Partial fraction decomposition hard examples

This section examines the antideral wings of rational functions. Remember that rational functions are functions of \(f(x)= \frac{p(x)}{q(x)}\\) on the form that contain \(p(x)\) and \(q(x)\) are polynomic and \(q(x)eq 0\). Such activities arise in many contexts, one of which is the resolution of certain basic difference equations. We begin with an example that shows the motivation behind this section. Consider integral \(\int \frac{1}{x^2-1}\ dx\). We do not have a simple formula for this (if the denominator was \(x^2+1\), we recognize the antiderivative as the arctangent function). It can be solved with a trigonometry replacement, but note that how to evaluate integral when we understand: $$\frac{1}{x ^2-1} = \frac{1/2}{x-1} - \frac{1/2}{x+1}.\] This is how \[\begin{align}\int\frac{1} {x^2-1}\ dx &= \int\frac{1/12}{x-1}\ dx - \int\frac{1/2}{x+1}\ dx \\ &= \frac12\ln|x-1| - \2}{x+1}\ dx \\ &= \frac12\ln|x-1| frac12\ln|x+1| + C.\end{align}\] This section teaches you how to $$\frac{1}{x^2-1}\quad \text{into}\quad \frac{1/2}{x-1}-\frac{1/2}{x+1}.\] We start with a rational function \(f(x)=\frac{p(x)}{q(x)}\) where \(p\) and \(q\) have no common factors and \(p\) degrees is less than \(q\) degrees. It can be demonstrated that any polynomic and therefore \(q\) can be considered as a product of a linear and irreversible quadratic term. The following key idea explains how to rot a rational function as the sum of rational functions whose names are all lower than \(q\). Key idea 15: Partial fractional explosion Enter \(\frac{p(x)}{q(x)}\) to be a rational function with a \(p\) degree less than the \(q\) degree. Linear terms: Allow \((x-a)\) to share \(q(x)\), where \(x-a)^n\) is the \(x-a)\) maximum power that distributes \(q(x)\). Then \(\frac{p(x)}{q(x)}\) degradation contains the sum of $$\frac{A_1}{(x-a)} + \frac{A_2}{(x-a)^2} + \cdots +\frac{A_n}} {(x-a)^n}.$$ CVADd terms: Allow \(x^2+bx+c\) to share \(q(x)\) with \(x^2+bx+c)^n\) having the highest power \(x^2+bx+c\), which distributes \(q(x)\). Then the breakdown of \(\frac{p(x)}{q(x)}\) contains the sum of $$\frac{B_1x+C_1}{x^2+bx+c}+\frac{B_2x+C_2}{(x^2+bx+c)^2}. +\cdots+\frac{B_nx+C_n}{(x^2+bx+c)^n}.$$ To find the odds \(A_i\), \(B_i\) and \(C_i\): Multiply all fractions \(q(x)\) and clear the names. Collect like terms. Chest\(x\) authority factors and resolve the resulting linear equation system. The following examples show how to put this key idea into practice. The example \(\PageIndex{1}\) highlights the degradation aspect of the key idea. Example \(\PageIndex{1}\): Degradation to stratum fractions Decompose \(f(x)=\frac{1}{(x+5)(x-2)^3(x^2+x+2)(x^2+x+7)^2}\) without resolving the resulting odds. Solution The denominator is already factored because both \(x^2+x+2\) and \(x^2+x+7\) cannot be Still. We need to rot \(f(x)\) correctly. Because \(x+5)\) is a linear term that divides the denominator, degradation means $$\frac{A}{x+5}\] terms. When \(x-2)^3\) shares the denominator, the imitation contains the following conditions: $$\frac{B}{x-2},\quad \frac{C}{(x-2)^2}\quad \text{and}\\quad \frac{D}{(x-2)^3}.\] The term \(x^2+x+2\) of the denominator leads to \(\frac{Ex+F}{x^2+x+2}\) Finally, the term \(x^2+x+7)^2\) leads to $$\frac{Gx+H}{x^2+x+7}\quad \text{and}\quad \frac{Ix+J}{(x^2+x+7)^2}.\] All together, we have \[\begin{align} \frac{1}{(x+5)).\] All together, we have \[\begin{align} \frac{1}{(x+5)).\] All together, we have \[\begin{align} \frac{1}{(x+5))x-2)^3(x^2+x+2)(x^2+x+7)^2} &= \frac{A}{x+5} + \frac{B}{x-2}+ \frac{C}{(x-2)^2}+\frac{D}{(x-2)^3}+ \\ & \frac{Ex+F}{x^2+x+2}+\frac{Gx+H}{x^2+x+7}+\frac{Ix+J }{(x^2+x+7)^2} \end{align}\] Solving odds \(A\), \(B\ldots J\) would be a little laborious, But not difficult. Example \(\PageIndex{2}\): Decomposition into partial fractions Run the partial fractional explosion of \(\frac{1}{x^2-1}\\. Solution Two linear terms of denominator odds: \(x^2-1 = (x-1)(x+1)\). Thus, $$\frac{1}{x^2-1} = \frac{A}{x-1} + \frac{B}{x+1}.\] To resolve \(A\) and \(B\), first multiply \(x^2-1 = (x-1)(x+1)\): \[\begin{align}11 &= \frac{A(x-1)(x+1)}{x-1}+\frac{B(x-1)(x+1)}{x+1} \\ &= A(x+1) + B(x-1)\\ &= Ax+A + Bx-B\end{align}\] Now collect the corresponding criteria. \[= (A+B)x + (A-B).\] The next step is the key. Note the equality we have: \[1 = (A+B)x+(A-B).\] For clarity, the left side is \[0x+1 = (A+B)x+(A-B).\] On the left, the factor in \(x\) is 0; on the right it is \((A+B)\). Because both sides are equal, we must have this \(0=A+B\). Similarly, on the left is the constant term 1; on the right, the standard name is \((A-B)\). That's why we have \(1=A-B\). We have two linear equations with two strangers. This one is easy to solve by hand, leading to $$\begin{array}{c} A+B = 0 \\ A-B = 1 \end{array} \Rightarrow \begin{array}{c} A=1/2 \\ B = -1/2\end{array}.$$ Thus $$\frac{1}{x^2-1} = \frac{1/2}{x-1}-\frac{1/2}{x+1}.\] Example \(\PageIndex{3}\): Integrating using partial fractions Use partial fraction decomposition to integrate \(\int\frac{1}{(x-1)(x+2)^2}\ dx.\) Solution We decompose the integrand as follows, as described by Key Idea 15: $$\frac{1}{(x-1)(x+2)^2} = \frac{A}{x-1} + \frac{B}{x+2} + \frac{C}{(x+2)^2}.\] To solve for \(A\), \(B\) and \(C\), we multiply both sides by \((x-1)(x+2)^2\) and collect like terms: \[\begin{align}1 &= A(x+2)^2 + B(x-1)(x+2) + C(x-1)\\ &= Ax^2+4Ax+4A + Bx^2 + Bx-2B + Cx-C \\ &= (A+B)x^2 + (4A+B+C)x + (4A-2B-C)\end{align}\] Note : Formula \(\PageIndex{22}\) provides a direct route to find \(A\), \(B\) and \(C\). Because the formula contains all \(x\) values, it includes in particular when \(x=1\). However, when the right side of \(x=1\) simplifies to \(A(1+2)^2 = Because the left side is still 1, we have \(1 = 9A\). Therefore\ (A = 1/9\). Equality also applies when \(x=-2\); This leads to \(1=-3C\). Therefore, \(C = -1/3\). Knowing \(A\) and \(C\), we can find \(B\) by selecting one more value \(x\), such as \(x=0\), and solving \(B\). We have $$0x^2+0x+ 1 = (A+B)x^2 + (4A+B+C)x + (4A-2B-C)\], resulting in equations $$A+B = 0, \quad 4A+B+C = 0 \quad \text{and} \quad 4A-2B-C = 1.\] These three equations of three strangers lead to a unique solution: $$A = 1/9,\quad B = -1/9 \quad \text{and} \quad C = -1/3.\] I saw $$\int\frac{1}{(x-1)(x+2)^2}\ dx = \int \int \frac{1/9}{x-1}\ dx + \int \frac{-1/9}{x+2}\ dx + \int \frac{-1/3}{(x+2)^^ 2}\ dx.\] Each can be integrated with a simple replacement \(u=x-1\) or \(u=x+2\) (or by using Key Idea 10 directly because the appointors are linear functions). The end result is $$\int\frac{1}{(x-1)(x+2)^2}\ dx = \frac19\ln|x-1| -\frac19\ln|x+2| +\frac1{3(x+2)}+C.\] Example \(\PageIndex{4}\): Integration with stratum fractions Integrate \(\int \frac{x^3}{(x-5)(x+3)}\ dx\). Key idea 15 assumes that the numerator's degree is lower than the denominator's degree. Since this is not the case here, we will start by using polynomol division to reduce the pointer's degree. We omitte the steps, but encourage the reader to ensure that $$\frac{x^3}{(x-5)(x+3)} = x+2+\frac{19x+30}{(x-5)(x+3)}.\] Using key idea 15, We can rewrite the new rational function as follows: $$\frac{19x+30}{(x-5)(x+3)} = \frac{A}{x-5} + \frac{B}{x+3}\] for appropriate values \(A\) and \(B\). Clearing the nominators, we have a note: \(A\) and \(B\) values can be quickly found using the technology described in the margin of the example \(\PageIndex{3}\). \[\begin{align}19x+30 &= A(x+3) + B(x-5)\\ &= (A+B)x + (3A-5B). B \\30&= 3A-5B.\\ \end{align}\] Resolving this system of linear equations gives \[\begin{align} 125/8 &=A\\27/8 &=B. \end{align}\] We can now integrate. \[\begin{align}\int \frac{x^3}{(x-5)(x+3)}\ dx &= \int\left(x+2+\frac{125/8}{x-8}{x-8 5}+\frac{27/8}{x+3}\right)\ dx \\ &= \frac{x^2}2 + 2x + \frac{125}{8}\ln|x-5| + \fra c{27}, <6>8\ln|x+3| + C.\end{align}\] Example \(\PageIndex{5}\): Integration using partial fractions Use partial fraction explosion \()\int\frac{7x^2+) 31x+54}{(x+1)(x^2+6x+11)}\ dx.\) Solution The pointer degree is less than the denominator degree, So we'll start by using Key Idea 15. We have: \[\frac{7x^2+31x+54}{(x+1)(x^2+6x+11)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+6x+11}. \] Clear the nominators now. \[\begin{align} 7x^2+31x+54 &= A(x^2+6x+11) + (Bx+C)(x+1)\\ &= (A+B)x^2 + (6A +B+C)x + (11A+C). that: \[\begin{align} 7&=A+B\\ 31 &= 6A+B+C\\ 54 &= 11A+C. \end{align}\] This system of linear equations comfortable result \(A=5\), \(B = 2\) and \(C=-1\). How to $$\int\frac{7x^2+31x+54}{(x+1)(x^2+6x+11)}\dx = \int\left(\\frac{5}{x+1} + \frac{2x-1}{x^2+6x+11}\right)\ dx.\] The first term in this new integer period is easy to evaluate. It leads to \(5\ln|x+1|\) Another term is not harsh, but takes several steps and uses replacement techniques. Integrand \(\frac{2x-1}{x^2+6x+11}\) the denominator has a quadrant and a linear term in the numerator. This leads us to try to replace. Enter \(u = x^2+6x+11\), so \(du = (2x+6)\ dx\). The numeral is \(2x-1\), not \(2x+6\), but we can get \(2x+6\) to the numera number by adding 0 in \(() 7-7\). \[\begin{align} \frac{2x-1}{x^2+6x+11} &= \frac{2x-1+7-7-1 7}{x^2+6x+11} \\ &= \frac{2x+2x 6}{x^2+6x+11} - \frac{7}{x^2 +6x+11}.\end{align}\] We can now integrate to replace the first term, resulting in \(\ln|x^2+6x+11|\) The last term can be integrated with the help of an arctangent. First fill in the denominator square: $$\frac{7}{x^2+6x+11} = \frac{7}{(x+3)^2+2}.\] The antiderative of the latter term can be found using Theorem 6.1.3 and replacement: $$\int \fra c{7}{x^2+6x+11}\ dx = \frac{7}{\sqrt{2}}\tan^{-1}\left(\frac{x+3}{\sqrt{2}}\right)+C.\] Let's start over and bring all the steps together. \[\begin{align}\int\frac{7x^2+31x+54}{(x+1)(x^2+6x+11)}\ dx &= \int\left(\frac{5}{x+1 } + \frac{2x-1}{x^2+6x+11}\right)\ dx \\ &= \int\frac{5}{x+1}\ dx + \int\frac{2x+6}{x^2+6x+11}\ dx -\int\frac{7}{x^2+6x+11}\ dx \\ &= 5\ln|x+1|+ \ln|x^2+6x+11 | -\frac{7}{\sqrt{2} }\tan^{-1}\left(\frac{x+3}{\sqrt{2}}\right)+C.\end{align}\] As with many other computing problems, it is important to remember that: that the final answer is not expected to be seen immediately after seeing the problem. Rather, given the original problem, we share it with smaller problems that are easier to solve. The final answer is a combination of answers to smaller problems. Partial fractional degradation is an important tool in the management of rational functions. Note that at its core is the technique of algebra, not counting, because we rewrite the fraction in a new format. Nevertheless, it is very useful in the world of computing, since it allows us to evaluate certain complex integratans. The following section introduces new functions called hyperbolic functions. They allow us to make similar exchanges to trigonometry substitution when we study, allowing us to approach even more integration problems. Assistants and attributes Gregory Hartman (Virginia Military Institute). Troy Siemers and Dimplekumar Chalishajar of the VMI and Brian Heinold of Mount Saint Mary's University spoke. integrated by Justin Marshall. Marshall.

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