Characteristics of functions worksheet pdf

Learning outcomes determine whether a relationship represents a function. Find function values. Determines whether a function is one-on-one. Use the vertical line test to identify functions. Charts the functions in the library of functions. A jetliner changes height as she increases distance from the starting point of a flight. The weight of a growing child increases with time. In each case, one quantity depends on another. There is a relationship between the two quantities we can describe, analyze and use to make predictions. In this section we will analyze such relationships. Properties of Functions A relationship is a set of ordered pairs. The set of the first components of each ordered pair is called the domain of the relationship and the set of the second components of each ordered pair is called the scope of the relationship. Consider the following set of ordered pairs. The first numbers in each pair are the first five natural numbers. The second number in each pair is twice the first. [latex]\left\{\left(1,2\right),\left(2,4\right), \left(3,6\right),\left(4,8\right),\left(45,10\right)\right\}[/latex] The domain is [latex]\left\{1,2,3,4,5\right\}[late].] The range is [latex]\left\{2,4,6,8,10\right\}[/latex]. Note the values in the domain are also known as an input values, or values of the independent variable, and are often marked with the lowercase [latex]x[/latex]. Values in the range are also known as an output values, or values of the dependent variable, and are often marked with the lowercase [latex]y[/latex]. A function [latex]f[/latex] is a relationship that assigns a single value in the range to each value in the domain. In other words, no [latex]x[/latex] values are used more than once. For our example associated with the first five natural numbers to numbers double their values, this relationship is a function because each element in the domain, [latex]\left\{1,2,3,4,5\right\}[/latex], is along with exactly one element in the range, [latex]\left\{2,4,6,8,10\right Now let's look at the set of ordered pairs that relate the terms even and oddly to the first five natural numbers. It would appear as [latex]\left\{\left(\text{odd},1\right),\left(\text{even},2\right),\left(\text{odd},3\right),\left(\text{even},4\right),\left(\text{odd},5\right)\right\}[/latex] Note that each element in the domain, [latex]\left\{\text{even,}\text{odd}\right\}[/latex] is not paired with exactly one element in the range, [latex]\left\{1,2,3,4,5\right\}[/latex]. For example, the term oddly corresponds to three values of the domain, [latex]\left\{1,3,5\right\}[/latex] and the term even corresponds to two values from the range, [latex]\left\{2,4\right\}[/latex]. It violates the definition of a function, so this relationship is not a function. This compare relationships that are functional and not functional. (a) (a) relationship is a function because each input is associated with a single output. Note that input [latex]q[/latex] and [latex]r[/latex] both give output [latex]n[/latex]. (b) This relationship is also a function. In this case, each input is associated with a single output. (c) This relationship is not a function because input [latex]q[/latex] is associated with two different outputs. A function is a relationship in which each possible input value results in exactly one output value. We say the output is a function of the input. The input values make up the domain, and the output values make up the range. How to: Given a Relationship Between Two Quantities, determine whether the relationship is a function. Identifies the input values. Identifies the output values. If each input value results in only one output value, the relationship is a function. If any input value results in two or more outputs, the relationship is not a function. The coffee shop menu consists of items and their prices. Is price a function of the item? Is the item a feature of the price? In a particular math class, the overall percentage grade corresponds to a grade-point average. Is grade point average a function of the percentage grade? Is the percentage grade a function of the grade point average? The table below shows a possible rule for assenting grade points. Percentage Grade 0–56 57–61 62–66 67–71 72–77 78–86 87–91 92–100 Degree Point Averaged 0.0 1.0 1.5 2.0 2.5 3.0 3.5 4.0 The table below lists the five greatest baseball players of all time in order to rank. Player Rank Babe Ruth 1 Willie Mays 2 Ty Cobb 3 Walter Johnson 4 Hank Aaron 5 Is the Rank a Function of the Player Name? Is the player name a function of the rank? Using Function Notation Once we determine that a relationship is a function, we need to display and define the functional relationships so that we can understand and use them, and sometimes also so that we can program it into computers. There are several ways to represent functions. A standard feature notation is one representation that makes it easier to work with features. To height is a function of age, we begin by identifying the descriptive variables [latex]h[/latex] for height and [latex]a[/latex] for age. The letters [latex]f[/latex], and [latex]h[/latex] are often used to represent functions just like us [latex]x,y[/latex], and [latex]z[/latex] to represent numbers and [latex]A,B[/latex], and [latex]C[/latex] to represent sets. [latex]\begin{align}&h\text{ is }f\text{ from }a&; text{We call the function }f;\text{ height is a function of age}. \\&h=f\left(a\right) &&\text{We use parentheses to indicate the function input}\text{. } \\ & left(a\right) & \text{We call the function the expression is read as }"f\text{ from }a". \end{align}[/latex] Remember, we can use any letter to name the function; we can use the [latex]h\left(a\right)[/latex] to show that [latex]h[/latex] depends on [latex]a[/latex]. The input value [latex]a[/latex] should be placed in the function [latex]h[/latex] to get an output value. The brackets indicate that age input is in the function; they do not indicate multiplication. We can also give an algebraic expression as the input of a function. For example [latex]f\left (a+b\right)[/latex] means first adding [latex]a[/latex] and [latex]b[/latex], and the result is the input for the function [latex]f[/latex]. We must perform the operations in this order to obtain the correct result. The notation [latex]y=f\left (x\right)[/latex] defines a function named [latex]f[/latex]. It is read as [latex]'y[/latex] is a function of [latex]x.''[/latex] The letter [latex]x[/latex] represents the input value, or independent variable. The letter [latex]y[/latex], or [latex]f\left(x\right)[/latex], represents the output value, or dependent variable. Use function notation to represent a function whose input is the name of a month and output is the number of days in that month in a non-leap year. A function [latex]N=f\left(y\right)[/latex] returns the number of police officers, [latex]N[/latex], in a town in year [latex]y[/latex]. What does [latex]f\left (2005\right)=300[/latex] represent? Instead of a notation such as [latex]y=f\left (x\right)[/latex], we can use the same symbol for the output as for the function, such as [latex]y=y\left(x\right)[/latex], meaning y is a function of x? Yes, this is often done, especially in applied subjects that use higher math, such as physics and engineering. However, in exploring maths itself, we like to maintain a distinction between a function such as [latex]f[/latex], which is a rule or procedure, and the output [latex]y[/latex] we get by applying [latex]f[/latex] to a particular input [latex]x[/latex]. This is why we usually use notation such as [latex]y=f\left (x\right), P = W \left (d\right) [/latex], and so on. Representing functions using tables A common method of representing functions is in the form of a table. The table rows or columns display the corresponding inputs and output values. In some cases, these values represent everything we know about the relationship; other times, the table provides some selected examples from a more complete relationship. The table below lists the input number of each month (January = 1, February = 2, and so on) and the output value of the number of days in that month. This information represents everything we know about the months and days for a given year (this is not a leap year). Note that in this table we have a days-in-a-month function [latex]f[/latex], where [latex]D=f\left (m\right)[/latex] months by an integer rather than by name. Month number, [latex]m[/latex] (input) 1 2 3 4 5 6 7 8 9 10 11 12 Days in month, [latex]D[/latex] (output) 31 28 31 30 31 30 31 31 30 31 31 31 The table below defines a function [latex]Q=g\left(n\right)[/latex]. Remember, this notation tells us that [latex]g[/latex] is the name of the function that takes the input [latex]n[/latex] and gives the output [latex]Q[/latex]. [latex]n[/latex] 1 2 3 4 5 [latex]Q[/latex] 8 6 7 6 6 8 The table below displays the age of children in years and their corresponding heights. This table displays just a few of the data available to the heights and ages of children. We can immediately see that this table does not represent a function because the same input value, 5 years, has two different output values, 40 in. and 42 in. Age in years, [latex]\text{ }a\text{ }[/latex] (input) 5 5 6 7 8 9 10 Height in inches, [latex]\text{ }h\text{ }[/latex] (output) 40 42 44 47 50 52 54 How to: Given a table of input and output values, or whether the tables represent a function. Identifies the input and output values. Check to see if each input value is paired with only one output value. If so, the table represents a function. Which table, A, B, or C represents a function (if any)? Table An input output 2 1 5 3 8 6 Table B Input Output –3 5 0 1 4 5 Table C Input output 1 0 5 2 5 4 When we know an input value and want to determine the corresponding output value for a function, we evaluate the function. Evaluation will always yield one result because each input value of a function matches exactly one output value. When we know an output value and want to determine the input values that will produce that output value, we set the output equal to the function's formula and resolve for the input. Solution can produce more than one solution because different input values can produce the same output value. Determines whether a function is one-on-one Some functions have a given output value that matches two or more input values. For example, in the next stock chart, the stock price was $1000 on five different dates, meaning there were five different input values that all resulted in the same output value of $1000. However, some functions have only one input value for each output value, as well as to have only one output for each input. We call these features one-on-one features. As an example, consider a school that uses only letter grades and decimal equivalents, as listed. Letter grade Grade point average A 4.0 B 3.0 C 2.0 D 1.0 This rating system represents a one-on-one function, because each letter input yields one particular grade point average output and each grade point average corresponds to one input letter. To visualize this concept, let's look again at the two simple features that are esched in (a) and (b) below. The function partially (a) shows a relationship that is not a one-on-one function because input [latex]q[/latex] and [latex]r[/latex] both output Give. The function partially (b) shows a relationship that has a because each input is associated with a single output. A one-on-one function is a function in which each output value matches exactly one input value. Is the area of a circle a function of its radius? If yes, is the feature one-on-one? Is a balance a function of the bank account number? Is a bank account number a function of the balance? Is a balance a one-on-one function of the bank account number? Evaluation and solution features When we form a function in formula, it's usually a simple matter to evaluate the function. For example, the function [latex]f\left (x\right)=5 - 3{x}^{2}[/latex] can be evaluated by squaeting the input value, multiplying by 3, and then subtractioning the product from 5. How to: EVALUATE A FUNCTION Given ITS FORMula. Replace the input variable in the formula with the value provided. Calculate the result. Given the function [latex]h\left(p\right)={p}^{2}+2p[/latex], evaluate [latex]h\left(4\right)[/latex]. For the function, [latex]f\left (x\right)={x}^{2}+3x - 4[/latex], evaluate each of the following. [latex]f\left(2\right)[/latex] [latex]f(a)[/latex] [latex]f(a+h)[/latex] [latex]\dfrac{f\left(a+h\right)-f\left (a\right)}{h}[/latex] Given the function [latex]g\left(m\right)=\sqrt{m - 4}[/latex], evaluate [latex]g\left(5\right)[/latex]. Given the function [latex]h\left(p\right)={p}^{2}+2p[/latex], resolve for [latex]h\left (p\right)=3[/latex]. Given the function [latex]g\left (m\right)=\sqrt{m - 4}[/latex], fix [latex]g\left (m\right)=2[/latex]. Evaluation of functions expressed in Formulas Some functions are defined by mathematical rules or procedures expressed in comparison form. If it is possible to express the function output with a formula involving the input quantity, we can define a function in algebraic form. For example, the comparison [latex]2n +6p=12[/latex] addresses a functional relationship between [latex]n[/latex] and [latex]p[/latex]. We can rewrite it to decide if [latex]p[/latex] is a function of [latex]n[/latex]. How to: Given a Function By Comparison Form, write its algebraic formula. Resolve the equation to isolate the output variable on one side of the equal sign, with the other side as an expression that involves only the input variable. Use all the usual algebraic methods to solve comparisons, such as adding or subtraining the same quantity at or from both sides, or multiplying or dividing both sides of the equation by the same amount. Express the relationship [latex]2n+6p=12[/latex] as a function [latex]p=f\left (n\right)[/latex], if possible. Does the comparison [latex]{x}^{2}+{y}^{2}=1[/latex] represent a function with [latex]x[/latex] as input and [latex]y[/latex] as output? If so, express the relationship as a function (x\right) [/latex]. If [latex]x - 8{y}^{3}=0[/latex], press [latex]y[/latex] as a function of [latex]x[/latex]. Have there been relationships expressed through a who does represent a function, but what still can't be represented by an algebraic formula? Yes, it can happen. For example, given the comparison [latex]x=y+{2}^{y}[/latex], if we want to express [latex]y[/latex] as a function of [latex]x[/latex], there is no simple algebraic formula involving only [latex]x[/latex] equal to [latex]y[/latex]. However, each [latex]x[/latex] determines a unique value for [latex]y[/latex], and there are mathematical procedures by which [latex]y[/latex] can be found to any desired accuracy. In this case, we say that the equation gives an implicit (implied) rule for [latex]y[/latex] as a function of [latex]x[/latex], even if the formula cannot be written explicitly. Evaluating a function given in Tabular Form As we saw above, we can represent functions in tables. Connively, we can use information in tables to write functions, and we can evaluate functions using the tables. For example, how well do our pets remember the sweet memories we share with them? There's an urban legend that a goldfish has a reminder of 3 seconds, but it's just a myth. Goldfish can remember up to 3 months, while the betafish has a reminder of up to 5 months. And while a puppy's memory team is no more than 30 seconds, the adult dog can remember for 5 minutes. This is meager compared to a cat, whose memory team lasts for 16 hours. The function that the type of pet relates to the duration of its memory team is more easily visualized with the use of a table. See the table below. Pet Memory Team in Hours Puppy 0.008 Adult Dog 0.083 Cat 16 Goldfish 2160 Beta fish 3600 At times, evaluating a function in table form can be more useful than using equations. Here we let mention the feature [latex]P[/latex]. The domain of the function is the type of pet and the range is a true number that represents the number of hours the pet's memory team lasts. We can evaluate the function [latex]P[/latex] against the input value of goldfish. We would write [latex] P\left (\text{goldfish}\right)=2160[/latex]. Note that, to evaluate the function in table form, we identify the input value and the corresponding output value of the appropriate row of the table. The table form for function [latex]P[/latex] seems ideally suited to this feature, more so than writing it in paragraph or function form. How to: Given a function represented by a table, identify specific output and input values. Find the given input in the row (or column) of input values. Identifies the corresponding output value along with that input value. Locate the given output values in the row (or column) of output values, and record each time that output value appears. the input value(s) corresponding to the given output value. Use the table below, Evaluate [latex]g\left (3\right)[/latex]. Resolve [latex]g\left(n\right)=6[/latex]. [latex]n[/latex] 1 2 3 4 5 [latex]g(n)[/latex] 8 6 7 6 8 Used table of the previous example, evaluate [latex]g\left(1\right)[/latex] . Evaluating a function using a chart also requires finding the corresponding output value for a given input value, only in this case, we find the output value by looking at the chart. Solving a function comparison using a chart requires finding all instances of the given output value on the chart and observing the corresponding input value(s). Given the chart below, Evaluate [latex]f\left(2\right)[/latex]. Fix [latex]f\left (x\right)=4[/latex]. Use the graph, fix [latex]f\left (x\right)=1[/latex]. Identify functions using graphs as we've seen in examples above, we can represent a function using a chart. Graphs display many input-output pairs in a small space. The visual information they provide often makes relationships easier to understand. We typically build graphs with the input values along the horizontal ale and the output values along the vertical avalanch. The most common graphs call the input value [latex]x[/latex] and the output value [latex]y[/latex], and we say [latex]y[/latex] is a function of [latex]x[/latex], or [latex]y=f\left (x\right)[/latex] when the function is called [latex]f[latex]f[latex][latex]. The graph of the function is the set of all points [latex]\left (x,y\right)[/latex] in the aircraft that meets the equation [latex]y=f\left(x\right)[/latex]. If the function is defined for only a few input values, charting the function is only a few points, where the x-coordination of each point is an input value and the y coordination of each point is the corresponding output value. For example, the black marks on the chart in the chart below tell us that [latex]f\left(0\right)=2[/latex] and [latex]f\left (6\right)=1[/latex]. The set of all points [latex]\left(x,y\right) [/latex] satisfactory [latex]y=f\left (x\right)[/latex] is a curve. The curve shown includes [latex]\left(0,2\right)[/latex] and [latex]\left (6,1\right)[/latex] because the curve passes through those points. The vertical line test can be used to determine whether a chart represents a function. A vertical line includes all points with a particular [latex]x[/latex] value. The [latex]y[/latex] value of a point where a vertical line crosses a chart represents an output for that input [latex]x[/latex] value. If we can draw any vertical line that crosses a chart more than once, the chart does not define a function because that [latex]x[/latex] value has more than one output. A function has only one output value for each input value. How to: Given a Chart, use the vertical line test to determine if the chart represents a function. Inspect the chart to see if any vertical line that is drawn the curve would cross more than once. If there is such a line, the chart does not represent a function. If no vertical line can cross the curve more than once, the chart does represent a function. What the graphs represent(e) a function [latex]y=f\left (x\right)? [/latex] Does the chart below represent a feature? The Horizontal Line Test Once we've established that a chart defines a function is an easy way to determine if it's a one-on-one function to use the horizontal line test. Draw horizontal lines through the graph. A horizontal line includes all points with a specific [latex]y[/latex] value. The [latex]x[/latex] value of a point where a vertical line crosses a function represents the input for that output [latex]y[/latex] value. If we can draw any horizontal line that crosses a chart more than once, the chart does not represent a one-on-one function because that [latex]y[/latex] value has more than one input. How to: Given a Chart of a Function, use the horizontal line test to determine if the chart represents a one-on-one function. Inspect the chart to see if any horizontal line drawn will cross the curve more than once. If there is such a line, the feature is not one-on-one. If no horizontal line can cross the curve more than once, the function is one-on-one. Consider the functions (a), and (b)shown in the graphs below. Is one of the features one-on-one? Identifying basic Toolkit Functions In this text, we explore features—the shapes of their graphs, their unique features, their algebraic formulas, and how to solve problems with them. When we learn to read, we begin with the alphabet. When we learn to do arcognistics, we start with numbers. When working with functions, it's similarly useful to have a base set of building block elements. We call it our toolkit features, which form a set of basic named features for which we know the chart, formula, and special features. Some of these features are programmed to individual buttons on many calculators. For these definitions, we will use [latex]x[/latex] as the input variable and [latex]y=f\left (x\right)[/latex] as the output variable. We'll see these toolkit features, combinations of toolkit features, their graphs, and their transformations often throughout this book. It will be very useful if we can quickly recognize these toolkit features and their functions by name, formula, chart, and basic table properties. The graphs and sample table values are included with each function shown below. Toolkit Functions Name Function Constant [latex]f\left(x\right)=c[/latex], where [latex]c[/latex] is a constant identity [latex]f\left (x\right)=x[/latex] Absolute value [latex]f\left(x\right)=|x| [/latex] Quadratic [latex]f\left(x\right)={x}^{2}[/latex] Cubic [latex]f\left(x\right)={x}^{3}[/latex] Mutual /Rational [latex]f\left(x\right)=\frac{1} {x}[/latex] Mutual/Rational square (x\right)=\frac{1}{{x}^{2}}[/latex] Square Root [latex]f\left(x\right)=\sqrt{x}[/latex] Cube Root Root Key concepts A relationship is a set of ordered pairs. A function is a specific type of relationship in which each domain value, or input, results in exactly one serial value or output. Function notation is a shorthand method for relating to the input of the output in the form [latex]y=f\left (x\right)[/latex]. In table form, a function can be represented by rows or columns associated with input and output values. To evaluate a function, we determine an output value for a corresponding input value. Algebraic forms of a function can be evaluated by replacing the input variable with a given value. To resolve for a specific function value, we determine the input values that return the specific output value. An algebraic form of a function can be written from a comparison. Input and output values of a function can be identified from a table. What input values relate to output values on a chart is another way to evaluate a function. A function is one-on-one if each output value matches only one input value. A chart represents a function if any vertical line drawn on the chart crosses the chart at no more than one point. A chart represents a one-on-one function if any horizontal line drawn on the chart crosses the chart at no more than one point. dependent variable an output value-horizontal line tests a method of testing whether a function is one-on-one by determining whether any horizontal line puts the graph more than once independent variable an input variable input each object or value in a domain associated with another object or value -one function a function for which each value of the output is associated with a unique input value output each object or value in the range produced when an input value is entered into a function, the set of output values arising from the input values in a relation to a set pairs of vertical line test varies a method to test whether a graph represents a function by determining whether a vertical line chart does not more than once