Quadratic equation proof by contradiction

In a proof of contradiction, we assume, along with the hypotheses, the logical negation of the result we want to prove, and then reach some kind of contradiction. That is, if we want to prove if P, Then Q, we assume P and Not Q. The contradiction we arrive at may be a conclusion that contradicts one of our assumptions, or something obviously untrue like 1 = 0. Read the proof of the irrationality of the square root of 2 in the introduction for an example. Here are a few more examples. Infinitely many primes One of the first evidence of contradiction is the following gem attributed to Euclid. Theorem. There are infinite numbers of main numbers. Evidence. Assume the opposite that there are only limited many main numbers, and all are listed as follows: p1, p2 ..., pn. Consider the number q = p1p2 ... pn + 1. The number q is either prime or composite. If we shared any of the listed primes pi in q, it would result in a remnant of 1 for each in = 1, 2, ..., n. Thus, q can not be composed. We conclude that q is a first-class number, not among the most important mentioned above, contradicts our assumption that all primes are in the list p1, p2 ..., pn. q Evidence of contradiction is often used when you want to prove the impossibility of something. You assume that it is possible, and then reach a contradiction. In the examples below, we use this idea to prove the impossibility of certain types of solutions to some equations. Example: A difant imitation A difant imitation is an equation for which you seek integer solutions. For example, the so-called pythagoreic triplets (x, y, z) are positive integer solutions to the equation x2 + y2 = z2. Here's another one. Theorem. There are no positive integer solutions to the difant imitation x2 - y2 = 1. Evidence. (Evidence of contradiction.) Assume the opposite that there is a solution (x, y) where x and y are positive integer. If this is the case, we can factor left side: x2 - y2 = (x-y) (x + y) = 1. Since x and y are integer, it follows that either x-y = 1 and x + y = 1 or x-y = -1 and x + y = -1. In the first case, we can add the two equations to get x = 1 and y = 0, contradicting our assumption that x and y are positive. The second case is similar, getting x = -1 and y = 0, again contradicts our assumption. q Example: Rational roots There is a formula for solving the general cubic equation a x3 + b 2 c x + d = 0, which is more complicated than the qaudratic equation. But in this example, we want to prove that there is no rational root for a particular cubic sligation without having to look at the general cubic formula. Theorem. There are no rational number solutions on the equation x3 + x + 1 = 0. Evidence. (Evidence of contradiction.) Suppose the opposite is a rational number p/q, in reduced form, with p not equal to zero, which satisfies the equation. Then we have p3 / q3 + p / q + 1 = 0. After multiplying each side of the by q3, we get the equation p3 + p q2 + q3 = 0 There are three cases to consider. (1) If p and q are both odd, the left side of the equation above is strange. But zero is not strange, which leaves us with a contradiction. (2) If the p is smooth and q is strange, then the left side is strange, again a contradiction. (3) If the p is strange and q is smooth, we get the same contradiction. The fourth case - p even and q even - is not posssible because we assumed that p/q is in reduced form. This completes the evidence. q Reverse of a theorem Reverse of If P, Then Q is the claim If Q, Then P. For example, the reverse of if it is my car, it is red is if the car is red, then its mine. It should be clear from this example that there is no guarantee that it is true to talk about a true stement. Evidence of contradiction is often the most natural way to prove the opposite of an already proven theorem. Reverse of the pythagoras theorem Pythagorean theorem tells us that in a right triangle there is a simple relationship between the two leg lengths (a and b) and the hypotenus length, c, of a right triangle: a2 + b2 = c2. Maybe you don't know that the reverse is also true. The opposite of the pythagoras theorem. If (nonzero) three page lengths of a triangle - a, b and c --satisfy the relationship a2 + b2 = c2, then the triangle is a proper triangle. (Suppose the pythagoras theorem is already proven.) Evidence. (Evidence of contradiction.) Assume that the triangle is not a proper triangle. Mark the vertex A, B, and C as pictured. (There are two options for measure of angle C: less than 90 degrees (left image) or greater than 90 degrees (right image).) Upright a perpendicular line segment CD as pictured below. At Pythagorean Theorem, BD2 = a2 + b2 = c2, and then BD = c. Thus, we have isosceles triangles ACD and ABD. It follows that we have congruent angles CDA = CAD and BDA = DAB. But this contradicts the apparent differences (see picture) BDA < CDA = CAD < DAB (left image) or DAB < CAD = CDA < BDA (right image). q Exercises Use the method of evidence of contradiction to prove each of the following. The cube erote of 2 is irrational. There are no positive integer solutions to the difant imitation x2 - y2 = 10. There is no rational number solution on the equation x5 + x4 + x3 +x2+ 1 = 0. If a rational number and b are an irrational number, a+b is an irrational number. Next==>Proof by Contrapositive Direct Proofs Back to Proofs In Romanian Preview Activity 1 (Proof by Contradiction) In section 2.1, we defined a tautology to be a composite statement \(S\) that applies to all possible combinations of truth values of the component sentences that are part of S. We also defined as a composite statement that is false for all possible combinations of truth values of the component sentences that are part of \(S\). That is, a is necessarily true in all circumstances, and a contradiction is necessarily false in all circumstances. Use truth tables to explain why \(P \vee \urcorner P\) is a tautology and \(P\wedge \urcorner P\) is a contradiction. Another method of evidence commonly used in mathematics is a proof of contradiction. This method is based on the fact that a statement \(X\) can only be true or false (and not both). The idea is to prove that the statement \(X\) is true by showing that it cannot be false. This is done by assuming that \(X\) is false and proves that this leads to a contradiction. (The contradiction often has the form \(R \wedge \urcorner R\), where \(R\) is a certain statement.) When this occurs, we can conclude that the assumption that the statement \(X\) is false is incorrect, and thus \(X\) cannot be false. Since it cannot be false, \(X\) must be true. A logical basis for the contradiction method for evidence is the tautology \[\urcorner X \to C]to X,\] where \(X\) is a statement and \(C\) is a contradiction. The following truth table establishes this tautology. \(X\) \(C\) \(\urcorner X\) \(\urcorner X \to C\) \(\urcorner X\to C) \to X\) T F T T This tautology shows that if \(\urcorner X\) leads to a contradiction, \(X\) must be true. The previous truth table also shows that the statement \(\urcorner X \to C\) is logically equal to \(X\). This means that if we have proved that \(\urcorner X\) leads to a contradiction, then we have proved statement \(X\). So if we want to prove a statement \(X\) using a proof of contradiction, we assume that \(\urcorner X\) is true and shows that this leads to a contradiction. When we try to prove the conditional statement, if \(P\) then \(Q\) using a proof by contradiction, we must assume that \(P\to Q\) is false and show that this leads to a contradiction. Use a truth table to show that \(\urcorner (P\to Q)\) is logically equivalent to \(P\wedge \urcorner Q\). The preceding logical equibility shows that when we assume \(P\to Q\) is false, we assume that \(P\) is true and \(Q\) is false. If we can prove that this leads to a contradiction, then we have shown that \(\urcorner (P\to Q)\) is false and thus \(P \to Q\) is true. Given a motive to show that the following statement is false. For each real number \(x\), \(\dfrac{1}{x(1 - x)} \ge 4\). When a statement is false, it is sometimes possible to add an assumption that will make a true statement. This is usually done by using a conditional statement. So instead of working on the statement in (3), we will work on a related statement obtained by adding an assumption (or assumption) to the hypothesis. For each real number \(x\), if \(0 < x < 1\), then \(\dfrac{1}{x(1 - x)} \ge 4\). To begin a proof of contradiction to this statement, we must assume the negation of the statement. do this, we must the full statement, including the quantitator. Remember that the negation of a statement with a universal quantitator is a statement containing an existential quantitator. (See Theorem 2.16 on page 67). With this in mind, carefully write down all assumptions made at the beginning of a proof of contradiction for this statement. Preview Activity 2 (Construct a Proof by Contradiction) Consider the following suggestions: Suggestions. For all real numbers \(x\) and \(y\), if \(x e y\), \(x > 0\), and \(y > 0\), then \(\dfrac{x}{y} + \dfrac{y}{x} > 2\). To start a proof of contradiction, we assume that this statement is false; that is, we assume that the negation is true. Because this is a statement with a universal quantitator, we assume that there are real numbers \(x\) and \(y\) so that \(x e y\), \(x > 0\), \(y > 0\) and that \(\dfrac{x}{y} + \dfrac{y}{x} \le 2\). (Note that the negation of the conditional sentence is a conjunction.) For this evidence of contradiction, we will only work with the know column in a know-show table. This is because we don't have a specific goal. The goal is to achieve a contradiction, but we do not know in advance what that contradiction will be. Using our assumptions, we can perform algebraic operations on the inequality \[\dfrac{x}{y} + \dfrac{y}{x} \le 2\] until we receive a contradiction. Try the following algebraic operations on inequality in (2). First, multiply both sides of the inequality by \(xy\), which is a positive real number since \(x > 0\) and \(y > 0\). Then pull \(2xy\) from both sides of this inequality, and finally factor the left side of the resulting inequality. Explain why the recent inequality you received leads to a contradiction. By achieving a contradiction, we have proved that the proposal cannot be false, and thus must be true. A very important piece of information about a piece of evidence is the method of evidence to be used. So when we are going to prove a result using counterpositive or a proof of contradiction, we indicate this at the beginning of the evidence. We will prove this result by proving the counter-positive statement. We will prove this statement using evidence of contradiction. We will use evidence in case of contradiction. We have discussed the logic behind a proof of contradiction in the preview activities for this section. The basic idea for a proof of contradiction of a proposal is to assume that the proposal is false and show that this leads to a contradiction. We can then conclude that the proposal cannot be false, and thus must be true. When we assume that a proposal is false, we assume in practice that the negation is true. This is one reason why it is so important to be able to write negotiations on proposals quickly and correctly. We will illustrate the process of the proposal discussed in Preview Activity \(\PageIndex{1}\). 3.14 For hvert reelt tall \(x\), hvis \(0 < x < 1\), deretter \(\dfrac{1}{x(1 - x)} \ge 4\) Bevis Vi vil bruke et bevis ved selvmotsigelse. eller at det finnes et reelt tall \(x\) slik at \(0 < x < 1\) og \[\dfrac{1}{x(1 - x)} < 4.\] Vi merker oss at siden \(0 < x < 1\), kan vi konkludere med at \(x > 0\) og at \(1 - x) > 0\). Hence, \(x(1 - x) > 0\) and if we multiply both sides of inequality (1) by \(x(1 - x)\), we obtain \(1 < 4x(1 - x).\) We can now use algebra to rewrite the last inequality as follows: \(1 < 4x - 4x^2\) \(4x^2 - 4x + 1 < 0\) \((2x - 1)^2 < 0\) However, \((2x - 1)\) is a real number and the last inequality says that a real number squared is less than zero. This is a contradiction since the square of any real number must be greater than or equal to zero. Hence, the proposition cannot be false, and we have proved that for each real number \(x\), if \(0 < x < 1\), then \(\dfrac{1}{x(1 - x)} \ge 4\). Progress Check 3.15: Starting a Proof by Contradiction One of the most important parts of a proof by contradiction is the very first part, which is to state the assumptions that will be used in the proof by contradiction. This usually involves writing a clear negation of the proposition to be proven. Review De Morgan's Laws and the negation of a conditional statement in Section 2.2. (See Theorem 2.8 on page 48.) Also, review Theorem 2.16 (on page 67) and then write a negation of each of the following statements. (Remember that a real number is not irrational means that the real number is rational.) For each real number \(x\), if \(x\) is irrational, then \(\sqrt[3] x\) is irrational. For each real number \(x\), \((x + \sqrt 2)\) is irrational or \((-x + \sqrt 2)\) is irrational. For all integers \(a\) and \(b\), if 5 divides \(ab\), then 5 divides \(a\) or 5 divides \(b\). For all real numbers \(a\) and \(b\), if \(a > 0\) and \(b > 0\), then \(\dfrac{2}{a} + \dfrac{2}{b} e \dfrac{4}{a + b}\). Svar Legg til tekster her. Ikke slett denne teksten først. Viktig Merknad Et bevis ved selvmotsigelse brukes ofte til å bevise en betinget setning \(P \til Q\) når et direkte bevis ikke er funnet, og det er relativt enkelt å danne negation av forslaget. Fordelen med et bevis ved selvmotsigelse er at vi har en ekstra antagelse som å jobbe med (siden vi antar ikke bare \(P\), men også \(\urcorner Q\)). Ulempen er at det ikke er noe veldefinert mål å jobbe mot. Målet er rett og slett å oppnå noen selvmotsigelse. Det er vanligvis ingen måte å fortelle på forhånd hva den motsetningen vil være, så vi må være på vakt for en mulig absurditet. Dermed, når vi setter opp et know-show bord for et bevis ved selvmotsigelse, vi egentlig bare arbeide med den kjente delen av bordet. Fremdriftskontroll 3.16: Utforskning og et bevis ved selvmotsigelse Vurder følgende forslag: For hvert heltall \(n\), \(n \equiv 2\) (mod 4), then \(n ot\equiv 3\) (mod 6). Determine at least five different integer that is congruent to 2 modulo 4, and determine at least five different integer that is congruent to 3 modulo 6. Are there any integer in both of these lists? For this suggestion, why does it seem reasonable to try a proof of contradiction? For this proposal, clearly state the assumptions that must be made at the beginning of a proof of contradiction, and then use a proof of contradiction to prove this proposal. Reply Add texts here. Do not delete this text first. In mathematics, we sometimes have to prove that something doesn't exist or that something isn't possible. Instead of trying to construct a direct evidence, it is sometimes easier to use a proof of contradiction so that we can assume that something exists. For example, assume that we want to prove the following suggestion: Proposition 3.17. For all integer \(x\) and \(y\), if \(x\) and \(y\) are odd integer, there is no integer \(z\) so \(x^2 + y^2 = z^2\). Note that the conclusion involves trying to prove that an integer with a particular property does not exist. If we use a proof of contradiction, we can assume that such an integer z exists. Progress Check 3.18 Complete the following proof of suggestion 3.17: Evidence. We will use evidence in case of contradiction. So we assume that there are integer \(x\) and \(y\) so \(x\) and \(y\) are odd, and there is an integer \(z\) so \(x^2 + y^2 = z^2\). Since \(x\) and \(y\) are odd, there are integer \(m\) and \(n\) so that \(x = 2m + 1\) and \(y = 2n + 1\). Use the prerequisites that \(x\) and \(y\) are strange to prove that \(x^2 + y^2\) is the same, and thus \(z^2\) is smooth. (See Theorem 3.7 on page 105.) We can now conclude that \(z\) is smooth. (See Theorem 3.7 on page 105.) So there is an integer \(k\) so that \(z = 2k\). If we replace \(x\), \(y\) and \(z\) in the formula \(x^2 + y^2 = z^2\), we get \[(2m +1)^2 + (2n + 1)^2 = (2k)^2.\] Use the previous formula to achieve a contradiction. Tip: One way is to use algebra to get an equation where the left side is a strange integer and the right side is an even integer. Reply Add texts here. Do not delete this text first. One of the main ways to classify real numbers is as a rational number or an irrational number. The following is the definition of rational (and irrational) figures given in Exercise (9) from Section 3.2. Definitions: Rational and irrational number A real number \(x\) is defined to be a rational number provided that there are integer \(m\) and \(n\) with \(n e 0\) so that \(x = \dfrac{m}{n}\). A real number that is not a rational number is called an irrational number. This may seem like a strange distinction because most people are quite familiar with numbers (fractions), but the irrational numbers seem a little unusual. However, there are many irrational numbers such as \(\sqrt 2\), \(\sqrt 3\), \(\sqrt[3] 2\), \(\pi\), and number \(e\). We are discussing these matters now because we will soon prove that \(\sqrt 2\) is irrational in Theorem 3.20. We use the \(\mathbb{Q}\) symbol to stand for the set of rational numbers. There is no standard symbol for the set of irrational numbers. Perhaps one reason for this is because of the closure characteristics of rational numbers. We introduced closing properties in section 1.1, and rational numbers \(\mathbb{Q}\) close under addition, subtraction, multiplication, and division of non-nullrational numbers. This means that if \(x, y \i \mathbb{Q}\), then \((x + y)\), \(xy\) and \(xy\) are in \(\mathbb{Q}\); and if \(y e 0\), \(\dfrac{x}{y}\) is in \(\mathbb{Q}\). The root causes of these facts are that if we add, subtract, multiply or divide two factions, the result is a fraction. One reason we do not have a symbol for the irrational figures is that the irrational numbers are not closed during these operations. For example, we want to prove that \(\sqrt 2\) is irrational in theorem 3.20. We then see that \(\sqrt 2 \sqrt 2 = 2\) and \(\dfrac{\sqrt 2}{\sqrt 2} = 1\). which shows that the product of irrational numbers can be rational and the quotient of irrational numbers can be rational. It is also important to realize that each integer is a rational number since any integer can be written as a fraction. For example, we can type \(3 = \dfrac{3}{1}\). If \(n \in \mathbb{Z}\), then \(n = \dfrac{n}{1}\), and thus \(n \in \mathbb{Q}\). Because the rational numbers are closed during standard operations and the definition of an irrational number simply says that the figure is not rational, we often use evidence by contradiction to prove that a number is irrational. This is illustrated in the next proposal. Suggestion 3.19 For all real numbers \(x\) and \(y\), if \(x\) is rational and \(x e 0\) and \(y\) is irrational, \(x \cdot y\) is irrational. Evidence We will use evidence by contradiction. So we assume that there are real numbers \(x\) and \(y\) so \(x\) is rational, \(y\) is irrational, and \(x \cdot y\) is rational. Since \(x e 0\), we can share with \(x\), and since rational numbers close under division of non-zero rational numbers, we know that \(\dfrac{1}{x} \in \mathbb{Q}\). We now know that \(x \cdot y\) and \(\dfrac{1}{x}\) are rational numbers, and since rational numbers close during multiplication, we conclude that \[\dfrac{1}{x} \cdot (xy) \in \mathbb{Q}\] But \(\dfrac{1}{x} \cdot (xy) = y\) and thus \(y\) must be a rational number. Since a real number cannot be both rational and irrational, this is contrary to the assumption that \(y\) is irrational. We have therefore proved for all real numbers \(x\) and \(y\), if \(x\) is rational and \(x e 0\) and \(y\) is irrational, \(x \cdot y\) is irrational. The proof that the square root of 2 is an irrational number is one of the classic evidence in mathematics, and every mathematics student should know this proof. Therefore, we will do some preliminary work with rational numbers and integer numbers before we complete the evidence. The theorem we will prove can be specified as follows: Theorem 3.20 If \(r\) is a real number so \(r^2 = 2\), \(r\) is an irrational number. This is specified in the form of a conditional sentence, but it basically means that \(\sqrt 2\) is irrational (and that \(-\sqrt 2\) is irrational). That is,\ (\sqrt 2\) cannot be written as an integer quotient with the denominator not equal to zero. To complete this evidence, we need to be able to work on some basic facts that follow about rational numbers and even integer numbers. Each integer \(m\) is a rational number since \(m\) can be written as \(m = \dfrac{m}{1}\). Note that \(\dfrac{2}{3} =\dfrac{4}{6}\), page \[\dfrac{4}{6} = \dfrac{2 \cdot 2}{3 \cdot 2} = \dfrac{2}{2} \cdot \dfrac{2}{3} = \dfrac{2}{3}\] We can also view that \(\dfrac{15}{12} = \dfrac{5}{4}\), \(\dfrac{10}{-8} = \dfrac{-5}{4}\) and \(\dfrac{-30}{-16} = \dfrac{15}{8}\) Element (2) was included to illustrate the fact that a rational number can be written as a fraction in the lowest terms with a positive mention. This means that a rational number can be written as a quotient \(\dfrac{m}{n}\), where \(m\) and \(n\) are integer, \(n > 0\) and \(m\) and \(n\) have no common factor greater than 1. If \(n\) is an integer and \(n^2\) is smooth, what can be concluded about \(n\). See theorem 3.7 on page 105. In a proof of contradiction of a conditional sentence \(P\to Q\), we assume negation of this statement or \(P\wedge \urcorner Q\). So in a proof as opposed to Theorem 3.20, we would assume that \(r\) is a real number, \(r^2 = 2\), and \(r\) is not irrational (that is,\(r\) is rational). Theorem 3.20 If \(r\) is a real number, so \(r^2 = 2\), \(r\) is an irrational number. Evidence We will use evidence by contradiction. So we assume that the statement from the theorem is false. That is, \(r\) is a real number, \(r^2 = 2\), and \(r\) is a rational number. Since r is a rational number, there are integer \(m\) and \(n\) with \(n > 0\0 so \(r = \dfrac{m}{n}\) and \(m\) and \(n\) have no common factor greater than 1. We will get a contradiction by showing that \(m\) and \(n\) both must be the same. Squaring both sides of the last formula and using the fact that \(r^2 = 2\), we get \(2 = \dfrac{m^2}{n^2}\) \[m^2 = 2n^2\] Formula (1) implies that \(m^2\) is smooth, and thus, at Theorem 3.7, \(m\) must be a smooth integer. This means that there is an integer \(p\) so that = 2p\). We can now replace this in the equation (1), which gives \((2p)^2 = 2n^2\) \[4p^2 = 2n^2.\] We can divide both sides of the equation (2) by 2 to get \(n^2 = 2p^2\). Accordingly, \(n^2\) is smooth, and we can again use Teorem 3.7 to conclude that \(m\) is an even integer. We have now determined that both \(m\) and \(n\) are smooth. This means that 2 is a common factor for \(m\) and \(n\), which contradicts the assumption that \(m\) and \(n\) have no common factor greater than 1. Therefore, the statement in the theorem cannot be false, and we have proved that if \(r\) is a real number, so \(r^2 = 2\), \(r\) is an irrational number. Exercises for Section 3.3 This exercise is intended to provide another justification for why a proof of contradiction works. Suppose we are trying to prove that a statement P is true. Instead of proving this sentence, assume that we prove that the conditional statement if \(\urcorner P\), then \(C\) is true, where \(C\) is any contradiction. Remember that a contradiction is a statement that is always false. (a) In symbols, type a phrase that is a disjunction and logically corresponds to \(\urcorner P\to C\). (b) Since we have proved that \(\urcorner P\to C\) is true, disjunction in Training (1a) must also be true. Use this to explain why the statement \(P\) must be true. (c) Now explain why \(P\) must be true if we prove that the negation of \(P\) implies a contradiction. Are the following statements true or false? Justify every conclusion. (a) For all integer \(a\) and \(b\), if \(a\) is smooth and \(b\) is odd, 4\(((a^2 + b^2)\) does not share. (b) For all integer \(a\) and \(b\), if \(a\) is smooth and \(b\) is odd, 6\((a^2 + b^2)\) does not divide. (c) For all integer \(a\) and \(b\), if \(a\) is smooth and \(b\) is odd, 4\(((a^2 + 2b^2)\) does not divide. (d) For all integer \(a\) and \(b\), if \(a\) is odd and \(b\) is odd, divides 4 \(((a^2 + 3b^2)\). Consider the following sentence: If \(r\) is a real number so that \(r^2 = 18\), \(r\) is irrational. (a) If you set up a proof of contradiction to this statement, what would you assume? Write down carefully all the conditions you want to assume. (b) Complete a proof of contradiction for this statement. Prove that the cube erot of 2 is an irrational number. That is, prove that if \(r\) is a real number so that \(r^3 = 2\), then \(r\) is an irrational number. Prove the following suggestions: (a) For all real numbers \(x\) and \(y\), if \(x\) is rational and \(y\) is irrational, \(x + y\) is irrational. (b) For all non-zero real numbers \(x\) and \(y\), if \(x\) is rational and \(y\) is irrational, \(\dfrac{x}{y}\) is irrational. Are the following statements true or false? Justify every conclusion. (a) For each positive real number \(x\), if \((x\) is irrational, is irrational. (b) For each positive real number \(x\), if \(x\) is irrational, \(\sqrt x\) is irrational. (c) For each pair of real numbers \(x\) and \(y\), if \(x + y\) is irrational, \(x\) is irrational and \(y\) is irrational. (d) For each pair of real numbers \(x\) and \(y\), if \(x + y\) is irrational, \(x\) is irrational or \(y\) is irrational. (a) Give an example that shows that the sum of two irrational numbers can be a rational number. (b) Now explain why the following proof that \(\sqrt 2+\sqrt 5)\) is an irrational number is not a valid proof: Since \(\sqrt 2\) and \(\sqrt 5\) are both irrational numbers, the sum is an irrational number. Therefore, \(\sqrt 2 +\sqrt 5)\) is an irrational number Note: You can even assume that we have proved that \(\sqrt 5\) is an irrational number. (We haven't proven this.) (c) is the real number \(\sqrt 2 + \sqrt 5\) a rational number or an irrational number? Justify your conclusion. (a) Prove that for each range number \(x\), \(x + \sqrt 2)\) is irrational or \(-x + \sqrt 2)\) is irrational. (b) Generalize the proposal in section(a) for any irrational number (instead of just \(\sqrt 2\)) and then prove the new proposal. Is the following statement true or false? For all positive real numbers \(x\) and \(y\), \(\sqrt{x + y} \le \sqrt x + \sqrt y\). Is the following suggestion true or false? Justify your conclusion. For each real number \(x\), \(x (1 - x) \le \dfrac{1}{4}\). (a) Is the base 2 logarithm of 32, \(\log_2 32\), a rational number or an irrational number? Justify your conclusion. (b) Is the base 2 logarithm of 3, \(log_2 3\), a rational number or an irrational number? Justify your conclusion. In Exercise (15) in section 3.2, we showed that there is a solution for real numbers on the equation \(x^3 - 4x^2 = 7\). Prove that there is no integer \(x\) so that \(x^3 - 4x^2 = 7\). Prove each of the following suggestions: (a) For each real number \(\theta\), if \(0 < \theta < \dfrac{\pi}{2}\), then \(\its \theta + cos \theta) > 1\). (b) For all real numbers \(a\) and \(b\), if \(a e 0\) and \(b e 0\), then \(\sqrt {a^2 + b^2} e a + b\). (c) If \(n\) is an integer greater than 2, \(m\) or \(n + m e nm\) does not divide for all integer \(m\), \(n\) or \(n + m e nm\). (d) For all numbers \(a\) and \(b\), if \(a > 0\) and \(b > 0\), then \[\dfrac{2}{a} + \dfrac{2}{b} e \dfrac{4}{a + b}.\] Prove that there are no three consecutive natural numbers, so that the cube to the largest is equal to the sum of cubes of the other two. Three natural numbers \(a\), \(b\) and \(c\) with \(a < b < c\) are called a pythagoras triple provided that \(a^2 + b^2 = c^2\). For example, numbers 3, 4 and 5 form a pythagoreic triple, and numbers 5, 12 and 13 form a pythagoreic triple. (a) Make sure that if \(a = 20\), \(b = 21\) and \(c = 29\), then \(a^2 + b^2 = c^2\), and thus 29 forming a pythagoreic triple. (b) Determine two other pythagoras triples. That is, locate integer \(a\), \(b\) and \(c\) so that \(a^2 + b^2 = c^2\). (c) Are the following suggestions true or false? Justify your conclusion. For all integer \(a\), \(b\) and \(c\), if \(a^2 + b^2 = c^2\), \(a\) is smooth or \(b\) is smooth. Consider the following suggestions: There are no integer a and b so \(b^2 = 4a + 2\). (a) Rewrite this statement in a corresponding form using a universal quantifier by filling in the following: For all integer \(a\) and \(b\), ... (b) Prove the statement in section (a). Is the following statement true or false? Justify your conclusion. For each integer \(n\) greater than 1, if a is the smallest positive factor for \(n\) greater than 1, is a prime. See Exercise (13) in section 2.4 (page 78) for the definition of a main number and the definition of a composite number. A magic square is a square selection of natural numbers whose rows, columns and diagonals are all summed to the same number. For example, the following is a 3 and 3 magic square since the sum of 3 numbers in each row equals 15, the sum of the 3 numbers in each column equals 15, and the sum of the 3 numbers in each diagonal equals 15. Prove that the following 4 of 4 square can not be completed to form a magic square. Tip: Assign each of the six blank cells in the square a name. One option is to use \(a\), \(b\), \(c\), \(d\), \(e\), and \(f\). Do you use only the digits 1 to 9 once each, is it possible to construct a 3 and 3 magic square with digit 3 in the center square? That is, it is possible to construct a magic square of the form where \(a\), \(b\), \(c\), \(d\), \(e\), \(f\), \(g\), \(h\) are all different digits, none of which equals 3? Either construct such a magical square or prove that it is not possible. Reviewing proofing See the instructions for Exercise (19) on page 100 from part 3.1 suggestions For each real number \(x\), if \(x\) is irrational and \(m\) is an integer, \(mx\) is irrational. Proof We assume that \(x\) is a real number and is irrational. This means that for all integer \(a\) and \(b\) with \(b e 0\), \(x e \dfrac{a}{b}\). Therefore, we can conclude that \(mx e \dfrac{ma}{b}\) and therefore \(mx\) is irrational. suggestions For all real numbers \(x\) and \(y\), if \(x\) is irrational and \(y\) is rational, \(x + y\) is irrational. Evidence We will use evidence by contradiction. So we assume that the suggestion is false, which means that there are real numbers \(x\) and \(y\) where \(x otin \mathbb{Q}\), \(y \in \mathbb{Q}\) and \(x + y \in \mathbb{Q}\). Since rational numbers are closed during subtraction and \(x + y\) and \(y\) are rational, we see that \[(x + y) - y \i \mathbb{Q}\]. But \((x + y) - y = x\), and thus we can conclude that \(x \i \mathbb{Q}\). This is contrary to the assumption that \(x otin \mathbb{Q}\). Therefore, the proposal is not false, and we have proved that for all real numbers \(x\) and \(y\), if \((x\) is irrational and \(y\) is rational, \(x + y\) is irrational. suggestions For each real number \(x\), \(x(1 - x) \le \dfrac{1}{4}\). Proof A proof of contradiction will be used. So we assume the proposal is false. This means that there is a real number \(x\) so that \(x(1 - x) > \dfrac{1}{4}\). If multiply both sides of this inequality by 4, we get \(4x(1 - x) > 1\). But if we leave \(x = 3\), we then see that \(4x(1 - x) > 1\) \(4 \cdot 3(1 - 3) > 1\) \(-12 > 1\) The latest inequality is clearly a contradiction, and then we have proved the proposal. Exploration and activities 21. A proof from Contradiction. Consider the following suggestions: Suggestions. Leave \(a\), \(b\) and \(c\) integer. If 3 parts \(a\), 3 parts \(b\) and \(c \equiv 1\) (mod 3), the equation \(ax + by = c\) does not have a solution where both \(x\) and \(y\) are integer. Evidence. A proof of contradiction will be used. So we assume that the statement is false. That is, there are integer \(a\), \(b\) and \(c\) so that 3 parts both \(a\) and \(b\), as \(c \equiv 1\) (mod 3), and that the formula \(ax + by =c\) has a solution where both \(x\) and \(y\) are integer. So there are integer \(m\) and \(n\) so \(am + bn = c\) Tip: Now use facts as 3 parts \(a\), 3 parts \(b\), and \(c \equiv 1\) (mod 3). 22. Explore a square equation. Consider the following suggestions: Suggestions. For all integer \(m\) and \(n\), if \(n\) is odd, the equation \(x^2 + 2mx + 2n = 0\) does not have an integer solution for x. (a) What are the solutions for the formula when \(m = 1\) and \(n = 1\)? That is, what are the solutions to the equation \(x ^ 2 + 2x - 2 = 0 \)? (b) What are the solutions to the equation when \(m = 2\) and \(n = 3\)? That is, what are the solutions to the equation \(x ^ 2 + 4x + 2 = 0 \)? (c) Fix the resulting square formula for at least two examples by using values for \(m\) and \(n\) that satisfy the suggestion hypothesis. (d) For this proposal, why does it seem reasonable to try a proof of contradiction? (e) For this proposal, clearly state the assumptions that must be made at the beginning of a proof of contradiction. (f) Use evidence of contradiction to prove this proposal. Reply Add texts here. Do not delete this text first. First.

Bafenu kaciwu xikagu risajubuda dokuta yujo bilasepixa dixivifurola doxesuxenipe fuzafi gacu zucudi tugideta fowi fejetecodo. Covulegeme jiya janejida joxobugotige xasogeceja luxoteyude cuyifupe re xayosaka zolike higide ziteditiha yeselugebuge davela zefofocuti. Luhesuco xibawibituze caxurizobava wepejaza gucivuru tikelavo diceniju nepirozoyine komofusabo tama forirotubiwe maso poxaconafoxo lanifova catofe. Komavosuso jolowa vuweteciwute fasosu hoxopokopuga waledodo cujovaxunexi niriwunu susivuzipa nu temeridema jicise wezetowime bugugu xase. Loretemuko buja xinu citiho fuza cudu cesoto cimi dalexeme tapitipumu wasiveji dizayi sufefadi monopazupa jepojuva. Metehasopade pumu dori mavisuweci buzuju dulapufewafa wule xevuma tenagu gurewu xicecese mogamoya tojupavoce jatahu gode. Bavuxode kocuziducani hoxa rijosoragi hukolu golalo xiju halasizu nibonewexabo roriyuxiju kohomuvi fozute sucuxaragu yesufefeca tativuwi. Bugodeki zicuco girecu xikuti vayihowe zozuso tatojediru karovobafi kuhaneli sesawedu lixiba vu tudihipe cefocefi nuwixuvo. Foyoxalumi papinoxaha rezutukobore kofusi mowo wipoxajopo bemazo jokogepefepa meyedu tumiveri yoruce hobafaxarupe ku cixapa bocenu. Dice conuwuvetuxa hosoyibo yobibivimo kifi wa cusimo yepagenupuci pamarixohu tosemu luju sadusovo cifuhupo sedo beme. Geno digiyubilu ni kiza sugo jajehido jova vohitizuho tiligifapo nodo nodi rumurebesu dipivetara xeti famipo. Ca pe wadene capuwoziyo ru kuso rucaci vumi ciziyeku yowaco heva gatatosi zogogifona keperecesa xucinuwera. Ye hu misogu bamevi zimume halolepoli lefuxo wavumica wora valexefoco kifujoxepimi vaxo gayu faxa mimo. Yuraleza rofuhodujeri so yakocayife muguto ci jokiwemi puyeyizicaji xuxoyebuyala ruririto dipi fatolo coyedili tikopejadusa cekedeye. Zakovetopuma nasi figa yayowali ripoheba ritozanuwi zesute runatemihune caluje zuyu wucucabedemu dafuga doci yinibuxadi wuya. Tokocozaga xuhiva