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Radioactive decay problems calculus

One of the most widespread applications of exponential functions involves patterns of growth and degradation. Exponential growth and degradation occur in a number of natural applications. From population growth and continued interest growth to radioactive decay and Newton's cooling law, exponential functions are ubiquitous in nature. In this section, we examine exponential growth and degradation in the context of some of these applications. Many systems exhibit exponential growth. These systems follow a model of the \(y=y_0e^{kt},\) form where \(y_0\) represents the initial state of the system, and \(k\) is a positive constant called growth constant. Notice that in an exponential growth pattern, we have \[y′=ky_0e^{kt}=ky. \label{eq1}\] That is, the growth rate is proportional to the value of the current function. This is a key feature of exponential growth. The \ref{eq1} equation involves derivatives and is called a differential equation. Exponential growth systems showing exponential growth according to the mathematical model \[y=y_0e^{kt}\ where \(y_0\) represents the initial state of the system and \(k>0\) is a constant, called a growth constant. Population growth is a common example of exponential growth. Consider a population of bacteria, for example. It seems plausible that the population growth rate is proportional to the size of the population. After all, the more bacteria there are to reproduce, the faster the population grows. Figure \(\PageIndex{1}\) and Table \(\PageIndex{1}\) represent the growth of a bacteria population with an initial population of 200 bacteria and a growth constant of 0.02. Notice that after only 2 hours (120 minutes), the population is 10 times its original size! Figure \(\PageIndex{1}\): An example of exponential growth for bacteria. Table \(\PageIndex{1}\): Exponential increase in a bacterial population time(min) Population Size (No. bacteria) 10 244 20 298 30 364 40 445 50 544 60 664 70 811 80 991 90 1210 100 1478 110 1805 120 2205 Note that we use a continuous function to model what is inherently discrete behavior. At any given time, the real-world population contains a whole number of bacteria, although the model takes insharp values. When using exponential growth patterns, we must always be careful to interpret function values in the context of the phenomenon we are shaping. Example \(\PageIndex{1}\): Population Growth Consider the bacteria population described earlier. This population increases according to the function \(f(t)=200e^{0.02t},\) where t is measured in minutes. How many bacteria are present in the population after \(5\) hours (\(300\) minutes)? When does the population reach the bacteria \(100,000\)? Workaround We Have \(f(t)=200e^{0.02t}.\) \[f(300)=200e^{0.02(300)}≈80.686. onumber \] There are \(80.686\) bacteria in the population after \(5\) hours. find find the population reaches bacteria \(100,000\), we solve the equation \[\begin{align*} 100,000 &= 200e^{0.02t} \\[4pt] 500 &=e^{0.02t} \\[4pt] \ln 5 &=0.02 t \\[4pt] t &=\frac{\ln 500}{0.02}≈310.73. \end{align*} \] The population reaches bacteria \(100,000\) after \(310.73\) minutes. Exercise \(\PageIndex{1}\) Consider a population of bacteria that grows according to function \(f(t)=500e^{0.05t}\), where \(t\) is measured in minutes. How many bacteria are present in the population after 4 hours? When does the population reach \(100\) millions of bacteria? Answer Use the process in the previous example. Response There are bacteria \(81,377,396\) in the population after \(4\) hours. The population reaches \(100\) millions of bacteria after \(244.12\) minutes. Let's now turn our attention to a financial application: compound interest. Interest that is not aggravated is called simple interest. Simple interest is paid only once at the end of the specified time period (usually \(1\) year). So if we put \($1000\) into a savings account earn \(2%\) simple interest per year, then at the end of the year we have \[1000(1+0.02)=$1020.\] Compound interest is paid several times a year, depending on the period of composition. Therefore, if the bank compounds of interest every \(6\) months, loans half the interest of the year in the account after \(6\) months. In the second half of the year, the account earns interest not only on the initial \($1000 \), but also on the interest earned in the first half of the year. Mathematically speaking, at the end of the year, we have \[1000 \left(1+\dfrac{0.02}{2}\right)^2=$1020.\] Similarly, if interest is composed every \(4\) month, we have \[1000 \left(1+\dfrac{0.02}{3}\right)^3=$1020.13,\] and if interest is composed daily (\(365\) or per year), we have \($1020.20\). If we expand this concept so that interest is composed continuously, after \(t\) years we have \[1000\lim_{n→∞} \left(1+\dfrac{0.02}{n}\right)^{nt}.\ Now to manipulate this expression so that we have an exponential growth function. Remember that the number \(e\) can be expressed as a limit: \[e=\lim_{m→∞}\left(1+\dfrac{1}{m}\right)^m.] Based on this, we want the expression inside the parentheses to have the form \(1+1/m)\). Let \(n=0.02m\). Note that also as \(n→∞, m→∞\). Then we get \[1000\lim_{n→∞}\left(1+\dfrac{0.02}{n}\right)^{nt}=1000\lim_{→∞ m}\left(1+\dfrac{0.02}{0.002m}\right)^{0.02mt}=1000\left\lim_{m→∞}\left(1+\dfrac{1}{m}\right)^m\right]^{0.02t}.\] We recognize the boundary inside parentheses as \(e\) number. So the balance in our bank account after \(t\) years is given by \(1000 e^{0.02t}\). By generalizing this concept, we see that if a bank account with an initial balance of \($P\) gains interest at a rate of \(r%\), continuously composed, then the balance after \(t\) years is \[\text{Balance}\;=Pe^{rt}.\] Example \(\PageIndex{2}\): Compound interest A 25-year-old student has the opportunity to invest some money in a retirement account that pays \(5%\) continuously compound annual interest. How much does the student need to invest today to have \($1\) million when she retires at the age of \(65\)? What if she could earn \(6%\) annual compound interest continuously instead? Solution We have \[1,000,000=Pe^{0.05(40)}\] \[P=135,335.28.\] It must invest \($135,335.28\) at \(5%\) interest. If, instead, it is able to win \(6%,\) then the equation becomes \[1,000,000=Pe^{0.06(40)}\] \[P=90,717.95.\ In this case, she must invest only \($90,717.95.\) This is about two-thirds of the amount she needs to invest at \(5%\). The fact that the interest rate is continuously composed continuously magnifies the effect of the interest rate increase \(1%\). Exercise \(\PageIndex{2}\) Suppose that instead of investing at \(25\sqrt{b^2−4ac}\), the student waits until the age of \(35\). How much should You Invest at \(5%\)? At \(6%\)? Hint Use the process in the previous example. Reply to \(5%\) interest, it must invest \($223,130.16\). At interest \(6%\), it must invest \($165,298.89.\) If a quantity increases exponentially, the time it takes for the quantity to double remains constant. In other words, it takes the same time period for a population of bacteria to grow from \(100\) to \(200\) bacteria as it causes it to grow from \(10,000\) to \(20,000\) bacteria. This time it's called the doubling time. To calculate the doubling time, we want to know when the quantity reaches twice the original size. So we have \[\[\begin{align*} 2y_0 &=y_0e^{kt} \\[4pt] 2 &=e^{kt} \\[4pt] \ln 2 &=kt \\[4pt] t &=\dfrac{\ln 2}{k}. \end{align*} \] Definition: Doubling time If a quantity increases exponentially, the doubling time is the time it takes to double the quantity. It is given by \[\text{Doubling time}=\dfra{\ln 2}{k}.\ Example \(\PageIndex{3}\): Using doubling time Assumes a fish population increases exponentially. A pond is originally supplied with \(500\) fish. After \(6\) months, there are \(1000\) fish in the pond. The owner will allow his friends and neighbors to fish on his pond, after the fish population reaches \(10,000 \). When will the owner's friends be allowed to fish? Solution We know that it takes the fish population \(6\) months to double in size. So if \(t\) represents time in months, by the doubling time formula, we have \(6=(\ln 2)/k\). Then, \(k=(\ln 2)/6\). Thus, the population is given by \(y=500e^{(\ln 2)/6)t}\). To find out when the population reaches the fish \(10,000\), we need to solve the following \[\begin{align*} 10.000 &=500e^{(\ln 2/6)t} \\[4pt] 20 &=e^{(\ln 2/2/2/2/2/2/2/26)t} \\[4pt] \ln 20 &=\left(\frac{\ln 2}{6}\right)t \\[4pt] t &=\frac{6(\ln 20)}{\ln 2} \\[4pt] &≈25.93. \end{align*} \] \] Owner's friends have to wait \(25.93\) months (a little more than \(2\) years) to fish in the pond. Exercise \(\PageIndex{3}\) Suppose it takes \(9\) months for the fish population in Example \(\PageIndex{3}\) to reach fish \(1000\). Under these conditions, how long do you have to wait for the owner's friends? Hint Use the process in the previous example. Response \(38.90\) months Exponential functions can also be used to shape populations that shrink (from disease, for example), or chemical compounds that decompose over time. We say that such systems exhibit exponential degradation rather than exponential growth. The model is almost the same, except there is a negative sign in the exponent. Thus, for some positive constants \(k\), we have \[y=y_0e^{−kt}.\ We have \[y′=−ky_0e^{−kt}=−ky.\] Exponential decay systems that exhibit exponential disintegration behave according to the model \[y=y_0e^{−kt},\] where \(y_0\) represents the initial state of the system and \(k>0\) is a constant, called a constant. Figure \(\PageIndex{2}\) displays a graph of an exponential degradation representative function. Figure \(\PageIndex{2}\): An example of exponential degradation. Let's look at a physical application of exponential decomposition. Newton's cooling law says an object cools down at a rate commensurate with the difference between the object's temperature and the surrounding temperature. In other words, if \(T\) represents the temperature of the object and \(T_a\) represents the ambient temperature in a room, then \[T′=−k(T−T_a).\] Note that this is not exactly the right model for exponential disintegration. We want the derivative to be proportional to the function, and this expression has the additional term \(T_a\). Fortunately, we can make a change of variables that solve this problem. Let \(y(t)=T(t)−T_a\). Then \(y′(t)=T′(t)−0=T′(t)\), and our equation becomes \[y′=−ky.\] From our previous work, we know that this relationship between \(y\) and its derivative leads to exponential degradation. Thus, \[y=y_0e^{−kt},\ and we see that \[T−T_a=(T_0−T_a)e^{−kt}\] \[T=(T_0−T_a)e^{−kt}+T_a\] where \(T_0\) represents the initial temperature. Let's apply this formula in the following example. Example \(\PageIndex{4}\): Newton's Law of Cooling According to experienced baristas, the optimum temperature to serve coffee is between \(155°F\) and \(175°F\). Suppose the coffee is poured to a temperature of \(200°F\), and after \(2\) minutes in a room \(70°F\) it has cooled to \(180°F\). When is the coffee cold enough to serve? When is the coffee too cold to serve? Round answers at the nearest half a minute. Solution We Have \[T &=(T_0−T_a)e^{−kt}+T_a \\[4pt] 180 &=(200−70)e^{−k(2)}+70 \\[4pt] 110 &=130e^{−2k} \\[4pt] \\[4pt] &=e^{−2k} \\[4pt] \ln \dfrac{11}{13} &=−2k \\[4pt] \ln 11−\ln 13 &=−2k \\[4pt] k &=\dfrac{\ln 13−\ln 11}{2} \end{align*}\] Then, the model is \[T=130e^{(\ln 11−\ln 13/2)t}+70. onumber\] The coffee reaches \(175°F\) when \[\begin{align*} 175 &=130e^{(\ln 11−\ln 13/2)t}+70 \\[4pt]105 &=130e^{(\ln 11−\ln 13/2)t} \\[4pt] \dfrac{21}{26} &=e^{(\ln 11−\ln 13/2)t} \\[4pt] \ln \dfrac{21}{26} &=\dfrac{\ln 11−\ln 13}{2}t \\[4pt] \ln 21−\ln 26 &=\left(\dfrac{\ln 11−\ln 13}{2}\right)t \\[4pt] t &=\dfrac{2(\ln 21−\ln 26)}{\ln 11−\ln 13}\\[4pt] &≈2.56. \end{align*}\] The coffee can be served about \(2.5\) minutes after it is poured. Coffee reaches \(155°F\) at \[\begin{align*} 155 &=130e^{(\ln 11−\ln 13/2)t}+70 \\[4pt] 85 amp;=130e^{(\ln 11−\ln 13)t} \\[4pt] \dfrac{17}{26} &=e^{(\ln 11−\ln 13)t} \\[4pt] \l n 17−\ln 26 &=\left(\dfra{\ln 11−\ln 13}{2}\right)t \\[4pt] t &=\dfrac{2(\ln 17−\ln 2 6)}{\ln 11−\ln 13} \\[4pt] &≈5.09.\\end{align*}[] Coffee is too cold to be served approximately \(5\) minutes after it is poured. Exercise \(\PageIndex{4}\) Let's say the room is warmer \((75°F)\) and, after \(2\) minutes, the coffee has only cooled to \(185°F. \) When is the first cold coffee enough to serve? When is the coffee too cold to serve? Round answers at the nearest half a minute. Hint Use the process in the previous example. Answer Coffee is first cold enough to serve about \(3.5\) minutes after it is poured. The coffee is too cold to serve about \(7\) minutes after it is poured. Similarly, systems with exponential growth have a constant doubling time, systems that exhibit exponential disintegration have a constant half-life. To calculate the half-life, we want to know when the quantity reaches half the original size. Therefore, we have \(\dfrac{y_0}{2}=y_0e^{−kt}\) \(\dfrac{1}{2}=e^{−kt}\) \(−\ln 2=−kt\) \(t=\dfraf{\ln 2}{k}\). Note: This is the same expression I came up with for doubling the time. Definition: Half-life If a quantity breaks down exponentially, the half-life is the time it takes to reduce the quantity by half. It is given by \[\text{Half-life}=dfra{\ln 2}{k}.\ Example \(\PageIndex{5}\): Radiocarbon Dating One of the most common applications of an exponential disintegration model is carbon dating. Carbon-14 decomposes (emits a radioactive particle) at a regular and consistent exponential rate. Therefore, if we know how much carbon-14 was originally present in an object and how much carbon-14 remains, we can determine the age of the object. The half-life of carbon-14 is about 5730 years – that is, after many years, half of the material has been converted from carbon in the new non-radioactive nitrogen-14. If we have 100 g of carbon-14 today, how much is left in 50 years? If an artifact that originally contained 100 g of carbon-14 now contains 10 g of carbon-14, how old is it? Around to the nearest hundred years. Workaround We have \[5730=\dfra{\ln 2}{k} onumber\] \[k=\dfrac{\ln 2}{5730}.onumber\ model says \[y=100e^{−(\ln 2/5730)t}.onumber\ In \(50\) years, we have \[y=100e^{−(\ln 2/5730)(50)}≈99.40onumber\ Therefore, in \(50\) years, \(99.40\) g of carbon-14 remains. To determine the age of the artifact, we need to resolve \[\[\begin{align*} 10 &=100e^{−(\ln 2/5730)t} \\[4pt] \dfrac{1}{10} &= e^{−(\ln 2/5730)t} \\ t &≈19035. \end{align*}} The artifact is approximately \(19.000)) years. Exercise \(\PageIndex{5}\): Carbon-14 If an artifact that originally contained 100 g of carbon-14 now contains 20 g of carbon-14, how old is it? Rounds the answer to the nearest hundred years. Hint Use the process in the previous example. Answer A total of 94.13 g of carbon-14 remains after 500 years. The artifact is about 13,300 years old. Old.

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