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Fourier series examples and solutions square wave

Цей документ походить від коефіцієнтів серії Фур'є для кількох функцій. Функції, показані тут, досить прості, але поняття поширюються на більш складні функції. Навіть функція імпульсу (серія козину) Розглянемо періодичну функцію пульсу, показану нижче. Це парна функція з періодом T. Функція є імпульсна функція з амплітудою A, і ширина імпульсу Tp. The function can be defined over one period (centered around the origin) as: $$x_T(t) = \left\{ {\matrix{ {A,\quad |t| \le {{{T_p}} \over 2}} \cr {0,\quad |t| \gt {{{T_p}} \over 2}} \cr } } \right.,\quad \quad - {T \over 2} < t \le {T \over 2}$$ The Fourier Series representation is $${x_T}(t) = {a_0} + \sum\limits_{n = 1}^\infty {\left({{a_n}\cos \left({n{\omega _0}t} \right) + {b_n}\sin \left({n{\omega _0}t} \right)} \right)} $$ Since the function is even there are only an terms. $${x_T}(t) = {a_0} + \sum\limits_{n = 1}^\infty {{a_n}\cos \left({n{\omega _0}t} \right)} = \sum\limits_{n = 0}^\infty {{a_n}\cos \left({n{\omega _0}t} \right)} $$ The average is easily found , $$a_0=A\frac{T_p}T$$ Інші терміни слідують від $${a_n} = {2 \over T}\int\limits_T {{x_T}\left(t \\ справа)\cos \left({n{\omega _0}t} \right)dt} ,\quad n e 0$$ Будь-який інтервал одного періоду дозволений, але інтервал від -T/2 до T/2 є простим у цьому випадку. $${a_n} = {2 \over T}\int\limits_{{-{T} \over 2}}^{T \over 2}} {{x_T}\left (t \ вправо)\ cos \ left({n{\omega _0}t} \right)dt} $$ Оскільки xT(t)=A від -Tp/2 до +Tp/2 і нуль в іншому місці інтеграл спрощує і може бути вирішений $$\begin{align} {a_n} &= {2 \over T}\int\limits_{ { {{T_p}} \over 2}}^{ + {{{T_p}} \over 2}} {A\cos \left({n{\omega _0}t} \right)dt} \cr & {{2 \over T}{A \over {n{\omega _0}}}\sin \left({n{\omega _0}t} \right)} \right|_{ - {{{T_p}} \over 2}}^{ + {{T_p}} \over 2}} \cr & {2 \over T} {A n{\omega _0}}} \left({\sin \left({ + n{\omega _0}{{{{T_p}} \over 2}} \right) - \sin \left({ - n{\omega _0}{{T_p}} \over 2}} \right)} \right) \cr\end{align} $$ 3 s sin(a)-sin(-a)=2sin(a) і використовує той факт, що ω0=2π/T і $${a_n} = {4 \over T}{A \over {n{\omega _0}}}\sin \left({n{\omega _0}{{t_p}} \over 2 }} \right) = 4{A \over {n2\pi }}\sin \left({n\pi {{{t_p}} \over T}} \right) = 2{A \over {n\pi }}\sin \left({n\pi {{{t_p}} \over T}} \right) = 2{A \over {n\pi }}\sin \left({n\pi {{{t_p}} \over T}} \right)$$ Цей результат буде додатково досліджено у двох прикладах. Розглянемо випадок, коли цикл мита становить 50% (це означає, що функція висока 50% часу, або Tp = T / 2), A = 1 і T = 2. У цьому випадку a0=середній=0.5 і n≠0: $$\displaylines{ {a_n} = 2{A \over {n\pi }}\sin \left({n\pi {{{t_p :}} \over T}} \right) = 2{A \over {n\pi }}\sin \left({{n\pi } \over 2}} \right) \cr n = 0,\,1,\,2 ,\,3,\,4,\,5,...\quad \sin \left({{n\pi } \over 2}} \right) = 0,1,0, - 1,0,1,... = \left\{ {\matrix{ { - {1^{{{n - 1} \over 2}}},\quad n\;odd} \cr {0,\quad n\;even} \cr } \right. \cr = \left\{ {\matrix{ {2{A \over {n\pi }}\left({ - {1^{{{n - 1} \over 2}}}} \right),\quad n\;odd} \cr {\quad 0,\quad n\;even,\;n 0\;} \cr } \right. \cr} $$ Values for them are listed in the table below. Note: This example was used on a page representing the Fourier series. Note also that in this case a (excluding n=0) is zero even n, and decreases by 1/n as n. n 0 0.5 1 0.6366 2 0 3 -0.2122 4 0 5 0.1273 6 0 7 -0.0909 Average + 1st harmonious to 3rd harmonious ... 5th harmonious ... 7th ... 21st Graph shows the function xT(t) (blue) and the partial sum of Fourieu (from n=0 to n=N) (red) $$$\sum\limits_{n = 0}^N {{a_n}\cos ({\omega _0}t)} $$ and the highest frequency harmony, $a_Ncos(N\omega_0t)$ (dotted purple). $a You can change n by pressing the buttons. As before, note: When you add sine waves of increasing frequency, approximation improves. The reasons for this are discussed below the Right button shows the sum of all harmonicas to the 21st harmonica, but not all individual sines are clearly shown in the story. In particular, harmonicas between 7 and 21 are not shown. Even Square Wave (Exploitation of Symmetry) In problems with even and odd functions, we can use the inherent symmetry to simplify the integral. We will use other symmetry later. Consider the problem above. We have an expression for a, n≠0 $$${a_n} = {2 \over T}\int\limits_{ { T \over 2}}^{T \over 2}} {{x_T}\left(t\right)\cos \left({n{\omega _0}t} \right)dt} $$ If xT(t) even, then the product xT(t)·cos(n·ρ0t) is paired (the product of two paired functions is even). Then we can use the fact that for the even function, e(t), $$\int\limits_{ - a}^{ + a} {e(t)dt} = 2\int\limits_0^a {e(t)dt} $$ so $${a_n} = {4 \over T}\int\limits_0^{{T \over 2}} {{x_T}\left(t \right)\cos \left({n{\omega _0}t} \right)dt} $$ which generates the same response, as before. It will often be easier to estimate than the original integrated media because one of the limits of integration is zero. Even Square Wave (Exponential Series) Consider, again, the pulse function. We can also represent the xT(t) exponential Fourier series $$${x_T}(t) = \sum\limits_{n = - \infty }^\infty {{c_n}} {e^{jn{\omega _0}t}}$$ We find cn $${c_n } = \int\limits_T {{x_T}(t){e^{ - jn{\omega _0}t}}dt} $$ As before, the gamer from -T/2 to +T/2 and use the facts, that the function is permanent for |t|<Tp/2 and в іншому місці, а T·ω0=2*·π. $$\begin{align} {c_n} &= {1

\over T}\int\limits_{ { T \over 2}}^{ + {T \over 2}} {{x_T}(t){e^{ - jn{\omega _0}}dt} = {1 \over T}\int\limits_{ { {{T_p}} \over 2}}^{ + {{{T_p}} \over 2}} {A{e^{ - jn{\omega _0}t}}dt} \cr &= \left. {{A \over { - Tjn{\omega _0}}}{e^{ - jn{\omega _0}t}}} \right|_{ - {{{T_p}} \over 2}}^{ { {{T_p}} \over 2}} = {A \over { - j2\pi n}}\left({{e^{ { { {T_p}} \over 2}}} - {e^{ + jn{\omega _0}{{{T_p}} \over 2}}}} \right) \cr\end{align} $$ Посвідчення Ейлера диктують, що e+jǎ-e-j|=2jsin(|) так e-jǎ-e=-2jsin(|) і $${c_n} = {{ - 2jA} \over { - j2\pi n}}\sin \left({n{\omega _0}{{{T_p}} \over 2}} \right) = {A \over {\pi n}}\sin \left({n{\omega _0}{{{T_p}} \over 2}} \) \right)$як і очікувалося, c0=a0 і cn=an/2, (n≠0) (оскільки це парна функція bn=0). Навіть трикутник хвиля (cosine Series) Розглянемо трикутник хвиля Середнє значення (тобто коефіцієнти серії 0-й Фур'є) становить a0 =0. For n>0 other coefficients the even symmetry of the function is exploited to give $${a_n} = {2 \over T}\int\limits_T {{x_T}\left(t \right)\cos \left({n{\omega _0}t} \right)dt} = {2 \over T}\int\limits_{ - {T \over 2}}^{ + {T \over 2}} {{x_T}\left(t \right)\cos \left({n{\omega _0}t} \right)dt} = {4 \over T}\int\limits_0^{ + {T \over 2}} {{x_T}\left(t \right)\cos \left({n{\omega _0}t} \right)dt} $$ Between t=0 and t=T/2 the function is defined by xT(t)=A-4At/T so $${a_n} = {4 \over T}\int\limits_0^{ + {T \over 2}} {\left(A - {{4A} \over T}t} \right)\cos \left({n{\omega _0}t} \right)dt} = {{4A} \over T}\left({\int\limits_0^{ + {T \over 2}} {\cos \left({n{\omega _0}t} \right)dt} - {4 \over T}\int\limits_0^{ + {T \over 2}} {t\cos \left({n{\omega _0}t} \ right) dt} } \right)$$ Виконайте інтеграцію (або вручну за допомогою інтеграції частинами, або з таблицею інтегралів, або комп'ютером) і використовуйте той факт, що ω0· T=2·π $${a_n} = {{4A} \over T}\left({{{T\sin (\pi n)} \over {2\pi n}} + {4 \over T}{{{T^2}{\mkern 1mu} \left({2{\mkern 1mu} \sin {{\left({{{\pi {\mkern 1mu} n} \over 2}} \right)}^2} - \pi {\mkern 1mu} n{\mkern 1mu} \sin \left({\pi {\mkern 1mu} n} \right)} \right)} \over {4{\mkern 1mu} {\pi ^2}{\mkern 1mu} {n^2}}}} \right)$$ Since sin(π·n)=0 this simplifies to $${a_n} = {{4A} \over T}{4 \over T}{{{T^2}{\mkern 1mu} 2{\mkern 1mu} \sin {{\left({{{\pi {\mkern 1mu} n} \over 2}} \right)}^2}} \over {4{\mkern 1mu} {\pi ^2}{\mkern 1mu} {n^2}}} = {{8A\sin {{\left({{{\pi {\mkern 1mu} n} \over 2}} \right)}^2}} \over {{\pi ^2}{\mkern 1mu} {n^2}}}$$ This answer is correct , але зазначивши, що $$n = 0,1,2,3,4,5,6,7,...\quad \quad \sin {\left({{{\pi {\mkern 1mu} n} \over 2}} \right)^2} = 0,1,0,1,0,1,0,... = {{{ 1 -{\left({ - 1} \right)}^n}} \over 2}$$ дає ще простіший результат $${a_n} = \left\{ {\matrix{ {4A{{1 - {{\left({ - 1} \right)}^n}} \over {{\pi ^2}{\mkern {n^2}}},\quad {\rm{n odd}}} \cr {0,\quad {\rm{n even}}}}\right.$$ If xT(t) is a wave triangle with A=1, the values for them are listed in the following table (note: this example was used on the previous page). n 0 0 1 0.8106 2 0 3 0.0901 4 0 5 0.0324 6 0 7 0.0165 Medium + 1st harmonious to 3rd harmonious ... 5th harmonious ... 7th ... 9th Note: this is similar, but not identical, to the triangle wave seen earlier. Note: When you add sine waves of increasing frequency, the approximation gets better and better, and these higher frequencies are better approximate details, (i.e. changing the slope) in the original function. Harmonica amplitudes for this example fall out much faster (in which case they go like 1/n2 (which is faster than the 1/n decay seen in the pulse function of the Fourier Series (above)). Conceptually, this is because the wave of the triangle looks much more like the 1st harmonious, so the contributions of higher harmonicas are less. Even with only the 1st few harmonies, we have a very good approximation to the original function. There's no gap, so there's no overloading Gibb. As before, the approximate function requires only odd harmonicas (1, 3, 5, ...); this is due to symmetry function. Until now, the features under consideration have been even. The following diagram shows the odd function. In this case, the Fourier Sine series fits $$${x_T}(t) = \sum\limits_{n = 1}^\infty {{b_n}\sin \left({n{\omega _0}t} \right)} \quad \quad {b_n } = {2 \over T}\int\limits_T {x_T}\left(t\right)\sin \left({n{\omega _0}t} \right)dt} $$ The easiest way to integrate from -T/2 to +T/2. During this interval $x_T(t)=2At/T$. $${b_n} = {2 \over T}\int\limits_{ { T \over 2}}^{ + {T \over 2}} {{x_T}\left(t \right)\sin \left({n{\omega _0}t} \right)dt} = {{right) \over T}\int\limits_{ - {T \over 2}}^{ + {T \over 2}} {{{{2At} \over T}\sin \left({n{\omega _0}t} \right)dt} $$ Implementation of integration (and use of the fact, that ρ0· T=2·π) the integral yields $${b_n} = {2 \over T}{{A{\mkern 1mu} T{\mkern 1mu} \left({\sin \left({\pi {\mkern 1mu} n} \right) - \pi {\mkern 1mu} n{\mkern 1mu} \cos \left({\pi {\mkern 1mu} n} \right)} \right)} \over {{\pi ^2}{\mkern 1mu} {n^2}}}$$ Using two simplification, sin(π·n)=0 and cos(π·n)=(-1)n gives $${b_n} = - {{2A{\mkern 1mu} } \over {\pi n}}{\left({ - 1} \right)^n}$$ Functions that are neither even nor odd So far, all of the functions considered have been either even or odd, but most functions are neither. This does not present conceptual difficulties, but may require more integration. For example, if the xT(t) function looks like the one below Because it doesn't have obvious symmetry, a simple Sine or Cosine series isn't enough. Fourie's trigonometry series requires three $$$\begin{align} $x_T}(t) &= {a_0} + \sum\limits_{n = 1}^\infty {\left({{a_n}\cos \left(_0}t} \right) + {b_n}\sin \left({n{\omega _0}t} \right)} \right)} \cr {a_0} &= {2 \over T}\int\limits_T {{x_T}\left(t \right)dt} \cr {a_n} &= {2 \over T}\int\limits_T {{x_T}\left(t \right)\cos \left({n{\omega _0}t} \right)dt} ,\quad n e 0 \cr {b_n} &= {2 \over T}\int\limits_T {{x_T}\left(t \right)\sin \left({n{\omega _0}t} \right)dt} \cr\end{align} $$ However, an exponential series requires only a single integral $$\displaylines{ {x_T}(t) = \sum\limits_{n = - \infty }^{ + \infty } {{c_n}{e^{jn{\omega _0}t}}} \cr {c_n} = {1 \over T}\int\limits_T {{x_T}\left(t \right){e^{ - jn{\omega _0}t}}dt} \cr} $$ For this reason , серед інших, експоненціальну серію Фур'є часто легше працювати, хоча вона не має простої візуалізації, яку надає тригонометрична серія Фур'є. Вплив симетрії функцій на коефіцієнти Якщо функція xT(t) має певні симетрії, ми можемо спростити розрахунок коефіцієнтів. Симетрія Тригонометрична серія та симетрія Симетрія Спрощення xT(t) становить навіть $$\displaylines{ {a_0} = середній \cr {a_n} = {4 \over T}\int\limits_0 ^{ {T \over 2}} {{x_T}(t)\cos \left({n{\omega _0}t} \right)dt} , \quad n eq 0 \cr {b_n} = 0 \cr} $$ xT(t) є непарним $$\displaylines{ {a_n} = 0 \cr {b_n} = {4 \over T}\int\limits_0^{ + {T \over 2}} {{x_T}(t)\sin \left({n{\omega _0}t} \right)dt} \cr} $$ xT(t) має функцію, яка може мати напівхвильову симетрію, не будучи навіть або непарною. $$\displaylines{ {a_n} = {b_n} = 0,\quad n\;even \cr {a_n} = {4 \over T}\int\limits_0^{ + {T \\ більше 2}} {{x_T}(t)\cos \left({n{\omega _0}t} \right)dt} ,\quad n\;odd \cr {b_n} = {04 \over T}\int\limits_0^{ + {T \over 2}} {{x_T}(t)\sin \left({n{\omega _0}t} \right)dt} ,\quad n\;odd \cr} $$ Перші дві симетрії обговорювалися раніше в обговореннях імпульсної функції (xT(t) парна) і розпилювана хвиля (xT(t) непарна). Напівхвильова симетрія зображена на схемі нижче. Верхня функція, xT1(t), непарна (xT1(t)=-xT1(-t)), але не має напівхвильової симетрії. Нижня функція, xT2(t) є чистою навіть і не дивною, але оскільки xT2(t)=-xT2(t-T/2), вона має напівхвилю симетрію. Щоб візуалізувати це уявіть собі зміщення функції на півставки (T/2); для напівхвильової симетрії зміщеною функцією має бути дзеркальне відображення оригінальної функції (про горизонтальну вісь), як показано нижче Причина, через яку коефіцієнти парних гармонік нульові, можна зрозуміти в контексті діаграми нижче. Верхній графік показує функцію, xT(t) з напівхвильовою симетрією разом з першими чотирма гармоніками серії Фур'є (потрібні лише гріхи, оскільки xT(t) є непарним). На нижньому графіку показано гармоніки, помножені на xT(t). Тепер уявіть собі інтеграцію умов продукту від -T/2 до +T/2. Непарні терміни (з 1-го (червоного) і 3-го (пурпурових) гармонік) будете мати позитивний результат (оскільки вони вище нуля більше, ніж нижче нуля). Парні терміни and lian) integrate to zero (because they are equally higher and below zero). Although this is a simple example, the concept applies to more complex functions and to higher harmonicas. The only function that was discussed with semi-wave symmetry was a triangle wave and indeed odds with even indicators equal to zero (like all terms of bn due to even symmetry). A square wave with a 50% duty cycle would have half the wave symmetry if it were in the center near zero (i.e. in the center of the horizontal axis). In that case, the term a0 will be zero, and we've already shown that all terms with pairs of indexes are zero, as expected. Simplifications can also be made on a basis - but they are not discussed here. Exponential series and symmetry Since cn series coefficients Exponential furies are associated with a trigonometry series using $$$\displaylines{ {c_0} = {a_0} \cr {c_n} = {{{a_n}} \over 2} - j {{{b_n}} \over 2}for\;n e 0 \cr {c_{ - n}} = c_n^* \cr} $$ (if xT(t) is real) we can use the trigonometric series symmetry to find and billion, and therefore cn. However, in addition, cn coefficients contain some symmetry of their own. Specifically, the value of cn terms even relative to n: |c-n|=|cn|. The angle of terms cn is odd in relation to n: ∠c-n=-∠cn. The real part cn even (Re(c-n) = Re(cn)) and the imaginary part of the odd (Im(c-n) =-Im(cn)) If xT(t) is even, then bn = 0 and cn is even and real. If xT(t) is odd, a=0 and cn is odd and imaginary. Some comments about the pulse function Let's look at the presentation of fourie series periodic function rectangular pulse, PT (t / Tp), more carefully. Since the function is uniform, we expect furie's exponential series coefficients to be real and even (from symmetry properties). In addition, we have already calculated the coefficients of the trigonometric series, and could easily calculate them from the Exponential Series. However, let's do it from the first principles. Furje's exponential series odds are given with $$$\displaylines{ {\Pi _t}\left({{t \over {{T_p}}}} \right) = \sum\limits_ T_p {{n = - \infty }^{ + \infty } {{c_n}{e^{jn{\omega _0}t}}} \cr with \cr {c_n} = {1 \over T}\int_T {{\Pi _t}\left({{t \over {{T_p}}}} \right){e^{ - jn{\omega _0 }t}}dt} \cr } $$ We can change the integration boundaries to -Tp/2 and +Tp/2 (because the function is zero elsewhere) and continue (function one in this interval, interval so we can throw it). We also use the fact that ņ0 = 2ρ /T and Eileer's identity for sine. $$\begin{align} {c_n} &= {1 \over T}\int\limits_{ { T \over 2}}^{ { {T \over 2}} {{\Pi _t}\left({{t \over {{T_p}}}} \right){e^{ -jn{{left \omega _0}}dt} = {1 \1 via T}\int\limits_{ { {{T_p}} \over 2}}^{ + {{T_p}} \over 2}} {{e^{ - jn{\omega _0}t}}dt} \cr & {1 \over { - jn{\omega _0}T}}\left({{e^{ \over 2}}} - {e^{ + jn{\omega _0}{{{T_p}} \over 2}}} \right) = {1 \over { - jn2\pi }}\left({{e^{ - j{{n\pi {T_p}} \over T }}} - {e^{ + j{{n\pi {T_p}} \over T }}} \right)\quad \quad {\omega _0} = {{2\pi } \over T} \cr &= {1 \over { - jn2 \pi }}2j\sin \sin on the left({ {{n\pi {T_p}} \over T}} \right) = {1 \over {n\pi }}\sin \left({{n\pi {T_p}} \over T}} \right) \cr &{{{T_p}} \over T}} \left({{{n\pi {T_p}} \over T}} \right)} \over {{n\pi {T_p}} \over T}} \end{align} $$ The last step in output is in progress, so that we can use the sinc() function (pronounced as a sink). This feature often appears in Fourier's analysis. $$${\mathop{\rm sinc}olimits} (x) = {{\sin \left({\pi x} \right)} \over {\pic}}$$ With this definition, the odds are simplified to $${c_n} = {{{{{$$ T_p}} \over T}{\mathop{\rm sinc}olimits} \left({{{n{T_p}} \over T}} \right)$$ The following diagram shows cn vs n for multiple duty cycle values, Tp/T. Duty Cycle = 0.1 ... 0.25 ... 0.5 ... 0.75 ... 0.9 The graph on the left shows the time domain function. If you press the middle button, you'll see a square wave with a duty cycle of 0.5 (i.e. it's a high 50% of the time). The square wave period is T=2·π;. The graph on the right shows cn vs n as red circles vs n (bottom of two horizontal axes; ignore top axis for now). The blue line passes through the horizontal axis every time the sinc() function argument is n· Tp/T is an integer (except when n=0.). In particular, the first intersection of the horizontal axis is given n· Tp/T=1 or n=T/Tp (note that this is not an integer Tp value). There are several important features to note as Tp is diverse. Because Tp decreases (along with the duty cycle, Tp/T), so does the c0 value. This is to be expected because c0 is only the average of the function, and it will decrease, as does the width of the pulse. As Tp decreases, the sinc() width function expands. This tells us that as the function becomes more time-localized (i.e. vuzher), it becomes less localized in frequency (wider). In other words, if the function occurs very quickly in time, the signal must contain high-frequency coefficients to enable rapid change. Let's call the function width sinc() the width of the main particle (i.e. between the first two zero intersections around ρ=0), Pn=2· T/Tp. If we call the pulse width Pt =tp, $$\left({\Delta n} \right) \cdot \left({\Delta t} \right) = \left({2{T \over {{T_p}}}} \right) \cdot \left({2 {T \over {{T_p}}}} \right) \cdot \left({2{T\} {T_p}} \right) = 2T = {\rm{constant}}$$ This clearly tells us, that the product frequency width (i.e. Pn) multiplied by the width of the time (Pt) is constant - if doubled, the other by half. Or - as one gets more localized in time, it's less localized in frequency. We'll discuss it later. Replace link

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