11.3 geometric sequences and series answers

Learning objectives Find a common ratio of the geometric sequence. List the terms of the geometric sequence. Use a recursive formula in a geometric sequence. Use an explicit formula in a geometric sequence. Many jobs offer an annual increase in the cost of living to keep wages in line with inflation. For example, assume that a recent university graduate finds a position as a sales manager who earns \($26,000\). He's promised that the cost of living will increase every year. His annual salary each year is obtained by multiplying his previous year's salary by \(102\%\). His salary is \($26,520\) after one year; \($27,050.40\) after two years; \($27,591.41\) after three years; And so on. When the salary rises at a fixed rate every year, the salary increases by a standard coefficient. In this section, we will look at the sequences grown in this way. The annual salary values described form a geometric sequence because they change with a constant factor each year. Each term in the geometric sequence grows or decreases with a constant factor called a common relationship. The sequence below is an example of a geometric sequence, since each term grows with a constant factor of 6. Numbering any term in a series with a common ratio of 6 creates the following term. Definition: GEOMETRIC SEQUENCE The A geometric sequence is one in which any term divided by the previous term is constant. This constant is called a sequence relationship. The common ratio can be found by dividing any term in the series by the previous term. If \(a_1\) is the first term of the geometric sequence and \(r\) is a global relationship, the series is \[\{a_1, a_1r,a_1r^2,a_1r^3,...\} .\] How: Use a number sequence to specify whether they represent a geometric sequence. Share each term with the previous term. Compare the subdi section. If they are the same, there is a common relationship and the sequence is geometric. Example \(\PageIndex{1}\): Find common ratios Is the sequence geometric? If so, find a common ratio. \(1\), \(2\), \(4\), \(8\), \(16\),... \(48\), \(12\), \(4\), \(2\),... Solution Share each term with the previous term and determine if there is a common relationship. \(\dfrac{2}{1}=2\) \(\dfrac{4}{2}=2\) \(\dfrac{8}{4}=2\) \(\dfrac{16}{8}=2\) The order is geometric because the ratio is shared. The global ratio is \(2\). \(\dfrac{12}{48}=\dfrac{1}{4}\) \(\dfrac{4}{12}=\dfrac{1}{3}\) \(\dfrac{2}{4}=\dfrac{1}{2}\) The order is not geometric because there is no common relationship. Analysis The chart for each sequence is shown in \(\PageIndex{1}\). From the charts, it appears that both (a) and (b) appear in the form of an exponential function chart in this viewer. However, we know that (a) is geometric, so this interpretation holds, but (b) is not. \(\PageIndex{1}\) Q&A If you are told that the sequence is share each term in the previous period in order to find a common relationship? Not. If you know that the sequence is geometric, you can choose any term in the series and divide it with the previous term to find a common relationship. Training \(\PageIndex{1A}\) Is the sequence geometric? If so, find a common ratio. \(5\), \(10\), \(15\), \(20\),... Answer Series is not geometric because \(\dfrac{10}{5}≠\dfrac{15}{10}\) Training \(\PageIndex{1B}\) Is the sequence geometric? If so, find a common ratio. \(100\), \(20\), \(4\), \(\dfrac{4}{5}\),... Answer The sequence is geometric. The common relationship is \(\dfrac{1}{5}\) Now that we can identify the geometric sequence, we will learn to find the terms of the geometric sequence if we are given the first term and the common relationship. The terms of the geometric sequence can be found by starting with the first term and multiplying the common relationship repeatedly. For example, if the first term of a geometric sequence is \(a_1=−2\) and the general ratio is \(r=4\), we can find the following terms by multiplying \(−2·4\) to get \(−8\) and multiplying the result \(8·4\) to get \(−32\) and so on. \[\begin{align*} a_1 &= −2 \\ a_2 &= (−2·4)=−8 \\ a_3 &= (−8· (4)=−32 \\ a_4 &= (−32·4)=128 \end{align*}\] The first four terms are \(\{2, –8, –32, –128\}\). How to: Considering the first term and common factor, look for the first four terms of the geometric sequence. Multiply the original term \(a_1\) by a common relationship to find the following term\(a_2\). Use \(a_n=a_2\) to repeat the process to search \(a_3\), and then use \(a_3 a_4\) \(a_4\) to find it. Type terms separated in parentheses by commons. Example \(\PageIndex{2}\): Enter geometric sequence list criteria with the first four terms of the geometric sequence \(a_1=5\) and \(r=–2\). To find \(a_1\), multiply \((−2\) and search \(a_2\). Repeat the process using \(a_2\) to find \(a_3\) and so on. \[\begin{align*} a_1 &= 5 \\ a_2 &= −2a_1=−10 \\ a_3 &= −2a_2=20 \20 \\ \ a_4 &= −2a_3=−40 \end{align*}\] The first four terms are \(\{5,–10.20,–40\}\). Training \(\PageIndex{2}\) List the first five terms of the geometric sequence with \(a_1=18\) and \(r=\dfrac{1}{3}\). Answer \(\left \{18, 6, 2, \dfrac{2}{3}, \dfrac{2}{9} \right \} \) We can use a recursive formula to find any geometric sequence term using the previous term. Each term is a product of a common ratio and a previous term. For example, assume that the global ratio is \(9\). Then each term is nine times the previous period. As with all recursive formulas, the first term must be entered. Note: GEOMETRIC SEQUENCE RECURSIVE FORMULA A recursive formula for a geometric sequence with a common relationship r and the first term \(a_1\) is n≥2\] How to: Consider the first several conditions, conditions, its recursive formula. The first term is. Find a common relationship by sharing any term with the previous term. Replace the common relationship with the geometric sequence recursive formula. Example \(\PageIndex{3}\): Use recursive formulas for geometric sequences Type a recursive formula for the following geometr sequence. \(\{6, 9, 13.5, 20.25, ...\} onumber\) Solution The first term is given as \(6\). The common ratio can be found by dividing the second season in the first period. \(r=\dfrac{9}{6}=1.5 onumber\) Replace the common relationship with the recursive formula of geometric sequences and specify \(a_1\). \[\begin{align*} a_n &= ra_{n−1} \\ a_n &= 1.5a_{n−1} \text{ for }n≥2 \\ a_1 &= 6 \end{align*}\] Analysis The order of data points follows the exponential formula. A global ratio is also the basis for an exponential function, as shown in \(\PageIndex{2}\). Picture \(\PageIndex{2}\) Q&A Do we need to share another term with the first term to find a common relationship? Not. We can share any term in the previous period. However, it is most common to share another term in the first term, since it is often the easiest way to find a common relationship. Training \(\PageIndex{3}\) Enter a recursive formula for the next geometric sequence. \(\{2, 43, 89, 1627, ...\}\) Reply \(\begin{align*}a_1 &= 2 \\ a_n &= \dfrac{2}{3}a_{n−1} \text{ for }n≥2 \end{*align}\) Because the geometric sequence is an exponential function, whose domain is a set of positive integers, and a common relationship is the basis of a function, we can write clear formulas that allow us to find specific terms. \[a_n=a_1r^{n−1}\] Let's take a look at \(\{18, 36, 72, 144, 288, ...\}\). This is a geometric sequence with a common relationship of \(2\) and an exponential function based on \(2\). The explicit formula for this series is \(a_n=18·2^{n−1}\) The chart for the series is shown in \(\PageIndex{3}\). Image \(\PageIndex{3}\) EXPLICIT FORMULA FOR GEOMETRIC SEQUENCE The term of the geometric sequence \(n^{th}\) is given by using an explicit formula: \[a_n=a_1r^{n−1}\] Example \(\PageIndex{4}\): Enter geometric sequence criteria Using an explicit formula Enter a geometric sequence with \(a_1=3\) and \(a_4=24\), locate \(a_2\). Solution The sequence can be written based on the original term and the common relationship \(r\). \(3\), \(3r\), \(3r^2\), \(3r^3\),... Use the fourth term entered to find a common relationship. \[\begin{align*} a_n&=a_1r^{n-1} \\ a_4&=3r^3 \qquad \text{Type the fourth term in the series as }\alpha_1 \text{ and } r \\ 24&=3r^3 \qquad \text{Substitute }24 \text{ for }a_4 \\ 8&=r^3 \qquad \text{Divide} \\ r&=2 \qquad \text{Solve for the common ratio} \end{align*}\] Multiply the first term by a common ratio. \[\begin{align*} a_2 &= \\ a_1 &= 2(3) \\ & = 6 6 Analysis The common ratio is multiplied by the first term once to find the second term, twice to find the third term, three times to find the fourth season, and so on. The tenth term could be found by multiplying the first term by a common ratio nine times or multiplying to ninth a common ratio. Training \(\PageIndex{4}\) Considering the geometric sequence \(a_2=4\) and \(2a_3=32\), look for \(a_6\). Answer \(a_6=16,384\) geometric sequence term Enter an explicit formula for the following geometric sequence n. \(\{2, 10, 50, 250, ...\}\) Solution The first term is \(2\). The common ratio can be found by dividing the second season in the first period. \(\dfrac{10}{2}=5\) The global ratio is \(5\). Replace the shared relationship and the first term in the series with the formula. \[\begin{align*}a_n &= a_1r^{(n−1)} \\ a_n &= 2·5^{n−1} \end{align*}\] The diagram in this series in image \(\PageIndex{4}\) displays an exponential pattern. Picture \(\PageIndex{4}\) Training \(\PageIndex{5}\) Type the explicit formula for the next geometr sequence. \(\{–1, 3, –9, 27, ...\}\) Response \(a_n=−{(−3)}^{n−1}\) In a real scenario with aritmetic sequences, we may need to use \(a_0\) instead of \(a_1\). For these issues, we can use the following formula to change the explicit formula slightly: \(a_n=a_0r^n\) Example \(\PageIndex{6}\): Solving application problems with geometric sequences In 2013, the number of students in a small school is \(284\). The student population is estimated to grow annually\ (4\%\). Enter a formula for the student population. Estimate the student population in 2020. Solution The situation can be modeled using a geometric sequence with an initial period of \(284\). The student population is \(104\%\) in the previous year, so the common ratio is \(1.04\). Enter \(P\) to be a student population and \(n\) after 2013. With the geometric sequence explicit formula, we get \(P_n =284·{1.04}^n\) We can find the number of years since 2013 by subtracting. \(2020−2013=7\) We will be looking for a population \(7\) in a year' s time. We can replace \(7\) \(n\) to estimate the population in 2020. \(P_7=284·{1.04}^7≈374\) The number of students will be approximately \(374\) in 2020. Training \(\PageIndex{6}\) The company will start a new site. Initially, there are \(293\) hits due to the curiosity factor. The business expects hits to increase by \(2.6%\) per week. Enter a formula for the number of hits. Evaluate hits \(5\) in weeks. Answer a \(P_n = 293·1.026a^n\) Answer b Hits are approximately \(333\). geometric sequence nth term repeating formula \(a_n=ra_{n−1}\), \(n≥2\), \(n≥2\) explicit formula for geometric sequence n: th term \(a_n=a_1r^{n−1}\) \(a_n=a_1r^{n−1}\)

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