Adding and subtracting rational expressions worksheet with answers

Adding and subtracting rational expressions is similar to adding and subtracting fractions. Remember that if the denominators are the same, we can add or subtract the numerators and write the result over the common denominator. When working with rational expressions, the common denominator will be a polynomial. In general, given the polynomies P, Q and R, where Q≠0, we have the following: In this section, assume that all variable factors in the denominator are not zero. Example 1: Add: 3y+7y. Solution: Add numerators 3 and 7, and write the result over the common denominator, y. Answer: 10y Example 2: Subtract: x−52x−1−12x−1. Solution: Subtract the numerators x−5 and 1, and write the result over the common denominator, 2x−1. Answer: x−62x−1 Example 3: Subtract: 2x+7 (x+5)(x−3)−x+10(x+5)(x−3). Solution: We use parentheses to remind us to subtract the entire numerator from the second rational expression. Answer: 1x+5 Example 4: Simplify: 2x2+10x+3x2−36−x2+6x+5x2−36+x−4x2−36. Solution: Subtract and add the numerators. Make use of parentheses and write the result on the common denominator, x2−36. Answer: x−1x−6 Try this! Subtracted: x2+12x2−7x−4−x2−2x2x2−7x−4. Answer: 1x-4 To add rational expressions with different denominators, first find equivalent expressions with common denominators. Do it like you did with fractions. If fraction denominators are relatively prime, then the least common denominator (LCD) is your product. For example, multiply each fraction by the appropriate shape of 1 to obtain equivalent fractions with a common denominator. The process of adding and subtracting rational expressions is similar. In general, given the polynomies P, Q, R and S, where Q≠0 and S≠0, we have the following: In this section, assume that all variable factors in the denominator are not zero. Example 5: Add: 1x+1y. Solution: In this example, LCD=xy. To obtain equivalent terms with this common denominator, multiply the first term by yy and the second term by xx. Answer: y+xxy Example 6: Subtract: 1y−1y−3. Solution: Since LCD=y(y−3), multiply the first term by 1 in the form of (y−3)(y−3) and the second term by yy. Answer: −3y(y−3) It is not always the case that the LCD is the product of the determined denominators. Typically, denominators are not relatively prime; thus, determining the LCD requires some thought. Start by factoring all denominators. THE LCD is the product of all factors with higher power. For example, given that there are three base factors in the denominator: x , (x+2) and (x−3). The greatest powers of these factors are x3, (x+2)2 and (x−3)1. Therefore, the general steps to add or subtract rational expressions are illustrated in the following example. Example 7: Subtract: xx2+4x+3−3x2−4x−5. Solution: Step 1: All denominators factor to determine LCD. LCD is (x+1)(x+3)(x−5). Step 2: Multiply by the appropriate factors to obtain equivalent terms with a common denominator. To do this, multiply the first term (x−5)(x−5) and the second term by (x+3)(x+3). Step 3: Add or subtract the numerators and place the result over the common denominator. Step 4: Simplify the resulting algebraic fraction. Answer: (x−9)(x+3)(x−5) Example 8: Subtract: x2−9x+18x2−13x+36−xx−4. Solution: It is better not to factor the numerator, x2−9x+18, because we will probably need to simplify after subtracting. Answer: 18(x−4)(x−9) Example 9: Subtract: 1x2−4−12−x. Solution: First, factor the denominators and determine the LCD. Notice how the opposite binomial property is applied to obtain a more viable denominator. The LCD is (x+2)(x−2). Multiply the second term by 1 in the form of (x+2)(x+2). Now that we have equivalent terms with a common denominator, add the numerators and write the result over the common denominator. Answer: x+3(x+2)(x−2) Example 10: Simplify: y−1y+1−y+1y−1+y2−5y2−1. Solution: Start by factoring the denominator. We can see that the LCD is (y+1)(y−1). Find equivalent fractions with this denominator. It then subtracts and adds the numerators and places the result over the common denominator. Finalize by simplifying the resulting rational expression. Answer: y−5y−1 Try this! Simplification: −2x2−1+x1+x−51−x. Answer: x+3x−1 Rational expressions are sometimes expressed using negative exponents. In this case, apply the rules to negative exponents before simplifying the expression. Example 11: Simplify: y−2+(y−1)−1. Solution: Remember that x−n=1xn. We started by rewriting negative exponents as rational expressions. Answer: y2+y−1y2(y−1) We can simplify sums or differences in rational functions using the techniques learned in this section. Result constraints consist of constraints to the domains of each role. Example 12: Calculate (f+g)(x), data f(x)=1x+3 and g(x)=1x−2, and declare constraints. Solution: Here the domain of f consists of all the actual numbers except −3, and the domain of g consists of all the actual numbers except 2. Therefore, the domain of f + g consists of all real numbers except −3 and 2. Answer: 2x+1(x+3)(x−2), where x≠−3, 2 Example 13: Calculate (f−g)(x), data f(x)=x(x−1)x2−25 and g(x)=x−3−5, and declare domain restrictions. Solution: The domain of f consists of all real numbers except 5 and −5, and the domain of g consists of all the actual numbers except 5. Therefore, the domain of f − g consists of all real numbers except −5 and 5. Answer: −3x+5, where x≠±5 Key Takeaways When adding or subtracting rational expressions with a common denominator, add or subtract the expressions in the numerator and write the result over the common denominator. To find rational expressions with a common denominator, first factor all denominators and determine the least common multiple. Then multiply each term's numerator and denominator by the appropriate factor to obtain a common denominator. Finally, add or subtract the expressions in the numerator and type the result over the common denominator. The domain constraints of a sum or difference of rational functions consist of constraints to the domains of each function. Part A: Add and Subtract with common denominators simplify. (Suppose all denominators are not zero.) 1. 3x+7x 9x−10x3. Xi3y 4. 4x−3+6x−3 5. 72x−1−x2x−16. 83x−8−3x3x−8 7. 2x−9+x−11x−9 8. y+22y+3−y+32y+3 9. 2x−34x−1−x−44x−1 10. 2xx−1−3x+4x−1+x−2x−11. 13y-2y-93y-13-5y3y 12. −3y+25y-10+y+75y−10−3y+45y-10 13. x(x+1)(x−3)−3(x+1)(x−3) 14. 3x+5(2x−1)(x−6)−x+6(2x−1)(x−6) 15. xx2-36+6x2-36 16. xx2-81−9x2-81 17. x2+2x2+3x−28+x−22x2+3x−28 18. x2x2−x−3−3−x2x2−x−3 Part B: Add and subtract with different denominators simplify. (Suppose all denominators are not zero.) 19. 12+13x 20. 15x2−1x 21. 112y2+310y3 22. 1x-12y 23. 1y-2 24. 3y+2−4 25. 2x+4+2 26. 2y-1y2 27. 3x+1+1x 28. 1x−1−2x 29. 1x−3+1x+5 30. 1x+2−1x−3 31. xx+1−2x−2 32. 2x−3x+5−xx−3 33. y+1y−1+y−1y+1 34. 3y−13y-y+4y-2 35. 2x−52x+5−2x+52x-5 36. 22x−1−2x+11-2x 37. 3x+4x−8−28−x 38. 1y−1+11-y 39. 2x2x2−9+x+159−x2 40. xx+3+1x−3−15−x(x+3)(x−3) 41. 2x3x−1−13x+1+2(x−1)(3x−1)(3x+1) 42. 4x2x+1−xx−5+16x−3 (2x+1)(x−5) 43. x3x+2x−2+43x (x−2) 44. −2xx+6−3x6−x−18(x−2)(x+6)(x−6) 45. xx+5−1x−7−25−7x (x+5)(x−7) 46. xx2−2x−3+2x−3 47. 1x+5−x2x2-25 48. 5x−2x2−4−2x-2 49. 1x+1−6x−3x2−7x-8 50. 3x9x2−16−13x+4 51. 2xx2−1+1x2+x 52. x(4x−1)2x2+7x−4−x4+x 53. 3x23x2+5x−2−2x3x−1 54. 2xx−4−11x+4x2−2x−8 55. x2x+1+6x−242x2−7x−4 56. 1x2−x−6+1x2−3x−10 57. xx2+4x+3−3x2−4x−5 58. y+12y2+5y−3−y4y2-1 59. y-1y2-25-2y2-10y+25 60. 3x2+24x2−2x−8−12x−4 61. 4x2+28x2−6x−7−28x-7 62. a4−a+a2−9a+18a2-13a+36 63. 3a-12a2-8a+16−a+24-a 64. a2−142a2-7a−4−51+2to 65. 1x+3−xx2−6x+9+3x2-9 66. 3xx+7−2xx−2+23x−10x2+5x−14 67. x+3x−1+x−1x+2−x(x+11)x2+x−2 68. −2x3x+1−4x−2+4 (x+5)3x2−5x−2 69. x−14x−1−x+32x+3−3(x+5)8x2+10x−3 70. 3x2x−3−22x+3−6x2−5x−94x2-9 71. 1y+1+1y+2y2-1 72. 1y−1y+1+1y-1 73. 5-2+2-174. 6-1+4-2 75. x−1+y−176. x−2−y−177. (2x−1)−1−x−2 78. (x−4)−1−(x+1)−1 79. 3x2(x−1)−1−2x 80. 2(y−1)−2−(y−1)−1 Part C: Add and subtract rational functions Calculate (f+g)(x) and (f−g)(x) and declare restrictions on the domain. 81. f(x)=13x g(x)=1x−2 82. f(x)=1x−1 and g(x)=1x+5 83. f(x)=xx−4 and g(x)=14−x 84. f(x)=xx−5 and g(x)=12x−3 85. f(x)=x−1x2−4 and g(x)=4x2−6x−16 86. f(x)=5x+2 and g(x)=3x+4 Calculate (f+f)(x) and declare domain restrictions. 87. f(x)=1x 88. f(x)=12x 89. f(x)=x2x−190. f(x)=1x+2 Part D: Discussion Board 91. Explain to a colleague why this is incorrect: 1x2+2x2=32x2. 92. Explain to a colleague how to find the common denominator by adding algebraic expressions. Give an example. 1: 10x 3: x−3y 5: 7−x2x−1 7: 1 9: x+14x−11: y−1y 13: 1x+1 15: 1x−6 17: x+5x+7 19: 3x+26x 21: 5y+1860y3 23: 1−2yy 25 : 2(x+5)x+4 27: 4x+1x(x+1) 29: 2(x+1)(x−3)(x+3) (x+15) 31: x2−4x−2 (x−2)(x+1) 33: 2(y2+1)(y+1)(y−1) 35 : −40x(2x+5)(2x−5) 37: 3(x+2)x−8 39: 2x+5x+3 41: 2x+13x+1 43: x2+4x+43x(x−2) 45: x−6x−7 47: −x2+x−5(x+5)(x−5) 49: −5x−8 51: 2x−1x(x−1) 53: x(x−4)(x+2)(3x−1) 55: x+62x+1 57: x−9(x−5)(x+3) 59: y2−8y−5(y+5)(y−5)2 61: 4xx+1 63: a+5a−4 65 : −6x(x+3)(x−3)2 67: x−7x+2 69: −x−54x−1 71: 2y−1y(y−1) 73: 2750 75: x+yxy 77: (x−1)2x2(2x−1) 79: x(x+2)x−1 81: (f+g)=2(2x−1)3x(x−2); (f−g) (x)=−2 (x+1)3x(x−2); x≠0, 2 83: (f+g)(x)=x−1x−4; (f−g) (x)=x+1x−4; x≠4 85: (f+g)(x)=x(x−5)(x+2)(x−2)(x−8); (f−g) (x)=x2−13x+16 (x+2)(x−2)(x−8); x≠−2, 2, 8 87: (f+f)(x)=2x; x≠0 89: (f+f)(x)=2x2x−1; x≠12 x≠12