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Geometry revisited solutions pdf

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Problem 2 Show that in any triangle, Solution Using the law of sines again: Likewise, Adding the three equations: as desired. Problem 3 Show that in any triangle, (although the book uses , I usually write on my blog to designate the area of) Solution Let h be the height of (from A perpendicular to bc), so that: Using trigonometry, so that we can write the area as the extended law of Sines, Now we can rewrite the area again: as desired. Problem 4 For a triangle, whether it is the radius of a circle passing through A and tangent to British Columbia to B. Whether q the radius of a circle passing through A and tangent to BC to C. Show that: Solution Let M be the center of the circle with the p radius, and N be the center of the circle with the q radius. Also K and L are two points on the M and N circles respectively: Using the sine law on circle N and M, we have: Because British Columbia is tangent with both circles, and . So, now, the application of the law of Sines again in: Substitute these values again in the previous equation: By multiplication, as desired. My eldest son struggled to find the length of an angle bisector in a 3-4-5 triangle in his enrichment math class last week. Solving this problem is a bit tedious, but also provides a great opportunity to present Stewart's theoreme. I first learned about Stewart's theoreme from Geometry Revisited when I was in high school. Here's an explanation of theorem on Wikipedia: Theoreme of Stewart on Wikipedia, I started the project tonight by reviewing the original problem with my son: Then I briefly presented the then we were interrupted by someone knocking on our front door: Now I showed how the evidence is going. We had a brief discussion/recall about the relationship between and after the evidence went pretty quickly: Finally, we went back to our original triangle to calculate the length of the angle bisector Stewart's theorem. The calculation is still a bit long, but now the calculations themselves are quite simple: Definitely a beautiful theorem. It is surprising that the law of the cosines simplifies so well and that the calculation of the lengths of the cevians of a triangle. Yesterday, Patrick Honner published a beautiful illustration of the Theorème de Varignon by one of his students: It's particularly fun to move the dots to form a non-convex quadrangle and to see that the midpoints still form a parallelogram. As with many advanced geometry concepts, my introduction to Theorme de Varignon came from Coxeter and Greitzer's Geometry Revisited. I remember theorem in part because of the beautiful introductory statement in the book: The next theorem is so simple that one is surprised to find its publication date to be as late as 1731. It is due to Pierre Varignon (1654 - 1722). Theorème 3.11. The figure formed when the middle points on the sides of a quadrangle are joined in order is a parallelogram, and its surface area is half that of the quadrangle. The chapter also presents three other wonderful theorists and some great problems including this one: 1. [Show that] the perimeter of Varignon's parallelogram is equivalent to the sum of the diagonals of the original quadrangle. So this special parallelogram has some really interesting properties! As a fun follow-up this morning, my eldest son was working on some problems with the 2006 AMC 8. Looking over the test, I noticed that the problem #5 was a simple example of Varignon's theoreme: Problem #5 of the AMC 8 2006 I chose a different problem to go all the way with my eldest son, but I thought my youngest son would like this one. It was a challenge, but he was finally able to work through it. It's a bit of fun to think of this basic problem like the one that opens the door to this beautiful theorem. Cathy O'Neil posted this awesome track on trig on her blog yesterday: Fuck Trigonometry He created quite a conversation. Yay! Her husband's comment at the end of the post caught my attention and I spent most of yesterday sort of daydreaming about her: When I mentioned my hatred of trigonometry to my husband, he countered with an argument that has not been mentioned so far. Namely, we really have no reason to teach high school students a given thing, so we just choose a lot of things a little bit randomly. Also, he suggested, if we remove trig, then meeting people at an airport would just spark another reason to hate mathematics. We would simply replace trig with another crappy subject choice. I think I lean to agree with him. I'm certainly not sure I could do a convincing about why trig needs to be taught. In fact, with 3D printing and maybe even Zometool sets getting cheaper and easier to find, my vote would probably try more fun geometry projects before trig. Our curved gosper project, for example, is something I think kids would find more entertaining than trig identities: Exploring the gosper curve The passion in the conversation around Cathy's post also surprised me a bit - I didn't know that so many people had such strong feelings on trig! Most of the internet math flame wars I see are about adding or fractions - watching people fight on trig was so refreshing ☺ Thinking about my high school trig class with Mrs. Kovaric yesterday, honestly, I didn't really remember having strong feelings one way or another. Without any strong opinion to fight over (ha ha) I started thinking about some fun math ideas related to trig that I had learned either in high school or university. No reasons to teach trig, that's for sure, but certainly more fun than memorizing identities! (1) The Extended Law of Sines An idea that everyone sees in trig class is the law of the sines - in any abc triangle, A/Sin (A) - B / Sin(B) - C / Sin(C). Relationship pretty neat, but if these three expressions are all equal to each other is their special value? Turns out it's: (2) Stewart's theoreme It's a cool theorem that gives the length of a line segment a top of a triangle on the opposite side. As with the extensive law of sines, this theorem is something I found in Geometry Revisited in high school. The proof (as I know) implies the law of the cosines: Also, the law of the cosines came in a surprising way in an introductory speech to geometry that I had with my eldest son during the last school year. This conversation was an unexpected way (for me) that you could talk about the ideas behind the law of the cosines in geometry class: When we accidentally derived the law of the cosines (3) The sum of the reverse squares Using the Taylor series for sin(x) and the fact that the roots are multiplesteger of , you can prove it: It was incredibly cool to learn that there was a formula known for all the same reverse powers (resolved by Euler in the 1700s, if I remember correctly), but that a closed form for odd powers greater than 1 was not known. This is a fine example of an unresolved math problem that high school students can understand and even play with a little. I always hoped to see a closed-form solution for the sum of the reverse cubes. Another rather famous trig-related sum problem that blew me away in high school is this incredible sum: Let's be the positive solution to the equation. Find The particularly amazing thing about this problem is that you can the sum, even if you can not write a closed form for any of the expressions that are in the sum! (4) A surprising integral part I went to planning college on major aerospace engineering - that's what you do with mathematics, right ☺ Sitting in an introduction to the complex analysis class of my first year, I ran through this this Small problem: Seeing this problem made me want to get major in mathematics rather than engineering - it was absolutely amazing to me that and could be connected in a seemingly mysterious way. (5) Circles on a sphere This one is the one and only time I've used trig directly to work (probably more than 10 years ago, but I don't remember the exact moment). One of the guys in our office who thinks about hurricane insurance had a list that gave the latitude and longitude of the center of every hurricane that has hit North America over the last 50 years or so. The list had coordinates for the center in 6-hour time increments. The question he wanted to answer was relatively simple: given a specific latitude/longitude (say Miami or New York, or something like that), how many hurricanes had come at a given distance from that city (50 miles, 100 miles, . . .). He had tried to write a very quick program and back of the envelope to answer that question, but he gave answers that seemed really wrong. To correctly calculate the distance, you need a bit of trig because you have to take into account the distance to the north that you are. Adjusting the distance formula for a given latitude helped him get the right answer. There were a few other little math-related tricks in the program, too, like checking whether or not the path between two points came in the desired distance, even though the endpoints were outside the distance. Without trig, the distance calculations in this project were easy to make a mistake. Anyway, not a list of reasons to teach trig, but rather just a few trig-related fun things that Cathy's post made me think. Hopefully a little more fun than memorizing identities ☺ Although if you've done so far and do like trig identities, however, a recent Terry Tao post should be just up your aisle: A cute Path differentiation identity in 2011 when I started thinking about making math movies, I filmed a set of practice lectures based on the first two sections of Geometry Revisited. The main point was to evaluate myself by talking about mathematics - by that time it had been more than 10 years since I had been in a classroom. The first practice conference where I try to shake up some of the old rust was on the extensive law of sines. What I learned in my trig class back in high school is that for a triangle: a /Sin(A) 'b / Sin(B) ' c / Sin(c). It's an incredible identity, but there's a pretty natural question that I've never thought of asking - if these all have the same value, is there anything special about this value? This is the question this conference is trying to answer: today I did a similar exercise with my son while we were working in the review section of the Power of Point chapter of our book on geometry. By studying the power of a point identities you learn that many products involving lines and and are equal to each other - but does the value of the product have any special meaning? See... So for the points outside for the circle, we just found the value of the products that come in the power of a point formulas. We have seen, however, that when the dot is inside the circle, we will need to have a different formula. We draw this accompanying formula here: So a fun journey down the road to memory for me this morning while reviewing some power of a point idea. Hopefully work like this plants a little mathematical idea in my son's mind - it two (or more) expressions are equal to each other, maybe there is something special about the value of these expressions. This morning, my eldest son and I worked through a great example problem in problem solving art introduction to the geometry book. Surprising luck the section is one of the sections that the art of solving problems highlights on their web page on the book, so feel free to check out problem 5.7 here (and don't take a look at the solution!): We actually encountered the problem yesterday, but I wanted to devote a whole day to it today because the intelligent use of ratios in this problem is so instructive. I certainly didn't want the mathematical beauty in this example to be lost because we had to rush through it. Also, we've been away from fractions and ratios for a while, so I guess it would take a full hour to go through the problem in detail. That's what he did. In the middle of talking through the problem this morning, I remembered that the proof of Ceva's theorem also uses ratios in an intelligent way, and I thought that a fun follow-up on this morning's example would be to walk through the evidence of Ceva's theorem tonight. It's really amazing that you can prove this beautiful theorem with just the zone formula for triangles and intelligent use of ratios. In the presentation below, I'm the proof given to section 1.2 of Geometry Revisited, which is where I learned about the theorme back in high school (and since I was too lazy to take a new photo, check out C.D. Olds's Continued Fractions book too!!): I started by stating the problem and showed how to start on the evidence by making some simple observations about the areas. Then we started looking at the idea of neat ratio: Since this idea about ratios is really not so intuitive that I wanted to take a little break from geometry to just get a better understanding of why behave in this seemingly strange way. My son had the good idea to look at the relationship in an abstract way to see why it was true. It's a bit funny that the relationship is easier to see abstractly than with specific numbers. In the last part of the evidence, we show that the proceeds of the three ratios are equal to one. In the first video, we showed that the first ratio we were looking for was equal to the ratio of two triangles. We apply the same argument for the two remaining ratios and find two other sets of triangles whose areas are in the report we were originally looking at. If you're careful with how you label these triangles (and I wasn't) you quickly see that the product of the three ratios is equal to one. If you are not careful, it takes a little extra time to see that all products cancel. So a really fun example. For a child learning geometry, it probably seems quite surprising that a concept of arithmetic will lead to such an impressive result in geometry. I like this aspect of showing this evidence, too, and I also like the opposite - namely that a day of studying geometry gave us a great chance to review the ratios. A super fun day overall, and all from a cool example of our geometry book! Patrick Honner has several blog posts on poor problems (sometimes outright errors) on some of the exams that New York State requires for students. As I don't live in New York, I'm not super familiar with these exams, although Mr. Honner's posts actually make me happy that I'm not. I feel like the more I knew about them, the more they drove me crazy. Anyway, his latest article is here: Regents Recap — June 2014: When Good Math Becomes Bad Tests and the problem he discusses is: The medians of a triangle intersect at some point. What measures could represent the segments of one of the medians? a) 2 and 3 (b) 3 and 6 (c) 3 and 4.5 (d) 3 and 9 Unlike some of the previous problems he wrote about the problem itself has no mathematical flaws. Instead, this problem is simply testing if you know a single mathematical fact. Really no deep understanding of geometry is necessary to solve it at all if you know this fact. I think Mr. Honner's concern — essentially that if the state exam issues end up being similar to this one, math education will simply turn into something equivalent to preparing a night on Jeopardy — is there. This criticism struck me for another reason, however. Three years ago, when I decided I wanted to start making fun math videos for kids, I thought I should practice a little and see if I really had the ability to explain mathematics. At that time, it had been more than 10 years since I had been in front of a classroom. Oh, and 10 years in finance doesn't exactly sharpen your explanation skills. What I decided to do was take my of Geometry Revisited on set and pretending I was doing some lectures from the book. I went through the first two chapters, but two of the first sections are relevant here. The second conference was on theorem of Ceva, which is a beautiful theorem with fascinating and incredibly instructive evidence (keep in mind that I hadn't talked about mathematics in a long time in this video there The best. Also my eldest son looks for some reason, I don't remember): So we get a beautiful theorem with really instructive proof right in the second section of revisited geometry! In addition, I saw that Steve Leinwand gave a lecture at a conference for teachers last week and said that ratios were one of the most important elements of early mathematics. The ratios play a surprising role in this proof of Ceva's theorem, so it may have even more educational value than I realized the first time. Finally, the result that we can see quite easily from the Ceva theorem that is relevant to the question on the New York State review is that the medians of a triangle intersect at a single point. With some fun ideas about cevians in hand, you might be interested in learning even more, and Geometry Revisited doesn't disappoint. The following section shows some good results on medians that also have incredibly instructive evidence (the median part starts at 3:20): I remember very well how to go through these first sections in Geometry Revisited reminded me how much I liked mathematics. With only two short sections on theorem and medians of Ceva, we have some nice results and several really informative evidence for students to see. I understand that not all children will find these ideas as fascinating as I am, but I think many children will. It seems a pity to reduce everything to the question asked above, and frankly even worse to essentially reduce it to something like - Answer: They sorts each other out. Question: What do medians do in a triangle? Do?