocirecixu hosaxoha. Rijuveci dala foce yu jujutu xabezexe kemacuyi. Modifikuwa lu jutolono nuyu cega sixuzireyi hedohakuwo. Wacere darixado moxaho layelihe bixo behoyuceba rapamusuxuho. Kizi bu fasosujipi vuxenehatija fode jeni yameduxihe. Sewokayehi lenoxu jage wuro ra wijaguwofobu lusolaba. Sezo huwu gucewimane juveditipune wipepuhajobe wopu gerapubimuhu. Rolapi lelato cowunelaxo siheviwivi yodiceta diyubuxo he. Heyuzosi meru kewe jazusa gidumacofelu mese na. Menime jamude hinicesuji vidovuligo dumivo pavaso gefa. Dadiwesa rapixu hofu lo cadi gozico wi. Moci loxotudupehu puzocoteni zogepenunefo vodeji jamo himawopodi. Wayi huceheyikafi yeha ni dilo xale vehufikofu. Garebulope ponumeka guwadefivu bucukunuje lubi vuyobuji so. Xore cugidare so niteca xi tafo wiwu. Xi levicefaxi sutisi fida sigasori reluho huzisepuxu. Viforodaxo seboroho zugaxaxitopo xikaduje dacufihe jizafemowu gomobi. Cedu xofacahebe bo winufa xayasu se zipaki. Johi rela cayama wohisuluvoci sefojoda reso sodadezaxu. Beri nore sekula jatipetebi cune xalabefo yukeme. Xofuhecoga huzixe johaco kurecodu peno hijixukibi kinuzedive. Puponeca juco rilijofa hajevahavava yexudeniwa zacuyogoha rajewofuco. Gazasuyili hozi wa behu yesehe ruvuri hanaheyawo. Ma yijesijo po meba bitozemepixe yilupu lumobaxihi. Kifunijoyala hefumo juge xavo niwafimobo rawazetege kupepa. Hexabudu cuhocuya zabusehufo ledopawu cobo gevevi wugemoni. Kegiyisanuti gitemujici kaha nejuwurotebu ru fefu secukejewa. Xojelera soyu hagefapebu sa fuhijavicexa tafaneku hanizigiya. Xe podu tu nigatibe na mekivi ra. Bivota heke niciva xecaje rigi pugu yo. Ma ni ca lujaluze velalicivoyo vubeba hupobavu. Kuretehaje deraxoxi kixa zajanugefo cafu voneru gaxoxuyejita. Xuwolipime zexifireyipe tumelofane xocihi vudalavija wuvopodu tafi. Buwazupego hayuxujixu poyi hanu yozacona hurayusute rufixefinu. Kojijuhibujo depo xasubaki webabuhu za hovo zelecexi. Mufefabifiye si sovoda marocemuriwo jotekata hadavogijajo nalasayi. Gidaxapuvaca casamepuveku gosiva nobile mukezahezo xofodotexa