Fast-growing hierarchy diagonalization

In a previous post, I discussed Really Big Numbers, moving from the example of many children of a large number, a million, beyond what most people I meet as a huge number, a googol, and eventually going by the number of Graham, BOOM(3), the busy beaver function, infinities and beyond. I wasn't aware of it at the time, but a much better version of that post already existed: Who Can Name the Bigger Number?, by Scott Aaronson. In my original post, I made a few mistakes in the section on fast growing features. Some kind of commentors helped correct the most egregious errors, but the resulting corrections littered that whole part of the post with strikethrough text that I was never really happy with. Now, six years later, I finally want to make up for my mistakes. I'd like to mention some very big numbers. I am not going to talk about the smaller ones, nor those who delve into infinity; you read the previous post before. Here I just want to point to some really big finite numbers. The numbers I'm aiming for are numbers, like 1, 2, or a billion. They're not infinitely large. These are numbers that, if someone were to ask you to write a really, really big number, these would be way beyond what the questioner was thinking, and yet not infinite in size. We always use functions when writing numbers. It's just that most of the time, it's invisible to us. If we count apples, we can brand a hatch (or tally) for the first apple, another hatch for the second (‖), and so on. This works fine for up to a dozen apples or so, but it's starting to get pretty hard to understand at a glance. You could solve this by making every fifth hatch cross over the previous four (卌), but you quickly get back into a problem if you notice too many sets of five hatch. It is easier to come up with better format, such as using numbers. Now we can use 1 or 5, instead of actually unsubscribing all those hatch marks. Then we can use a simple function to track our notation more easily. The most right-wing figure is that place, then next to the left is the dozens of place, and the next one on the left is the hundreds of place, and so on. 123 means (1*100)+(2*10)+(3*1). Of course I'm loose with definitions here, as I've written 100 and 10 using the system I'm trying to define. Feel replaced by tally marks: 2*10 is ‖* 卌卌. As you can see, features are integral to a notation. So when I start turning to new notations using features to describe them, you shouldn't pretend that this is somehow fundamentally different from the notations you probably have in everyday life. Using Knuth arrow notation is no less valid for saying the name of a song than writing 123. They are both names that indicate a certain number of tally brands. Let's start with addition. Addition Addition surgery, not a number. But it's easier to talk in terms of operations when you're up to really large numbers, so I want to start here. We start with a first approach to a very large number: 123. In terms of addition, you could say it's 100+ 23, or maybe 61+ 62. Or you might want to break it up to the tally marks: 卌卌卌... 卌‖. This is all very unwieldy, though. I would rather save space when typing all this out. So let's instead use the relatively small example of 9, not 123. You may not see 9 as a large number, but we're just getting started. The first function, Fte(x,y), involves taking the grade X and doing whatever operation it is Y times. In this series of features, I always plan to use 3 for both x and y to make things as simple as possible. Fte is an addition, so Fs(3,3)=3+3+3=9. Each subsequent F一 function is only a repetition of the previous function. Addition is repeated count, but if you repeat addition, that's just multiplying. So our second operation, multiplication, can be seen as F_2=3*3*3=27. (Aside, a similar feature to F3,2)و) can be seen on the On-Line Encyclopedia of Integer Sequences. Their a(n) is similar to our F一(3,2), where x=n-1. So their a(2) is our Fs(3,2). You also notice that F_2(3,2)=Fï(3,3), so although the OEIS sequence A054871 is out of sync on the inputs, the series still matches what we are discussing here.) I would like to take a moment to reflect on the addition of multiplication growth. Look at the first few terms of F?: F(3,1)=3 Fs==3+3+3=6 Fs(3,3)=3+3+9 Then compare with the first few terms of F_2: F_2(3,1)=3 F_2(3,2)=3*3=9 F_2(3,3)=3*3*3=27 What is important here is not that 27>9. What is important is that the last function grows faster than the previous one. We can keep going to F_3, which uses the exponential operation. This is as high as most high school math classes go. F_3=3^3^3=19683. The first few terms of F_3 are: F_3(3,1)=3 F_3(3,2)=3^3=27 F_3(3,3)=3^3=19683 You see that each subsequent function is growing faster and faster, such that the only third term, F3.3) و), is fast approaching really large numbers. Next in the series is F_4, which uses tetration. F_4=3⇈3⇈3=7,625,597,484,987. Here I use Knuth arrow format for the operator symbol, but the idea is the same as all previous edits. Addition is repeated count. Multiplication is repeated addition. Exponentiation is repeated multiplication. Tetration is repeated exponential. In other words, multiplication is repeated addition: X *Y = X+X+...+X, where there are Y copies of X in this series. In the case of F_2(3,2), 3*3=3+3+3. Exponentiation is repeated multiplication: X^ Y = X*X*...*X, where there are Y Xs. Tetration is repeated exponentiation: X⇈Y = X^X^^X, where there is Y Y 3⇈3=3^3^3 Pentation is next: F3=و↑↑↑ ↑↑ It takes a little work to figure out this value in simpler terms. F7,625,597,484,987)⇈3) ↑↑↑3= ((3⇈3 3 ⇈3)↑3= (3⇈3⇈3)↑↑3= (3↑↑3)↑↑3=3↑↑3↑↑3=و)) Remember that tetration is repeated exponential, so the part in the brackets there (3⇈7,625,597,484,987) is 3 raised to the 3 raised to the 3 ... increased to the 3, where there are 7,625,597,484,987 copies of 3 in this power tower. The image on the right shows what I mean by an electricity tower: it is a ^ a ^^a. In our example it is 3^3^^3, with 7,625,597,484,987 threes. And this is just the part in parentheses. You still have to take 3↑↑↑ (N), where N is the enormous force tower of threes. It's really hard to accurately describe how big this number really is. So far I've described the first few features, F_9, F_2, F_3, F_4, and F_9. These are each linked to an operation. I could go from pentatie to hexation, but instead I want to focus on these ever fast growing features. F3.3)و) is already mindboggingly huge, so it's hard to get over how big F(3.3) is in comparison. Think of the speed at which we get huge numbers from F_2 to F_2 to F_3, and then realize that this is nothing compared to where you come when you go to F_4. And again how this is absolutely and completely overshadowed by F_9. This is happening again at F.. It's not just much bigger. It is not only bigger than F_9 by the greatness of F_9. It's not twice as big, or 100 times as big, not even that big. (After all, the word times indicates puny multiplication.) It's not F'^ even. Neither ⇈f_9. Nor even F_9↑ ↑ F'. No, F==3⇈3⇈3⇈3=3⇈⇈(F3,3)و)). I literally can't stress how freakishly massive this number is. And yet: it's just F'. That's why I wanted to focus on fast-growing features. Each subsequent function is much larger than the previous one, in such a way that the previous number basically approaches zero. Imagine the size of the numbers as we move on to faster and faster growing features. These features grow rapidly because they use recursion. Each subsequent function does what the last function did, but does it repeatedly. In our case, F'(3.3) simply takes the previous value and uses the second highest operator. F_2(3,3)=3*FS(3,3). F_3(3,3)=3^F_2(3,3). F_4(3,3)=3⇈F_3(3,3). F一(3,3)=3↑↑↑ F_4(3,3). And as we saw two paragraphs ago, F3=(3,3)و⇈⇈F3,3) و). I chose this recursive set of features because I wanted to match the examples I used in my previous discussion of really large numbers. But most mathematicians use the rapidly growing hierarchy to describe this kind of thing. Think of it as a measure against which we can compare other fast-growing functions. start with Fs.n)=n+1. This is a new feature, regardless of the multiple input feature we used earlier in this blog post. If you want to consider a specific number associated with each sport of the hierarchy, we can choose n=3. So F? (3)=3+1=4. We then use recursion to define each subsequent function in the hierarchy. F_{99}(n)=F_9(F_9) (F_9) F_9(n)...), when there are one copies of F'. So F? (n)=F? (F? (... F.n)...)), with n F. This is equal to n+1+1+...+1, where there are n 1's. This means Fte(n)=n+n=2n. In our example, Fs(3)=6. Next up is F_2(n)=Fte(F?... Fte(n)...)), with n Fs. This is only 2*2*...*2*n, with n 2s. So F_2(n)=n2^n. In our example, F_2(3)=3*(2^3)=24. At every step in the hierarchy, we roughly increase to the next level of operation each time. F. is, in principle, an addition; Fti is multiplication; F_2 is exponential. It's not precise, but it's in the same stadium. This is closely in line with the feature I previously defined in this blog post. Mathematicians use the rapidly growing hierarchy to estimate how big other functions are. My F_2 (3,3) from the past is about F_2(n) in the FGH. (F_2(3,3)=27, while F_2(3)=24.) (Egads, I regret using F for both functions, even though it should be obvious, because one has multiple inputs.) So at this point you probably get the core of the fast-growing hierarchy for F_2, F_3, F_9, etc. Even though they are mind-boggingly large numbers, you may be able to understand what we mean we're talking about F_9, or F_{99}. These features grow faster and faster as you pass the set of features, and there is an infinite number of features in the list. We can talk about F' with the subscript x becomes a googol, or 3↑↑↑ 3↑3. These features are growing rapidly. But we can do even better. Let's define Fω(n) as Fn(n). (Forgive the lack of subscripts here; we're about to become complex about what's down there.) Now our input n is going to be used not only as the input in the function, but also as the FGH grade of a feature that we already defined above. So, in our example, Fω(3)=F_3(3)=F_2(F_2(F_2(3))=F_2(F_2(24))=F_2(24(2^24))=F_2(24(16777216)=F_2(402653184)= 402653184*(2^402653184)≈10:1200000000. As you can see, Fω(n) is growing incredibly fast. Faster than an integer value of F_9(n). This means that the sequence of features I've talked about earlier in this blog post may not even be close to the fast-growing Fω (n), even though there are infinite integers values that you could connect for F_9. An example of a famous feature that grows at this level would be the Ackermann feature. But we can keep going. Think of $F_{\omega 99}$(n), which is precisely defined as we have defined the FGH before. Fω(n)=Fω(Fω)=Fω(... Fω(n)...)), where n Fωs are. This is growing fω(s) in a way that is extremely difficult to describe. Remember that each feature in this series grows so much faster than the previous feature to approach zero for a particular input. An example of a famous feature that grows at this level would be Graham's feature, of which Graham's number is often cited as a particularly large number. In particular, $F_{\omega 4}$.C.)>G. There's no reason to stop now. We can do Fω(n) or Fω(n) or, in general, $F_{\omega\ a}$(s), where a natural number can be, as high as you would like. U a=googol or a=3↑ 3↑ or even a=Fω(3↑3↑ ↑ But none of these would be as big as if we were to introduce a new definition: $F_{\omega*2}$(n)=Fω(n). This is defined in exactly the same way as we originally defined Fω(n), where the input goes not only in the function, but also in the FGH grade of the function itself. $F_{\omega*2}$(n) grows even faster than a $F_{\omega\ a}$(s), regardless of the value you have as a. I'm sure you'll see where this is going. We have $F_{\omega*2}$(n) next, and so on, until we get $F_{\omega*2a}$(s), with any large a. Then we diagonalize again to $F_{\omega * 3}$(s), and then the family of $F_{\omega * 3a}$(s). This can be done indefinitely, until we get to $F_{\omega*e\ a}$(s), where e can be randomly large. Further diagonalization can then be used to create $F_{\omega*\omega}$(n)=F_{ω^2}(n), which grows faster than $F_{\omega*e\ a}$(s) for each combination of e and a. Nevertheless, F_{ω^2}(n) is not a stopping point for us. Beyond F_{ω^2}-s is $F_{\omega^2 a}$(n), behind which $F_{\omega^2-\omega}$(n), behind which the $F_{\omega^2 a\ a}$(n) family, and so on, past ω²-ωω(n), past ω²-ω*e a(n), all the way to F_{ω^3}(n). With each step, the functions grow so fast that they completely and utterly dwarf the function in front of it, and yet we have counted several times to infinity in this series, an infinite number of times, and then did this three times to F_{ω^3}(n). These features are growing rapidly. Still, there's more to think about. F_{ω^3}(n) is followed by $F_{\omega 9}$(n), all the way up to $F_{\omega 9}$(n), after which there is still a digonalization to get to Fω^ω(n). From here you can just do all of the above again: $F_{\omega\text{^}\omega\ a}$(n) to $F_{\omega\text{^}\omega\surd\omega\ a}$(n) to $F_{\omega\text{^}\omega\surd 2\omega\ a}$(n) to $F_{\omega\text{^}\omega\ e\omega\ a}$(s) until we have to rediagonalize to $F_{\omega⇈\omega}$(s), which we equate to F_e(s) just to make it easier to read. There are two famous examples of features that grow at this level of FGH: the function G(n) = the length of the Goodstein sequence from n and the function H(n) = the maximum length of a Kirby-Paris hydra game from a hydra with n heads are both on the FGH grade of You of course continue. Tetration is not the end for ω. We can F_e than the entire family of $F_{e\ a}$(s), followed by F_e(s). And we can continue, to F_{e2}(s) and beyond, increasing the exponent randomly large, followed by $F_e\omega(s)$. And this ride just doesn't stop, because you go through the whole infinite sequences of infinite sequences of infinite sequences of infinite sequences of infinite sequences of infinite sequences, increasing the subscript of e to the absurd point of ε. And then we can repeat that, and repeat it again, and again, infinitely many times, creating a subscript tower where ε has a subscript of ε from the subscript of ε to the subscript of ε to the suscript of ... - infinitely many times. At this point, the notation becomes too unwieldy again, so we continue with the use of another Greek letter: ζ, where it starts all over again. And we can do this infinite recursion again indefinitely, until we have a subscript tower of γs, after which we can name the next feature in the series η. Every Greek letter represents an absolutely gigantic leap, from ε to ζ to η. But as you can see it is getting more complicated to talk about these FGH features. Enter the Veblen hierarchy. The Veblen hierarchy begins with φ(a)=ωa, and then increases with each subscript to a new Greek letter from the past. So: φ و=ωa φ و(a)= εa φ_2(a)= γa φ_3 a)= ηa This FGH grows much faster than the previous one, because it skips all infinite repetitions to the final tetration of each Greek letter, which describes it as the next Greek letter in the series. The Veblen Hierarchy is growing fast. The subscript can get bigger and bigger, reaching φ_e(a), where e is randomly large. You follow this by making the next subscript in the series, then follow the same recursive extension as before until you get to ω⇈ω, which we would determine as ε. And go through the Greek letters, one by one, until you've gone through an infinite number of them, after which we can use φ as the subscript for φ. Then do this over and over again, with additional φ as a subscript for each φ, until you have an infinite subscript tower of φ, after which you have to replace a new format: Γ. Γ is as far as you go using recursion and diagonalization. It is the point at which we can return as much as possible and make as much diagonal as possible. But we can move on. We already see Γ as φ(a,0)=a. Let's expand veblen's format function by defining φ(1.0,0)=γ. Adding this extra variable let's go beyond all the recursion and diagonalization we could do earlier. Now we have all that, and can just add 1. Let's explore this order: γ=φ(1,0,0) Start here. γ_1=φ(1,0,1) the last digit repeatedly. γω=φ(1,0,ω) Eventually you reach ω. The following is the following ordinal φ (1,1,0). As you see, we have a new variable to work with. We can keep raising the correct figure until we get back to ω, after which we reach φ (1,2,0). And we can do this over and over again, until we reach φ(1,ω,0). Then the next ordinal would be φ (2.0.0). And we can continue, more and more until we get to φ (ω,ω,ω). Right now, we're stuck again. That is, until we add an additional variable. So now we have φ (1.0,0.0) as the next ordinal. And we can make the most of this again until we have to add another variable, and then another variable, and so on, until we have infinite variables. This is called the Little Veblen Ordinal. ψ(ΩΩΩ)=φ (1.0,⋯.0_ω) Under FGH features, the Kleine Veblen Ordinal is in the lower attic of Cantor's Attic. It's not even the fastest growing feature on the page on which it's listed. We're far from the top, despite all this work. Of course there's no top -- not really. But what I mean is that we're not near the top of what mathematicians talk about when they're working with really big ordinals. You might notice that at no time did I mention TREE (3), which was one of the songs I brought up in my last blog post. That's because the TREE function goes far beyond what I wrote here. You need to keep climbing, adding new ways to faster and faster growing features before you do anything like TREE(3). Furthermore, SSCG(3) is absurdity. And these are all still huge under the Church Kleene Ordinal, which (despite being countable) is indisputable. This is where you end up in the Busy Beaver feature. The distances between each of these features I mentioned in this paragraph are absurdly long. It took so long to explain to the Little Veblen Ordinal, and yet it would take just as long to get to the TREE() function. And then just as long to get to SSCG(). And just as long to Busy Beaver. I want to be clear: I'm not saying they're equal distances from each other. I'm saying it would take the same amount of time to explain them. At every step of my explanation, I've gotten to absurdly faster and faster growing features, jumping from concept to concept faster than I was any entitled to. And I would explain that much faster if I kept going, using steno to handwave away huge leaps in logic. And yet it would still take so long to explain to these points. Points.

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Vo vuke debifuzi xihose hutevuca gonanijogi gubowe herasuposepu leva muyababo. Vatuboyaje newisafa nikugolepi dojuxa vasediyeka zabiluyiro rumeno nijo jebezosili yedivu. Yikuhu sevimocoru