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Math pick up lines

In our SAT guide to lines and angles, we dealt with parallel lines, angular lines, and the many different ways to find angle measures with two or more lines. Now we will look at the second aspect of lines, namely their slopes and equations. This will be your complete guide to lines and slopes - what slopes mean, how to find them, and how to solve the many types of slope and line-crossing issues you'll see on the SAT. What are lines and slopes? Before we start, take a moment to familiarize yourself with our guide to SAT coordinate points to update yourself on the basics of coordinate geometry. Basically, coordinate geometry takes place in the room where $x $-axis and y axis meet. Any location in this area gets a coordinate point — written as (x, y)– that indicates where the point is along each axis. A line (or line segment) is a perfectly straight marker without curvature. It consists of (and connects) a number of points together. A slope is a measure of a line's slope (steepness). A slope exists by finding the change in the distance along the y axis above the change in the distance along the x axis. You are probably most familiar with this concept by finding ladder over the run to find the slope of a line. $${\change \in y}/{\change \in x}$$ Here is a typical line that appears on the coordinate grid. To find our slope, you must first start by marking the points where the line hits the grid at perfect hero coordinates. This will make life simpler as we go to find the slope. No matter where the net meets in a corner, we will have coordinates that are integers. We can see here that our line hits the coordinates: $(-3, 4)$, $(0, 2)$, and $(3,0)$. Now let's find the rise and run off the line. Our rise will be -2 as we need to move down 2 units to reach the next coordinate point in our line. Our race will be +3 as we need to move 3 units to the right to reach the next coordinate point in our line. So our final slope will be: $\rise/\run$ $-{2/3}$ Properties of slopes A slope can either be positive or negative. A positive slope increases from left to right. A negative slope falls from left to right. A straight horizontal line has a slope of zero. It is defined by only one axis. $y=2$ A straight vertical line has an undefined slope (because the race will always be 0 and you cannot divide by 0). It is defined by only one axis. $x=2.5$ The steeper the line, the bigger the slope. The red line is steepest, with a slope of $4/$, or 4. The blue line is not so steep, with a slope of $4/9$ San Francisco at a bit on steep slopes. Line and slope formulas Find slope $${y_2 - y_1}/{x_2 - x_1}$$ To find the slope of a line that connects two points, you must find the change in y values relative to the change in x values. (Note: does not matter which points you award as $(x_1.y_1)$ and as long as you are consistent.) Given the coordinates $(2, 2)$ and $(-1,0)$, you need to find the slope of the line. Now we can solve this question in one of two ways – by drawing a graph and counting or by using our formula. Since we've already seen how it's possible to count our tilt on a graph, let's use our formula to see how it's done. Of the two coordinate points, we assign one set to be x_1 and y_1 and the other to be x_2 and y_2. Let's choose (2,2) to be our x_1 and y_1 and (-1,0) to be our x_2 and y_2. ${y_2 - y_1}/{x_2 - x_1}$ $(0 - 2)/(-1 - $2-2}/{-3}$ $2/3$ But what would have happened, if we had awarded $(-1,0) $ to be our x_1 and y_1 and $ (2, 2) $ to be our x_2 and $y_2 $? We would have had the same results either way! ${y_2 - y_1}/{x_2 - x_1}$ $(2 - 0)/(2 - -1)$ $2/3$ No matter what coordinates we assign to be the first or second value for x and y, we end up with the same slope as long as we are consistent. Equation of a line $ $y = mx + b $ $ This is called the equation of a line or a line in the slope-intersection shape, and it shows exactly how a line is placed along $x $ - and $y $-axes as well as how steep it is. This is the main formula you need when it comes to lines and slopes, so let's break it into pieces. y is your y coordinate value for a certain value of x. $x $ is your x coordinate value for a certain value of $y $. m is a measure of your inclination. b is the y-tapping value of your line. This means that this is the value where the line hits y axis (remember that a straight line only hits each axis at most once). Find the equation of the line from the graph. We use the same graph from above and we can see that the line cuts with $y the $axis of about $y = 0.5.$* We have also established previously that the slope is $2/3$. So when we put these two pieces of information together, the equation of our line would be: $y = {2/3}x + $0.5* If the question wants you to be more specific than about a half about what fraction of a number line hits $x =0$or $y $0 on, it will have a more detailed drawing. In this question, while the actual eavesdropping for a line of coordinates $ (2.2) $ and $ (0.-1) $ would actually be $2/3 $ or $0.66 $, not $0.5 $, graphene is not on a scale where you could reasonably visually estimate that. Remember to always rewrite any line equations you get in the right shape! Often, the test will try to stand you up by giving you an equation that is not written in correct form and asks for the slope of the line. This is to get people to make a mistake if they go through the test too quickly. $$tx+12y=-3$$ The equation above is the equation of a line in the xy plane, and t is a constant. If the line slope is -10, what is the value of t? Let's Put this equation into the correct equation of a line. Line. = −3$ $12y = −tx−3$ $y = - {tx}/{12} -3/12$ Now we only don't care about finding the slope, let's find the value of t (our slope) by using our given. $-{t/12} = -10$ $-t = -120$ $t = 120$ Our final answer is $t = 120$. Remember to always create your equation as your first step and you will be able to solve most any slope problem quickly and easily. Perpendicular lines When two lines meet perpendicular, the lines are called perpendicular. Perpendicular lines will always have slopes that are negative mutual of each other. This means that you must reverse both the sign of the slope as well as the fraction. For example, if a two lines are perpendicular to each other and one has a slope of $3/4$, the other line will have a slope of $-{4/3}$. Perpendicular lines with slopes of $3/4$ and $-{4/3}$ And if a line has a slope of −5, the line that meets it perpendicular will have a slope of $1/5$. Parallel lines When two lines will never meet, no matter how infinitely long they stretch, the lines are said to be parallel. That means they're the body all the time. If two lines are parallel, they will also have the same slope. You can see why this makes sense, since the increase over run will always be the same to ensure that the lines will never touch. Parallel lines with slopes of $4/3$ When lines turn devious. Typical Line and Slope Questions Most line and slope questions on the SAT are quite basic in their core. You will generally see two questions on the slopes per test, and almost all of them will simply ask you to find the slope of a line or the equation of a line. The test may try to complicate the issue using other shapes or shapes, but the questions always boil down to these simple concepts. Just remember to rewrite a given equations in the correct slope-tapping form and keep in mind your rules to find slopes (and your rules for parallel or perpendicular lines). With this knowledge in hand, you will be able to solve these types of problems quickly and easily. From the graph, we can see that y-tapping the line is (0.1). The line also passes through the point (1.2). This is enough information for us to figure out the slope of the line, which we know is ${\change \in y}/{\change \in x}$. ${2-1}/{1-0}={1}/{1}=1$ Now we know the line slope is 1. In slope-cutting form, the equation for l is $y=x+1$ or choice D. Our final answer is D. We know that the slope is ${\change \in y}/{\change \in x}$. The equation $y = k-x +4$ is already in slope-cutting form, so we know that the slope of the line is $k $. We also know that the line contains the point $ (c, d)$ which means we can replace these variables for $ (x, y) $ in the equation. This gives us $d = kc + 4 $ Solution for this equation for the slope, $k $, gives us $k = {d-4}/c$ Our final answer is A, $ {d-4}/c$ How to solve a line and Problem As you go through your line and slope problems, keep in mind these tips: #1: Always rearrange your equation in $y =mx +b$ The test makers will often present you with an equation of a line that is not in proper shape, for example: $4y + 3x = $12. If you go too fast through the test, or if you forget to rearrange the given equation in proper slope-cutting form, you will misidentify the slope of the line. So remember to always rearrange your equation in proper shape as your first step. $4y + 3x = 12$ => $y = −{3/4}x + 3$ #2: Always remember your $\rise/\run$ It can be easy to make a mistake trying to find the change in x before you find the change in y, as our brains are used to doing things right. Keep a close handle on your variables to reduce careless errors of this kind. Remember the mantra of ladder over run and this will help you always by finding your change in y over your change in $x $. #3: Create your own graph and/or numbers to find your slope Because the slope is always over-running, you can always find the slope with a graph – whether it's from a given graph or from your own. It's never a bad idea to take a second and make your own graph if you're not provided with one. This will help you better visualize the problem and avoid errors. If you forget your formulas (or simply don't want to use them), simply count how the line rises (or falls) and track its run, and you'll always find your inclination. Oh, for the days when all we needed to know about lines was how to color in (or out of) them.... Test your knowledge Now that we've gone through the typical slope issues you'll see on the test (and a few basic tips you'll need to solve them), let's put your knowledge to the test. 1. 2. 3. 4. Answer: D, A, B, D Answer explanations: 1. We know that the slope is ${\change \in y}/{\change \in x}$. We are also told that our slope is -2, which means it must be $-{2/1}$. This means that for every time our y value decreases by 2, our x value increases by 1. And every time our y value increases by 2, our x value will decrease by 1. If we use our rectangle, we also have a reference point on the line. We can see that the rectangle has a length of 3 (because it ranges horizontally from $x = 0$ to $x = 3 $) and a length of 4 (because it ranges vertically from $y = 0 $ to $y = 4 $). This means that the rectangle hits the line in the upper right corner at the coordinates (3, 4). Now we can just count where the line will hit the y-axis. Because the slope rises horizontally (along the x-axis) one unit at a time, we can see that there will be $3/1 = 3$ straight points along the line needed to find the y-intersection. Basically, this means we take the slope, $-{2/1}$, and multiply it by 3 to get $-{6/3}$. In other words, we add 6 to the change in y and subtract 3 from in x because we take takes Backwards. So now we can find our new point by saying that we increased our y-value from $4 to $4 + 6 = 10 $, and we have reduced our x-value from 3 to $3−3 = 0$, which would give us the new slope of: $ (4 + 6) / (3 - 3) = 10 /0$ So our new coordinate point is (0,10), which means that our y-interception is 10. Our final answer is D, 10. 2. We know we can find the slope of the line using ${y_2 - y_1}/{x_2 - x_1}$, let's connect our coordinates (0.r) and (s.0) for these values. ${y_2 - y_1}/{x_2 - x_1}$ $(0 - r)/(s - 0)$ r/s Our s remains unchanged, but our r value will be negative as it is deducted from zero. Our final answer is A. 3. If you count to the point where the line crosses y-cutting, you can see the hits on $y = $3 In the equation $y = mx + b$, b is y-intersection. That means our b will be 3. We can therefore cross response choices A and D so that we can go down to B, C and E. We can also see that our line is falling from left to right, so our slope will be negative. This means that we can eliminate answer choice E, leaving us between choice B and C. Now let's take the two points where the line hits the axes. We've already seen that the line hits y-tapping on $y =3$ and we can also see that the line hits the x-axis on $x = 2$. This means that our line hits coordinates (0, 3) and (2,0). This means that the change in our y value is -3 and the change in our x value is +2. (Why? Because we reduced our y value by 3 and we increased our x value by 2.) So our slope should be $-{3/2}$, which means that our final equation will be: $y = -{3/2}x + 3$ Our final answer is B. 4. We are told that the equation of the line is $y = 2x-5 $. This means that the slope of the line is 2 and $y $-interception is on (0.-5). A slope of 2 means that for each increase in x of 1, y increases by 2. Looking at each of the graphs, options C and D are the only lines with a slope of 2. (Don't be fooled by choice B, which has a slope of -2.) Next, we can look at y intercepts. The line in Choice C has a $y $interception of ($0.5) $, which does not correspond to the equation of the line we got. But Choice D's line has a $y $-tapping of $ (0.-5)$, which is exactly what we're looking for. Choice D is the only line with both the correct slope and y-intercept. Our last answer is D. Hurray, hooray! You've found your slopes, you know your lines! Congratulations, congratulations. Take-Aways Once you've familiarized yourself with the basics of coordinate geometry, slopes shouldn't be too far off the track. Although the SAT will try to complicate problems as much as they are reasonably capable of, issues on lines and slopes are almost always easier than they look. Keep your important formulas close to your heart and be vigilant with your negative signs and you'll do just fine when it comes to slopes and interceptions. What's next? You have learned all there is to learn about slopes angles. Fortunately for you (for a certain definition of luck), there is so much more to learn! Before proceeding, make sure you have a firm understanding of all the subjects covered by SAT math so you can see what to prioritize. If you're looking for a specific math subject, the scope out of our SAT math archive for each subject guides like this. We have mastered solid geometry, probability, key figures and more! If you don't know where to start, be sure to take a practice test and see how your score ranks. This will give you a good sense of where your strengths and weaknesses are and how you should focus your studying. Want to get a perfect score? Check out our guide to getting an 800, written by a perfect scorer! Want to improve your SAT score by 160 points? Check out our best in class online SAT prep program. We guarantee your money back if you do not improve your SAT score by 160 points or more. Our program is completely online and it adapts what you study to your strengths and weaknesses. If you liked this Math strategy guide, you will love our program. Along with more detailed lessons, you get thousands of practice problems organized by individual skills so you learn most effectively. We will also give you a step-by-step program to follow so you will never be confused about what to study next. See our 5-day free trial: trial period: